

Multi-Objective Fast Satellite Selection Method Based on NSGA-II Algorithm (Postprint)

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Abstract

The multi-system satellite selection problem in Global Navigation Satellite Systems (GNSS) can be formulated as a constrained multi-objective optimization problem, enabling simultaneous optimization of Geometric Dilution of Precision (GDOP) and the number of selected satellites, thereby reducing receiver computational load while maintaining good positioning accuracy. This paper proposes a multi-objective fast satellite selection method based on the NSGA-II algorithm, which leverages the sequential nature of the satellite selection problem to generate the initial population, improves constraint handling methods, and employs appropriate genetic operators and utility functions for decision-making. Simulation results demonstrate that the proposed method exhibits excellent reliability and real-time performance, is independent of satellite geometric distribution, and is applicable to complex environments with obstacles or occlusions.

Full Text

Multi-objective and Fast Satellite Selection Method Based on the NSGA-II Algorithm

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Abstract

For multi-constellation Global Navigation Satellite Systems, the satellite selection problem can be formulated as a constrained multi-objective optimization problem that simultaneously optimizes the Geometric Dilution of Precision (GDOP) and the number of selected satellites, thereby reducing receiver computational load while maintaining good positioning accuracy. This paper proposes a multi-objective fast satellite selection method based on the NSGA-II

algorithm. The method leverages the sequential nature of satellite selection to generate the initial population, improves constraint handling techniques, and selects appropriate genetic operators and utility functions for decision-making. Simulation results demonstrate that the proposed method exhibits high reliability and real-time performance, and does not depend on the geometric distribution of satellites, making it suitable for complex environments with obstacles and occlusions.

Keywords: GNSS; satellite selection; multi-objective optimization; NSGA-II; Geometric Dilution of Precision

The rapid development of Global Navigation Satellite Systems (GNSS) has broken the monopoly of the United States GPS. With the rise of Russia's GLONASS, Europe's Galileo, and China's BeiDou (BDS) systems, a new era of four coexisting satellite navigation systems has essentially taken shape. Multi-constellation integrated navigation has become the main development trend, providing users with more accurate and reliable positioning, navigation, and timing services [1-2]. Today, even low-cost receivers can simultaneously receive signals from multiple GNSS constellations. However, excessive redundant information increases computational load and power consumption, seriously affecting the real-time performance of receiver positioning [3]. Therefore, the satellite selection step plays a crucial role in balancing positioning accuracy and computational complexity.

Satellite selection is the process of choosing the optimal subset of visible satellites, typically using Geometric Dilution of Precision (GDOP) as the criterion for evaluating results. The key steps involve determining the number of satellites to select and designing the selection method [4]. Multi-constellation GNSS selection algorithms have evolved from single-system algorithms, such as the maximum convex polyhedron volume method based on satellite geometry [5] and algorithms based on elevation and azimuth angles [4,6]. Reference [7] proposed a satellite selection algorithm suitable for typical urban obstacle environments, which constructs an initial set through single-system selection and then gradually expands the selection range to obtain results with minimum GDOP. Reference [8] achieves selection by calculating each satellite's contribution to GDOP and eliminating the least contributing satellites. Evolutionary algorithms and machine learning have also been extensively applied to satellite selection, including particle swarm optimization [9], simulated annealing [10], neural networks [11], genetic algorithms [12-14], and support vector machines [15]. Reference [9] argues that evolutionary algorithms are faster than neural networks for satellite selection because the latter requires extensive training. Reference [14] compared various selection methods and found that support vector machine and genetic algorithm-based approaches offer superior performance.

Most existing selection methods determine the number of satellites a priori and then select the visible satellite combination with optimal GDOP, making them

single-objective optimization methods. In today's multi-constellation navigation era, the average number of visible navigation satellites at any location exceeds 40, making the selection of how many satellites to use for positioning calculations an increasingly important consideration. Previous algorithms cannot flexibly determine the number of satellites based on geometric layout quality, suffering from inherent limitations. When too few satellites are selected, GDOP increases and affects positioning accuracy; when too many are selected, GDOP decreases but computational load and power consumption increase dramatically. GDOP and the number of selected satellites are two conflicting objectives where optimization of one inevitably degrades the other. Therefore, the satellite selection problem can be transformed into a multi-objective optimization problem. This approach reduces the subjective influence of a priori satellite number determination, removes constraints on GDOP extremes imposed by fixed selection numbers, and improves selection flexibility. This multi-objective approach yields optimal satellite selection schemes that balance both GDOP and selection number, better satisfying receiver positioning accuracy and real-time requirements.

This paper innovatively treats the multi-constellation GNSS satellite selection problem as a constrained multi-objective optimization problem and provides its mathematical formulation. Furthermore, it proposes a fast satellite selection method based on the Non-dominated Sorting Genetic Algorithm-II (NSGA-II) that simultaneously optimizes both GDOP and selection number under various GNSS combination conditions. Simulation results validate the effectiveness and practicality of this method for solving multi-system satellite selection problems.

1 Mathematical Formulation of Multi-objective Satellite Selection

Multi-objective satellite selection involves choosing the optimal satellite combination to simultaneously minimize both the number of selected satellites and the GDOP value. To transform this into a solvable mathematical problem while reducing computational complexity, this paper establishes a mathematical model based on the inherent characteristics of satellite selection.

The decision vector $\mathbf{X} = (x_1 x_2 x_3 \dots x_n)^T$ represents different selection schemes in the decision space, where n is the total number of visible navigation satellites. Here, x_j represents the j -th visible satellite, with $x_j = 1$ indicating selection and $x_j = 0$ indicating non-selection. This design facilitates binary encoding of selection schemes. Two objective functions $f_1(\mathbf{x})$ and $f_2(\mathbf{x})$ in the objective space represent the GDOP value and the number of selected satellites for scheme \mathbf{X} , respectively. The constraint functions $g_i(\mathbf{x})$ define the feasible solution region. For k satellite navigation systems, $k + 3$ visible satellites are needed for positioning. Considering receiver autonomous integrity monitoring requirements for fault detection and exclusion, more satellites must be selected. While more satellites generally yield higher positioning accuracy, an upper limit m_{\max} must be set for the selection number due to receiver computational capacity. Therefore, satellite selection can be solved as a constrained multi-objective optimization

problem.

Multi-system positioning must account for inter-system inconsistencies. At current measurement precision levels, coordinate reference frame deviations between systems are negligible, so inter-system time biases become the primary concern. For users, time biases can be incorporated into receiver clock errors, with each additional system adding one clock error parameter and requiring corresponding expansion of the observation matrix \mathbf{H} . The multi-system GDOP and measurement matrix \mathbf{H} can be expressed as [15]:

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_A & \mathbf{1}_A & \mathbf{0}_A & \mathbf{0}_A & \dots \\ \mathbf{H}_B & \mathbf{0}_B & \mathbf{1}_B & \mathbf{0}_B & \dots \\ \mathbf{H}_C & \mathbf{0}_C & \mathbf{0}_C & \mathbf{1}_C & \dots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}, \quad (2)$$

where the subscript $\gamma = A, B, C, \dots$ denotes different navigation systems (e.g., GPS, GLONASS, Galileo, BDS). The total number of satellites used for positioning is $n = n_A + n_B + n_C + \dots$. In the observation matrix \mathbf{H}_n , \mathbf{H}_γ is an $n_\gamma \times 3$ direction cosine matrix, while $\mathbf{1}_\gamma$ and $\mathbf{0}_\gamma$ are n_γ -dimensional column vectors used to solve for system time biases.

2 Fast Satellite Selection Method

Multi-objective optimization aims to obtain a set of compromise solutions called the Pareto optimal set or non-dominated solution set, from which the most satisfactory scheme is selected using certain decision rules [16]. Genetic Algorithms (GA), as heuristic random search algorithms simulating natural selection, are widely used in multi-objective optimization. Among existing multi-objective genetic algorithms, NSGA-II is most suitable for multi-system satellite selection, employing elite preservation mechanisms, fast non-dominated sorting, and diversity maintenance strategies based on crowding distance calculation [17]. The basic NSGA-II process generates an initial population of size N , then in each generation: (1) parent population P_t undergoes selection, crossover, and mutation to produce offspring Q_t ; (2) parent and offspring populations are merged ($P_t \cup Q_t$) and fast non-dominated sorting is performed; (3) crowding distance calculation determines virtual fitness; (4) the best N individuals are selected to form new population P_{t+1} . This cycle repeats until termination conditions are met.

NSGA-II's main advantages include reduced computational complexity through fast non-dominated sorting and elimination of external archives for storing non-dominated individuals. Consequently, initial population size greatly affects algorithm performance and requires careful analysis [18]. However, NSGA-II's standard genetic operators—Simulated Binary Crossover and Polynomial Mutation—are unsuitable for binary-encoded satellite selection problems, necessitating the design of more appropriate operators. Moreover, obtaining the Pareto optimal set is only the first step; equally important is selecting a satisfactory

solution from this set using appropriate decision rules. Based on these NSGA-II characteristics, this paper proposes an improved multi-objective satellite selection method that follows the NSGA-II framework but: (1) utilizes the sequential nature of satellite selection for initial population generation, (2) improves constraint handling, (3) selects appropriate genetic operators, and (4) designs a utility function for final decision-making.

2.1 Population Initialization

Multi-objective satellite selection can conveniently employ binary encoding. Assuming n visible satellites at a given moment, each satellite is assigned as a gene in sequence, marked as 1 when selected and 0 otherwise. All visible satellites after encoding form a length- n binary string called a chromosome, with each chromosome representing a potential selection scheme. Typically, the initial population consists of randomly generated chromosomes.

Continuous satellite selection can be viewed as a sequential decision-making process requiring optimal decisions at each moment in a dynamic environment based on current state and historical records. Reference [19] provides detailed analysis confirming that the optimal satellite combination at each moment remains optimal within a certain time interval, satisfying this pattern 95% of the time when the interval is set to 1 minute. [Figure 1: see original paper] compares optimal GPS/BDS selection schemes at two adjacent moments with a 5° elevation mask and selection number of 7. Within this time span, 5 of 7 selected satellites remain matched while only 2 change. Changes occur because satellites appear or disappear from view over time, happening in only 5% of cases. This proves that previous selection results can generate next-moment selection schemes.

Based on this sequential property, the Pareto optimal set from the previous moment should be identical or highly similar to the next moment's optimal set. Therefore, the previous moment's Pareto optimal set can be decoded to obtain optimal selection results, then updated by removing satellites that become invisible to generate the next moment's initial population P_0 . This process is illustrated in [Figure 2: see original paper]. With this initialization, the population already approaches the optimal solution set, enabling satisfactory selection results with smaller population sizes and fewer evolutionary generations. This significantly reduces computational complexity and improves continuous selection efficiency, an improvement applicable to other selection methods as well.

2.2 Constraint Handling

The convergence and distribution quality of constrained multi-objective optimization algorithms depend on constraint handling techniques and diversity maintenance strategies. Existing techniques include penalty functions, stochastic ranking, feasibility rules, and hybrid methods [20]. Penalty functions require appropriate penalty coefficients to avoid premature convergence or stag-

nation; stochastic ranking lacks stability; feasibility rules based on constraint dominance relationships [21] state: (1) feasible solutions dominate infeasible ones; (2) among feasible solutions, higher fitness dominates lower fitness; (3) among infeasible solutions, smaller constraint violation dominates larger violation. However, this approach overemphasizes feasible solutions over infeasible ones, easily causing local convergence.

Since optimal solutions for satellite selection often lie on constraint boundaries, maintaining some infeasible solutions with small constraint violations is necessary to preserve population diversity. This paper proposes a new selection operator that maintains a certain proportion of infeasible solutions. Let ρ represent the threshold proportion of infeasible solutions in the population, expressed as a monotonically decreasing function:

$$\rho(t) = a \cdot \exp\left(-\frac{t}{G_{\max}}\right) + b, \quad (1)$$

where t is the generation number, G_{\max} is the maximum generation number, and a and b are adjustable parameters in $[0, 1]$, set as $a = 0.5$ and $b = 0.2$ for satellite selection. Based on this threshold, the selection operation for generating new population P_{t+1} proceeds as follows:

1. Merge parent and offspring populations of size N to form $R_t = P_t \cup Q_t$ of size $2N$; initialize $P_{t+1} = \emptyset$, $i = 1$.
2. If the number of infeasible solutions $N_{\text{infeasible}} < \text{int}(\rho N)$ in R_t , select all infeasible solutions $x_{\text{infeasible}}$; if $N_{\text{infeasible}} \geq \text{int}(\rho N)$, calculate constraint violation degrees and select the $\text{int}(\rho N)$ infeasible solutions with smallest violations; update $P_{t+1} = P_{t+1} \cup x_{\text{infeasible}}$.
3. Perform fast non-dominated sorting on feasible solutions in R_t and mark non-domination ranks F_i , $i = 1, 2, 3, \dots$. While $|P_{t+1}| + |F_i| < N$, select individuals by non-domination rank: $P_{t+1} = P_{t+1} \cup F_i$, $i = i + 1$. Then fill remaining slots by crowding distance sorting: $P_{t+1} = P_{t+1} \cup F_i[1 : N - |P_{t+1}|]$.

This improved constraint handling enhances search capability near feasible region boundaries, maintains population diversity, and avoids premature convergence.

2.3 Genetic Operators

Genetic operators include selection, crossover, and mutation. Mutation changes gene values at certain positions with a given probability. For satellite selection, basic bit mutation can be used: specify mutation points according to mutation probability and flip the gene values at those positions. Crossover operator selection generally depends on encoding scheme. Common binary crossover operations include single-point, multi-point, uniform, cycle, and partially matched crossover. To effectively preserve excellent genes, this paper adopts the XOR

crossover operator using roulette wheel selection to randomly choose parents, as illustrated in [Figure 3: see original paper].

2.4 Utility Function

Multi-objective evolutionary algorithms typically involve optimization and decision-making steps, with techniques classified into three categories: a priori (decision \rightarrow optimization), a posteriori (optimization \rightarrow decision), and interactive (optimization \leftrightarrow decision) methods [22]. This paper employs the a posteriori approach: after obtaining the Pareto optimal set using the improved NSGA-II algorithm, a higher-level utility function selects a preferred solution. For the bi-objective satellite selection problem, the utility function is designed as:

$$U(\mathbf{x}_j) = w_1 \cdot \frac{f_1(\mathbf{x}_j) - f_1^{\min}}{f_1^{\max} - f_1^{\min}} + w_2 \cdot \frac{f_2(\mathbf{x}_j) - f_2^{\min}}{f_2^{\max} - f_2^{\min}}, \quad (4)$$

where \mathbf{x}_j is the j -th selection scheme in the Pareto optimal set, f_i^{\max} and f_i^{\min} are the maximum and minimum values of the two objective functions, and w_1 and w_2 are design weights. After calculating utility values for all schemes in the Pareto set, the scheme with minimum utility value is selected as the final result. This decoded result can then be used for positioning calculations.

This a posteriori approach effectively controls subjective influences from decision-maker preferences, leverages genetic algorithm capabilities for parallel objective processing, and efficiently identifies the best solution from globally optimal candidates.

Based on the above analysis and improvements, the flowchart of the NSGA-II based multi-objective fast satellite selection method is shown in [Figure 4: see original paper].

3 Simulation Results and Analysis

To evaluate the performance of the proposed NSGA-II based multi-objective fast satellite selection method, custom software was developed for simulation analysis. The simulation environment models fully operational GNSS constellations. Real ephemerides were used for GPS and GLONASS (already deployed). China's BeiDou system was simulated with 35 satellites (5 GEO, 3 IGSO, and 25 MEO). Two combination schemes were adopted for generality: GPS+BDS and GPS+BDS+GLO.

[Figure 5: see original paper] shows worldwide visible satellite numbers and GDOP values for the GPS+BDS combination at a specific moment with $5^\circ \times 5^\circ$ resolution and 5° elevation mask. [Figure 6: see original paper] shows the selected satellite numbers and post-selection GDOP with maximum selection number $m_{\max} = 15$. Detailed results are listed in . At this moment, the global

average visible satellite number is approximately 23. Selecting 7 satellites from 23 yields $C_{23}^7 = 245,157$ combinations, growing geometrically with more available satellites—an enormous computational burden making exhaustive search impractical. After selection, the mean selected satellite number is 10.5953 (46.7% of pre-selection), reducing positioning computation by approximately 60% and significantly improving receiver efficiency.

The minimum pre-selection GDOP is 0.9652. GDOP values greater than 1 amplify positioning errors, while values less than 1 indicate good satellite geometry that can reduce errors. Post-selection GDOP maximum remains below 3, maintaining good availability, with mean value 1.5682—only about 0.2 higher than pre-selection mean. This demonstrates that the selection method effectively preserves positioning accuracy.

For GPS+BDS+GLO combination navigation, simulations were conducted on an Intel Core i7-5500U (2.4GHz/L3 4M) processor over 24 hours with 60s sampling interval at location P (39.9°N, 116.3°E, 0m). Maximum selection number m_{\max} was set to 60% of visible satellites at each moment. Elevation mask settings primarily control received satellite numbers. Low-elevation satellites suffer greater interference from multipath effects and obstructions, producing less accurate positioning results. Therefore, receivers should avoid low-elevation satellites. In practice, elevation mask angles should be adjusted based on environmental openness—more open environments permit smaller mask angles. This paper tested elevation masks of 5°, 15°, and 30°, with results shown in and [Figure 7: see original paper].

At 5° elevation mask, the mean selected satellite number is 40% of pre-selection, with post-selection GDOP maximum below 1.7 (comparable to pre-selection maximum) and mean only 0.26 higher than pre-selection. Post-selection GDOP shows little difference from pre-selection throughout the simulation period, maintaining excellent availability. At 15° elevation mask, post-selection GDOP is smaller than pre-selection during some periods, demonstrating that redundant information exists in positioning calculations and confirming the superior performance of the proposed method in identifying appropriate selection schemes. This advantage becomes more prominent at 30° elevation mask, where post-selection GDOP outperforms pre-selection during 30% of the time period, further illustrating the necessity of satellite selection. [Figure 7: see original paper] shows selection time distributed between 10ms and 70ms with mean approximately 30ms, proving the algorithm's real-time capability.

As a global search algorithm independent of satellite geometric distribution, the proposed method has no applicability limitations and is particularly suitable for complex environments with obstructions. Algorithm performance can be enhanced by adjusting key parameters and steps such as population size, generation number, constraint handling methods, utility functions, and termination conditions to meet user requirements for positioning accuracy or real-time performance.

With the rapid development of multi-GNSS constellations and increasing numbers of visible satellites alongside enhanced receiver capabilities, the number of selected satellites has become an important consideration. This paper formulates multi-system satellite selection as a constrained multi-objective optimization problem and provides its mathematical description. This multi-objective approach simultaneously optimizes both GDOP and selection number, meeting receiver requirements for positioning accuracy and real-time performance while reducing a priori assumptions and offering greater flexibility than traditional fixed-number methods.

Building on this foundation, this paper proposes an NSGA-II based multi-objective satellite selection method that leverages the sequential nature of satellite selection to generate initial populations from previous results, significantly reducing computational complexity and improving efficiency—an improvement applicable to other selection methods. The constraint handling approach is enhanced by retaining certain infeasible solutions to maintain population diversity, prevent local convergence, and improve edge search capability. Appropriate genetic operators and decision-making utility functions are selected based on satellite selection characteristics, ensuring algorithm usability. Simulations confirm the method's real-time performance and effectiveness, achieving good positioning accuracy while reducing receiver burden. The selection process is independent of satellite geometric distribution, making it particularly suitable for obstructed environments.

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