Postprint: Heuristic Road Traffic Compensation Strategy Based on Delay Function Subgradient

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Abstract

Congestion pricing is considered an effective method for solving traffic congestion. The main idea to address road traffic congestion is to charge for roads that are prone to congestion while providing appropriate compensation for underutilized roads. To this end, a subgradient heuristic road traffic compensation strategy based on delay functions is proposed. First, a nonlinear programming model for tolling/subsidy of the road set is presented, primarily implemented based on Beckmann' s minimization objective function; then, the model' s conditional constraints are established using Kuhn-Tucker conditions and Lagrange multipliers. Second, a pricing and compensation strategy for road traffic is established based on a heuristic algorithm; a delay function analysis model is constructed using marginal cost, and then the model is optimized based on the subgradient method. Finally, through simulation experiments on real-world road networks, it is shown that the proposed algorithm exhibits good performance in terms of travel time, traffic flow, convergence, and other metrics, validating the effectiveness of the algorithm.

Full Text

Preamble

Subgradient Heuristic Method for Road Traffic Compensation Based on Delay Function

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Abstract: Congestion pricing is considered an effective method for solving traffic congestion. The main idea for addressing road traffic congestion is to charge fees for roads that are prone to congestion while providing appropriate compensation for underutilized roads. This paper proposes a subgradient heuristic road traffic compensation strategy based on delay functions. First, a nonlinear programming model for tolling/subsidizing road sets is presented, primarily based on the Beckmann minimization objective function. Kuhn-Tucker conditions and Lagrange multipliers are then used to establish the model' s constraints. Second, a pricing compensation strategy for road traffic is developed based on a heuristic algorithm, utilizing marginal costs to construct a delay function analysis model, which is then optimized using the subgradient method. Finally, simulation experiments on real road networks demonstrate that the proposed algorithm exhibits good performance in terms of travel time, traffic flow, and convergence, thereby verifying its effectiveness.

Keywords: delay function; subgradient; heuristic; road traffic; compensation strategy

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0 Introduction

Traffic congestion causes enormous economic and social losses. In 2016, urban congestion fees in the United States reached $101 billion. A recent study estimates that congestion costs in Australian capital cities amounted to approximately $1.75 billion in 2015, a significant increase from the previous estimate of $1.28 billion. Therefore, research on solving traffic congestion problems holds substantial academic and practical value.

Using supply-demand analysis, congestion occurs when demand exceeds supply. Thus, congestion can be addressed by either increasing supply or reducing demand. On the supply side, more roads can be built to divert traffic. On the demand side, congestion pricing strategies from economic literature can be employed to balance supply and demand. For instance, reference [5] proposes a time-varying pricing scheme that includes both tolls and subsidies on the same route. Reference [6] discusses a redistribution scheme that can shift flows collected from ideal routes to less desirable ones. Reference [7] examines Pareto-improving traffic pricing for multi-user scenarios, showing that under restricted Pareto-improvement conditions, solutions always exist when significant gaps are present. Reference [8] studies the use of incentive measures to guide travelers toward less congested road segments. Reference [9] investigates traffic guidance strategies based purely on subsidy mechanisms (without any charges) to reduce congestion in urban networks.

The above approaches focus on highly congested roads, aiming to redistribute traffic loads uniformly across the entire network, including uncongested roads. These methods resemble expert strategies but lack well-defined analytical models, resulting in suboptimal optimization outcomes. To address this limitation, this paper establishes a mathematical programming model for traffic pricing compensation and achieves optimization of road traffic compensation.

1.1 Problem Description

Given a set of roads with upper capacity limits, a traffic pricing scheme involving tolls or subsidies must be implemented. This problem is essentially a Capacity-Constrained Traffic Assignment Problem (CTAP). Consider $G(N, A)$ as a traffic network graph with N nodes, where A represents the set of roads connecting source and destination nodes. The tolls/subsidies for the road set can be denoted as β_a for each road $a \in A$. The CTAP can be formulated as the following nonlinear programming problem:

$$
\begin{aligned}
\min_{x} \quad & z(x) = \sum_{a \in A} \int_0^{x_a} t_a(\omega)\, d\omega \\
\text{s.t.} \quad & \sum_{p \in P_r} f_{pr} = q_r, \quad \forall r \in R \\
& f_{pr} \geq 0, \quad \forall p \in P_r, r \in R \\
& x_a = \sum_{r \in R} \sum_{p \in P_r} \delta_{ap} f_{pr}, \quad \forall a \in A \\
& x_a \leq C_a, \quad \forall a \in A
\end{aligned}
$$

where $z(x)$ is the Beckmann minimization objective function; x_a is the traffic flow on road a; q_r is the travel demand corresponding to OD pair r; f_{pr} is the traffic flow on path p connecting OD pair r; P_r is the set of all paths connecting OD pair r; δ_{ap} is the road-path incidence indicator ($\delta_{ap} = 1$ if road a is on path p, otherwise 0); and C_a is the traffic capacity of road a.

1.2 Kuhn-Tucker Conditions and Lagrange Multipliers

It has been proven that CTAP is a strictly convex problem under linear constraints, yielding a unique global optimal solution for road flows. Let u_r and β_a be the Lagrange multipliers for travel demand and capacity constraints, respectively. The Kuhn-Tucker (KKT) conditions can be constructed as follows:

$$
\begin{aligned}
& \sum_{a \in A} \delta_{ap}(t_a(x_a) + \beta_a) - u_r = 0, \quad \forall p \in P_r, r \in R \\
& \sum_{a \in A} \delta_{ap}(t_a(x_a) + \beta_a) - u_r \geq 0, \quad \forall p \in P_r, r \in R \\
& (C_a - x_a)\beta_a = 0, \quad \forall a \in A \\
& C_a - x_a \geq 0, \quad \forall a \in A \\
& f_{pr} \geq 0, \quad \forall p \in P_r, r \in R \\
& \sum_{p \in P_r} f_{pr} = q_r, \quad \forall r \in R
\end{aligned}
$$

From equations (7)-(9), if the capacity constraint is saturated (i.e., $x_a = C_a$), then the corresponding Lagrange multiplier β_a is non-zero. Equation (11) contains two terms that contribute to increasing travel time: the normal or cruise travel time $t_a(x_a)$ and the β_a term. The β_a term can be interpreted as the toll/subsidy value or as a mechanism to prevent road transport delays from exceeding saturation levels. In the literature, the β_a term can also be interpreted as waiting time caused by queue buildup on oversaturated roads. As shown in the figure, the β_a term representing toll or subsidy value is positive, while subsidies should be negative. Next, a strategy will be proposed to prevent any negative cycles.

1.3 Travel Time Enhancement

First, assume only tolls exist without subsidies. The model will later be expanded to include subsidies. Consider the delay function $t_a(x_a)$, where t_a^0 is the free-flow travel time on road a, and $t_a(x_a)$ is a non-decreasing convex function satisfying $t_a(0) = t_a^0$. Intuitively, travel time equals free-flow time on empty or uncongested roads. As traffic volume x_a builds up, travel time $t_a(x_a)$ increases accordingly, becoming higher than free-flow time. Based on equation (11), the travel time on saturated road segments can be expressed as:

$$\hat{t}_a = t_a(x_a) + \beta_a$$

where β_a is the additional penalty to free-flow time in equation (14). In other words, β_a is the beta value when $x_a = C_a$. Note that it has been previously proven that the additional penalty term in travel time is equivalent to the Lagrange value of the respective capacity constraint, as shown in equation (11). Therefore, in CTAP, the enhanced travel time replaces the original travel time, i.e., $\hat{t}_a$ is used instead of $t_a(x_a)$. The total travel time on path p connecting OD pair r becomes $\hat{u}_{pr} = \sum_{a\in A} \delta_{ap}\hat{t}_a$, while the capacity constraints are relaxed. If the global optimal value of β_a (in CTAP) is known, the TAP problem can be solved using the inflated travel time based on any known algorithm or software.

2.1 Heuristic Beta Value Update Algorithm

The primary function of equations (14)-(15) is to relieve delay functions on oversaturated roads until traffic volumes stabilize within capacity levels. The initial beta value $\beta_a^{(0)}$ in the delay function is updated iteratively. The main challenges are convergence and user equilibrium.

There is a strong correlation between capacity constraints and traffic assignment. The general theory of congestion pricing is based on marginal cost pricing strategies. Travel time can be expressed as:

$$\hat{t}_a(x_a) = t_a(x_a) + x_a \cdot \frac{\partial t_a(x_a)}{\partial x_a}$$

The parameter settings are as follows: $b_a = x_a \cdot \frac{\partial t_a(x_a)}{\partial x_a}$. Define $\hat{t}_a$ as the corresponding path travel time. Substituting equation (11) into equations (6) and (8) yields:

$$\beta_a = \hat{t}_a - t_a(x_a)$$

Using equations (12)-(13), it can be proven that β_a represents the marginal cost or additional travel time for extra commuters who have already traversed road a. The term $x_a \cdot \frac{\partial t_a(x_a)}{\partial x_a}$ is the additional travel time experienced by each driver on road a. System-optimal flows are achieved on the network through marginal costs, and underutilized roads are expanded. Therefore, using β_a as a template, the initial test is updated as follows:

$$\nabla \beta_a^{(i)} = \frac{\hat{t}_a(C_a) - \hat{t}_a(x_a^{(i)})}{C_a - x_a^{(i)}}$$

where the superscripts i and a denote the current iteration and the respective (over)saturated road; $\hat{t}_a(C_a)$ represents the travel time at capacity on the inflated delay function; and t_a^0 is the free-flow travel time. $\nabla \beta_a^{(i)}$ is the update rate for the initial beta value in the current iteration. Equation (18) satisfies the marginal cost principle, i.e., it uses the slope of the inflated travel time minus the capacity value normalized by the step size.

In this progressive approach, if a saturated road is found to be unsaturated during intermediate iterations, its corresponding penalty becomes ineffective, as shown in Figure 1(b). The computation process can be summarized by two rules:

The above settings satisfy the KKT stability point conditions for the capacity allocation problem. As previously stated, the beta term can be interpreted as waiting time caused by queue buildup on oversaturated roads. Therefore, the beta value for unsaturated road segments is ultimately set to 0.

2.2 Pricing Compensation Strategy

The model can now include pricing subsidies. As mentioned earlier, a negative cycle may disrupt the entire pricing process. To avoid this, road travel times must remain positive. To ensure this, first cancel the free-flow times on all toll/subsidy roads, then initialize the initial beta to the corresponding free-flow travel time. In practice, this means no change–the delay function remains unchanged:

$$t_a^0 = 0, \quad \beta_a^0 = 0$$

The actual toll/subsidy value is constrained by: $-t_a^0 \leq \beta_a \leq +\infty$.

Given the output from the final iteration (iteration n), the toll/subsidy value β_a can be calculated as:

$$\beta_a = \hat{t}_a^{(n)} - t_a^0$$

As specifically shown in Figure 1(c), in the final step of iterative computation, based on the obtained beta value, the delay function on A or B can be derived. The beta value minus the free-flow time can produce either positive values (delay function A) or negative values (delay function B), representing tolls or subsidies, respectively. Furthermore, the absolute value of a subsidy cannot exceed the free-flow travel time. In other words, no road should have negative travel time; therefore, no negative cycles will occur.

2.3 Algorithm Flow

Figure 2 presents the flowchart for the toll/subsidy pricing scheme. The algorithm can be easily integrated into a conventional TAP solution, such as the Frank-Wolfe method. Given the gradual accumulation of initial beta values, the convergence criterion can be defined as a common value below the step size, specifically:

$$\max_a \left| \frac{\nabla \beta_a^{(i)}}{\hat{t}_a} \right| \leq \epsilon$$

Computational results show that $\epsilon = 1\%$ is sufficient to obtain reliable results. Therefore, the algorithm does not terminate until both the relative gap and relative step size values are satisfied. Attempting to solve for the Lagrange multiplier values of capacity constraints through iterative methods is essentially equivalent to solving the partial dual transformation of the original CTAP. Results indicate that the partial dual gradient problem leads to excessive traffic volume vectors, and the subgradient method is considered an effective solution.

3.1 Experimental Setup

The method is coded using a macro, which is a specialized programming language for the transportation planning software Emme 3, where the uncapacitated TAP process is solved using the Frank-Wolfe algorithm. The experiments are conducted on a desktop PC with a 3.60 GHz CPU and 16 GB RAM. A large-scale real-life road network from Shanghai is used to validate the algorithm' s performance, as shown in Figure 3.

The case study consists of 154 zones, 943 nodes, and 3,075 directional roads, with an hourly vehicle travel demand of 56,219. The city is divided into two parts by a river located in the middle. Based on traffic surveys and field observations, the transportation authority proposes an ideal traffic allocation to avoid major congestion during morning peak hours. In Figure 4, bridges 1-11 are marked on the river.

In this study, the local and target flow values for tolls/subsidies are known and can be given externally. Table 1 provides the characteristic values of transit points, including transit traffic volume, where t_a^0 is the free-flow travel time, C_a is the physical capacity of the road, and $\hat{x}_a$ is the upper limit of traffic volume.

3.2 Results Analysis

The algorithm runs for 237 iterations until the relative gap and relative step size values drop below 0.0001 and 0.01, respectively. The computation time is only 15 seconds. The results, including traffic volume, inflated travel time, and toll/subsidy values, are shown in Table 2. To satisfy traffic volume constraints, minor violations in the $x_a \leq C_a$ column are less than 1%, which can be attributed to ITT being a heuristic algorithm with some allowable overshoot.

Figure 5 shows the changes in traffic volume and inflated travel time across consecutive iterations for roads 1, 2, and 3. The results demonstrate that variations are unstable in early iterations, causing non-constant changes in traffic volume and travel time. However, as the iterative process continues, the algorithm gradually converges and eventually stabilizes, indicating that it achieves the desired control effect on the example road network.

If travel demand is projected to increase by 5%, reaching 59,029.94 trips, this growth can be considered typical for urban traffic over one or two years, providing insights into annual changes in toll and subsidy amounts. Table 2 presents comparative results for the projected demand scenario.

As shown in Table 2, the total traffic flow on toll/subsidy roads increases from 24,407.3 to 25,609.1, representing a 4.9% growth. Subsidy and toll amounts also exhibit moderate changes, supporting the stability of the final results.

For beta value convergence verification, four road segments are selected from Figure 3: bridges 1, 5, 10, and 11. Bridge 5 is located in the city center with saturated traffic, while bridges 1, 10, and 11 are in suburban areas with unsaturated traffic. The experimental results for beta value convergence are shown in Figure 6.

Figure 6 demonstrates that all four selected bridges converge through the iterative process. Bridges 1, 10, and 11 converge to beta values near 0, while bridge 5 converges to a non-zero beta value, consistent with previous statements.

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4 Conclusion

This paper proposes a subgradient heuristic road traffic compensation strategy based on delay functions. The approach is implemented using the Beckmann minimization objective function, with Kuhn-Tucker conditions and Lagrange multipliers establishing the model constraints. A heuristic-based pricing compensation strategy for road traffic is developed, utilizing marginal costs to construct a delay function analysis model optimized via the subgradient method. Experimental results show that the proposed algorithm effectively optimizes traffic volume and travel time. The total traffic flow on toll/subsidy roads increases from 24,407.3 to 25,609.1 (a 4.9% improvement), enhancing traffic throughput. Theoretical and experimental analysis of beta value convergence is also provided. Future work will focus on practical application of the algorithm.

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Note: Figure translations are in progress. See original paper for figures.