Postprint of Fast Compressed Sensing Reconstruction Algorithm Based on Separable Dictionary Construction

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Date: 2018-05-20T00:00:00+00:00

Abstract

To address the issues of low reconstruction quality and excessive reconstruction time in existing compressed sensing reconstruction algorithms, this paper proposes a block compressed sensing reconstruction algorithm based on separable dictionary construction via matrix manifold. First, the algorithm trains a separable sparse representation matrix using the matrix manifold model and orthogonalizes it. Second, a random measurement matrix is constructed and combined with the obtained sparse representation matrix through matrix operations to build a set of separable dictionaries. Finally, these dictionaries are applied to signal compressed sensing, enabling fast signal reconstruction through linear operations. Experimental results show that compared with current mainstream compressed sensing reconstruction algorithms, the proposed algorithm achieves improvements in both reconstruction accuracy and speed, exhibiting significant application potential in domains with stringent real-time requirements.

Full Text

Preamble

Fast Compressive Sensing Reconstruction Algorithm Based on Separable Dictionary Construction

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Abstract: Existing compressive sensing reconstruction algorithms suffer from low reconstruction quality and long reconstruction times. To address these issues, this paper proposes a block compressive sensing reconstruction algorithm based on matrix manifold separable dictionary construction. The algorithm first trains separable sparse representation matrices using a matrix manifold model and orthogonalizes them. Second, it constructs a random measurement matrix and combines it with the obtained sparse representation matrices through matrix operations to generate a set of separable dictionaries. Finally, the algorithm applies these dictionaries to signal compressive sensing and achieves fast reconstruction through linear operations. Experimental results demonstrate that compared with current mainstream compressive sensing reconstruction algorithms, the proposed algorithm offers improvements in both reconstruction accuracy and reconstruction time, showing promising application value in fields with high real-time requirements.

Keywords: compressive sensing; matrix manifold; separable dictionary; fast reconstruction

0 Introduction

Compressive sensing (CS) [?, ?, ?] has emerged as a revolutionary theory in signal processing, attracting significant attention from researchers. Traditional signal sampling must adhere to the Nyquist sampling theorem, which results in considerable redundancy among sampled data and necessitates subsequent compression. This approach of sampling first and compressing later is not only time-consuming but also requires substantial storage space. Compressive sensing addresses this limitation by sampling signals at frequencies far below the traditional Nyquist rate, enabling simultaneous sampling and compression. This avoids excessive waste of resources such as sensor elements and sampling time caused by large data volumes, giving CS crucial importance and broad application prospects in signal processing.

The design of compressive sensing reconstruction algorithms represents a core problem in CS theory and plays a vital role in signal recovery, attracting extensive research efforts. Some scholars have focused on signal reconstruction, proposing algorithms such as Orthogonal Matching Pursuit (OMP) [?] and Gradient Pursuit (GP) [?]. However, these methods often suffer from low reconstruction accuracy and long reconstruction times. Li et al. [?] proposed a non-local regularized compressive sensing image reconstruction algorithm that introduces local regression models and non-local self-similarity to establish a CS image reconstruction model, thereby improving image reconstruction quality, but the reconstruction process remains time-consuming. Lu et al. [?] proposed a compressive image sensing fast recovery (CISFR) algorithm that constructs separable dictionaries using linear operators for compressive sensing. This approach enables fast approximate image reconstruction through simple linear operations, significantly reducing reconstruction time. However, since the dictionaries are randomly generated, the algorithm lacks specificity for the images being processed.

Recently, sample-trained learned dictionaries have demonstrated excellent adaptability and can more fully capture sparse representations of images, drawing considerable research attention. K-SVD [?], a sparse dictionary training method based on singular value decomposition, is a typical sample-trained dictionary learning approach. However, due to the computational complexity of singular value decomposition, the dictionary training process is time-consuming, memory-intensive, and exhibits only linear convergence, resulting in low training efficiency. Hawe et al. [?] proposed a separable dictionary learning method (SeDiL) that combines the matrix manifold structure of dictionaries with geometric conjugate gradient methods. This approach can train signals of large dimensions while achieving superlinear convergence, substantially improving dictionary training efficiency and enhancing reconstruction quality.

1 2D Separable Dictionary Learning Method SeDiL

The traditional dictionary training optimization model [?] is expressed as:

where $\{X_i\}_{i=1}^N$ represents N training samples, $X_i \in \mathbb{R}^{m\times n}$; $D \in \mathbb{R}^{m\times d}$ is the dictionary; $\{S_i\}_{i=1}^N$ are the sparse coefficients, $S_i \in \mathbb{R}^{d\times n}$ for $i = 1, 2, \dots, N$; λ is a regularization parameter; and $g(\cdot)$ is a sparsity measure function. By solving this optimization model, we obtain the dictionary matrix D and the sparse coefficient matrices $\{S_i\}_{i=1}^N$.

Since traditional dictionary training methods suffer from low efficiency, SeDiL [?] significantly improves training efficiency. The optimization model established by this method is as follows:

where $L \in \mathbb{R}^{m\times h}$ and $R \in \mathbb{R}^{n\times k}$; $\{S_i\}_{i=1}^N$ are the sparse coefficients, $S_i \in \mathbb{R}^{h\times k}$ for $i = 1, 2, \dots, N$; λ is a weighting factor; $g_h(\cdot)$ is a non-uniformity formula [?], with $g_h(L) = \sum_{i,j=1,i\neq j}^{h} \ln(1+\frac{(l_i^T l_j)^2}{\rho})$, where $\rho > 0$. Adding this constraint term to the optimization problem makes the correlation between dictionary columns smaller; $g_k(\cdot)$ is defined similarly; and ddiag$(\cdot)$ denotes a diagonal matrix composed of the diagonal elements of a matrix. The matrices L and R are called a set of separable dictionaries, which have the following manifold structure:

2 Block Compressive Sensing Fast Reconstruction Based on Manifold Separable Dictionary Construction

This paper proposes a block compressive sensing fast reconstruction algorithm. The algorithm first divides an image into patches and uses the SeDiL algorithm to train separable sparse representation dictionaries according to the patch size. Since these dictionaries are redundant and cannot guarantee orthogonality, we select a subset of column vectors to form square matrices and orthogonalize them to obtain a set of sparse representation matrices. Second, through matrix operations, we combine the obtained sparse representation matrices with randomly generated measurement matrices to construct a set of separable dictionaries. Finally, we apply these dictionaries to image compressive sensing and reconstruct image patches through linear operations, then reassemble the patches to obtain the final reconstructed image.

Let p be the sampling rate of image patches, $x \in \mathbb{R}^{m\times n}$ be the original image, and b be the block size. The specific processes of image blocking, separable dictionary construction, and compressive sampling are as follows:

a) **Image Blocking**: Divide image x into patches using a window of size $b \times b$ with stride s, obtaining the image patch collection $\{x_{ij}\}_{i,j=1}^{m,n}$, where $x_{ij} \in \mathbb{R}^{b\times b}$.

b) **Separable Dictionary Construction**: First, select an image library of the same class as x as the training set, denoted as S, and train two separable sparse representation dictionaries using the SeDiL algorithm. After orthogonalization, denote them as L_1, R_1 and L_2, R_2. Then construct a random measurement matrix $\Phi \in \mathbb{R}^{pb\times b}$. Apply L_1, R_1 and L_2, R_2 to the measurement matrix Φ to obtain Φ_{L_1,R_1} and Φ_{L_2,R_2}. To accelerate reconstruction speed, discard high-frequency coefficients in Φ_{L_1,R_1} and Φ_{L_2,R_2} by setting the last $b-pb$ columns of Φ_{L_1,R_1} to zero (denoted as $\tilde{\Phi}_{L_1,R_1}$) and the last $b-pb$ rows of Φ_{L_2,R_2} to zero (denoted as $\tilde{\Phi}_{L_2,R_2}$). The constructed separable dictionary is then obtained through inverse transformation.

c) **Compressive Sampling of Image Patches**: Perform compressive sensing on each image patch x_{ij} using the separable dictionary to obtain measurement values y_{ij}.

Taking image patch x_{ij} as an example, the reconstruction formula is derived as follows:

Since L_2 and R_2 are invertible matrices after truncating certain rows and columns, and the last $b - pb$ column values of z_1 and the last $b - pb$ row values of z_2 are zero vectors, we can reconstruct using:

where $\hat{x}_{ij}$ is the reconstructed image patch. Finally, all reconstructed image patches are sequentially concatenated, with overlapping regions between adjacent patches averaged.

The pseudocode of the proposed reconstruction algorithm is as follows:

Algorithm: Block Compressive Sensing Fast Reconstruction Based on Manifold Separable Dictionary Construction

Input: Original image x, image patch sampling rate p, window size b, stride s, training set S.

Output: Reconstructed image $\hat{x}$.

1. Divide the original image x into patches using a $b \times b$ window with stride s to obtain the image patch collection $\{x_{ij}\}$.
2. Use the SeDiL algorithm to train two separable dictionaries and orthogonalize them, denoted as L_1, R_1 and L_2, R_2.

3. Construct a random measurement matrix Φ and use it to construct a set of separable dictionaries via the described matrix operations.
4. Perform compressive sampling on each image patch using the separable dictionaries: $y_{ij} = \Phi_{L_1,R_1} \cdot x_{ij} \cdot \Phi_{L_2,R_2}$.
5. Reconstruct each image patch using the derived reconstruction formula.
6. Reassemble the image patches, averaging overlapping regions to obtain the final reconstructed image $\hat{x}$.

3 Experimental Results and Analysis

To verify the reconstruction performance of the proposed algorithm, we select five standard test images: Lena (256 × 256), Barbara (512 × 512), Cameraman (256 × 256), Man (512 × 512), and Biker (256 × 256). We randomly choose 12,000 images from the "Cropped Labeled Faces in the Wild Database" [?, ?] as the training set. Sampling rates are set to 1/16, 9/64, 1/4, and 9/16, with a window size of 16 × 16 and stride of 2. The hardware environment is a PC with Intel Core i5-3470 CPU, 8 GB RAM, and 64-bit Windows 7 Ultimate OS, using MATLAB R2010a for simulation. Peak Signal-to-Noise Ratio (PSNR) is adopted to evaluate reconstruction quality. We compare the proposed algorithm with current mainstream compressive sensing reconstruction algorithms: BCS-SPL-DCT [?], TSBS-VB [?], s-HM [?], and CISFR [?]. Table 1 presents the reconstruction performance of different algorithms on the five test images at various sampling rates, where BCS-SDT (Block Compressive Sensing with Separable Dictionary Training) denotes the proposed algorithm.

As shown in Table 1, for all five selected images, the proposed algorithm achieves superior reconstruction performance regardless of sampling rate. For Lena and Cameraman images, the PSNR values of the proposed algorithm are 2.26–3.83 dB higher than BCS-SPL series algorithms, 1.96–5.32 dB higher than TSBS-VB, and 3.00–5.16 dB higher than s-HM. This demonstrates that the BCS-SDT algorithm better recovers images with complex edge information. For Barbara images, which contain rich textures, the proposed algorithm consistently outperforms others by 1.48–4.69 dB over BCS-SPL, 2.88–3.98 dB over TSBS-VB, and 2.71–5.02 dB over s-HM, clearly showing its superiority for texture-rich images. Even for images with simpler texture structures like Man and Biker, the proposed algorithm maintains a significant advantage. These comparisons confirm the outstanding reconstruction performance of BCS-SDT across different image types.

To further validate the superiority of the proposed algorithm, Figure 3 [Figure 3: see original paper] compares the average PSNR values of reconstructed images across different sampling rates. The results show that the average PSNR of the proposed algorithm is significantly higher than other algorithms at all sampling rates, maintaining excellent reconstruction accuracy even at low sampling rates. Moreover, the algorithm demonstrates strong stability and robustness across images with different characteristics.

For visual comparison, Figure 1 [Figure 1: see original paper] shows the reconstruction results of various algorithms on the Cameraman image at a fixed sampling rate of 1/4, while Figure 2 [Figure 2: see original paper] displays results on the Barbara image at 1/16 sampling rate. Subjectively, the proposed algorithm produces clearer reconstructions with reduced block artifacts and better preservation of edge and texture information compared to other methods.

To compare reconstruction speed, Table 2 lists the reconstruction times for the Cameraman image. The proposed algorithm requires significantly less time, with an average of only 3.83 seconds, demonstrating substantially reduced computational complexity. Since the computational complexity of the proposed algorithm arises solely from matrix multiplications, its complexity is $O(m^2)$, where m is the dimension of measurement values. This makes the algorithm highly suitable for real-time applications.

4 Conclusion

This paper proposes a block compressive sensing fast reconstruction algorithm. The method first trains a set of separable sparse representation dictionaries based on matrix manifold learning. Since these redundant dictionaries cannot guarantee orthogonal columns, we extract the non-redundant column vectors to form matrices and orthogonalize them to obtain sparse representation matrices. Second, we construct random measurement matrices and combine them with the sparse representation matrices through matrix operations to generate separable dictionaries. Finally, we apply these dictionaries to compressive sensing, enabling image reconstruction through simple matrix operations.

The proposed algorithm achieves high reconstruction quality for images with various edge and texture structures while substantially reducing reconstruction time. Experimental results demonstrate that compared with existing compressive sensing algorithms, the proposed method offers higher reconstruction quality and shorter reconstruction time. Future work will focus on further reducing reconstruction time and improving reconstruction quality.

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Note: Figure translations are in progress. See original paper for figures.