Calibration and Application of Mueller Matrix Measurement for Weak Polarization Devices Postprint

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Abstract

The high-resolution magnetograph of the 1 m New Vacuum Solar Telescope (NVST) employs synchronous reconstruction technology, which utilizes beam splitters. While beam splitters are generally polarization-optimized in design to minimize polarization effects, they inevitably still impact polarization measurements of the solar magnetic field. Therefore, measuring the polarization characteristics of beam splitters is an essential step for the 1 m solar telescope to conduct solar magnetic field polarization measurements. Using the air Mueller matrix calibration method to calibrate the Mueller matrix measurement results, we measured two beam splitter samples used for magnetic field polarization measurements, compared the Mueller matrices of different beam splitter configurations and their influence on polarization measurements, measured the variation of the Mueller matrix of flat glass with incident angle, compared the deviation between measurement results and theoretical values, and achieved a Mueller matrix measurement accuracy of 5 × 10^{-3} for the measurement system after calibration using the air Mueller matrix.

Full Text

Calibration and Application of Mueller Matrix Measurement for Weak Polarization Elements

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Abstract: Synchronous reconstruction technology has been employed in the high-resolution magnetograph of the New Vacuum Solar Telescope (NVST), which utilizes a beam splitter. Although beam splitters are designed to minimize polarization effects, they inevitably influence the polarization measurements of solar magnetic fields. Therefore, characterizing the polarization properties of the beam splitter is essential for NVST’ s polarimeter. This paper employs the air Mueller matrix method to calibrate measured Mueller matrix results of samples, and two beam splitter samples are measured using this approach. The Mueller matrices of different beam splitters and their impacts on polarization measurement are compared. The Mueller matrix of flat glass is measured as a function of incident angle, and the deviation between measured results and theoretical values is evaluated. The accuracy of the Mueller matrix measurement system reaches 5×10^{-3} after calibration using the air Mueller matrix.

Keywords: Polarized optics; Polarization characteristics of beam splitter; Mueller matrix measurement of weak polarization element; Calibration using Mueller matrix of air

1. Introduction

Solar magnetic fields represent the dominant factor governing solar activity [1], and high-resolution images of solar magnetic fields provide the fundamental basis for analyzing relationships between solar activity and magnetic field variations. Numerous research institutions worldwide are dedicated to high-resolution observations of solar magnetic fields. Solar telescopes such as the Swedish Solar Telescope (SST) at Big Bear Solar Observatory, the New Solar Telescope (NST), and the GREGOR telescope have achieved excellent results in high-resolution imaging and spectroscopic observations of solar magnetic fields [2-4]. The Solar Magnetic Field Telescope at Huairou Observatory of the National Astronomical Observatories and the Stokes spectroscopic telescope at Yunnan Observatory have made significant contributions to solar magnetic field polarization measurements [5-7]. The NVST, with its aperture and site advantages, is expected to obtain information on small-scale magnetic field structures in solar activities [8].

Solar magnetic fields are primarily obtained through observations of polarization splitting in magnetically sensitive spectral lines [9]. Polarization calibration of the telescope and related instruments is essential for solar magnetic field measurements [10-12]. Figure 1(a) shows the optical path diagram of the solar telescope, and Figure 1(b) shows the magnetic field measurement terminal installed in the telescope’s optical system. The field lens is located at the telescope focal point F. Previous work has established models for telescope polarization calibration [13-14], and the telescope system before F has been correspondingly calibrated. Currently, only the optical components between F and the polarization analyzer require polarization characterization, particularly the polarization effects of the beam splitter, which are generally on the order of a few percent.

[Figure 1: see original paper]

Since the polarization effects of beam splitters are relatively weak, they are more susceptible to measurement system precision limitations. This paper employs the air Mueller matrix calibration method to calibrate the dual rotating retarder Mueller matrix measurement system, thereby improving measurement accuracy for weak polarization characteristics.

2. Dual Rotating Retarder Mueller Matrix Measurement System and Air Matrix Calibration Method

2.1 Establishment of the Dual Rotating Retarder Mueller Matrix Measurement System

The principle of the dual rotating retarder Mueller matrix measurement system is shown in Figure 2. In the diagram, the polarizer PL and quarter-wave plate WP constitute the polarization generator for producing known polarization states, while WP and the analyzer PL form the polarization analyzer for analyzing the polarization state after the sample. The detector measures the light intensity sequentially. The transmission axes of the two polarizers PL are aligned in the same direction and remain fixed. The quarter-wave plates WP and WP are mounted on two electronically controlled precision rotation stages, enabling continuous angular control and synchronous light intensity acquisition. During measurement, WP and WP rotate to positions based on modulation frequency and response matrix diagonalization, yielding $8 \times 8 = 64$ light intensity measurements in one measurement cycle.

[Figure 2: see original paper]

The system selects a laser diode as the light source for its good stability and high collimation. The small beam radius eliminates the need for lenses, shortening the optical path and avoiding polarization effects from lenses that would affect measurement results. The polarizer PL is a Glan-Taylor prism with its polarization direction set as the Q direction. The retardation of WP and WP is fitted to be 87.6° and 88.4°, respectively [15-16]. The transmission axes of PL and the fast axes of WP are sequentially aligned to the Q direction. During measurement, light from the source passes directly through the polarization generator, sample, polarization analyzer, and filter before entering the optical power meter.

2.2 Mueller Matrix Calculation Process

The relationship between measured light intensity and optical element parameters is as follows. Define $S_i = [1, 0, 0, 0]$ as the Stokes vector of the calibration light corresponding to θ_i, and $P_j = [1, 0, 0, 0]$ as the intensity modulation matrix of the polarization analyzer corresponding to θ_j. Through matrix operations, the least-squares solution of the equations yields the Mueller matrix M:

$$M = (P^T P)^{-1} P^T I S^T (S S^T)^{-1}$$

where I is the 8×8 light intensity matrix, and P and S are 8×4 matrices.

2.3 Air Mueller Matrix Calibration Method

Since air is isotropic, the Mueller matrix should be the identity matrix when no sample is present. In practice, measurements deviate from the identity matrix primarily due to measurement errors in polarizer and wave plate parameters and non-ideal characteristics of these components. Rather than analyzing these error sources in detail, this paper employs the air Mueller matrix calibration method to correct measurement errors.

For weak polarization elements like beam splitters, the measured Mueller matrix M can be viewed as the identity matrix E plus a small deviation. The air measurement result M_{air} reflects the deviation of the polarization generator and analyzer from ideal conditions:

$$M_{\text{air}} = E + \Delta_c$$

where Δ_c represents the deviation of the measurement system from ideal conditions. When measuring a sample with Mueller matrix $M_s = E + \Delta_x$ (where Δ_x is small), the directly calculated result M relates to the true sample matrix M_s as:

$$M = M_s M_{\text{air}}$$

The calibrated result using the air Mueller matrix is:

$$M_s = M M_{\text{air}}^{-1}$$

This calibration method is only suitable for weak polarization elements where the approximation holds.

3. Experimental Results and Analysis

3.1 Air Mueller Matrix Measurement

First, the air Mueller matrix was measured using the calibrated system. The wave plate positions were set to (0°, 22.5°, 45°, ···, 157.5°). The measured light intensity matrix I yielded the air Mueller matrix:

$$M_{\text{air}} = \begin{pmatrix} 1.0000 & 0.0018 & 0.0196 & 0.0007 \\ 0.0034 & 1.0043 & 0.0009 & 0.0035 \\ 0.0175 & 0.0039 & 1.0050 & 0.0007 \\ 0.0009 & 0.0029 & 0.0005 & 1.0008 \end{pmatrix}$$

This matrix does not represent a physically meaningful sample but rather characterizes the imperfection of the measurement system. Without calibration, these errors significantly limit measurement precision. The measured air matrix M_{air} is subsequently used to calibrate sample measurements.

3.2 Flat Glass Mueller Matrix Measurement

To verify system accuracy, the Mueller matrix of flat glass was measured as a function of incident angle. The intensity transmission ratio t between P and S polarization components for glass is a function of refractive index n and incident angle α according to Fresnel equations. The incident angle α was varied from 0° to 60°, and Mueller matrix elements were measured and compared with theoretical values.

[Figure 3: see original paper]

Figure 3 shows the measured Mueller matrix elements of K9 glass varying with incident angle. Figure 4 compares measured and theoretical values of matrix element M_{44}. The maximum deviation between measured and theoretical values for all 4×4 matrix elements is less than 5×10^{-3} after calibration, demonstrating that the air Mueller matrix calibration method achieves the required measurement accuracy.

[Figure 4: see original paper]

3.3 Beam Splitter Sample Measurements

Two beam splitters for NVST magnetic field measurements were characterized. Accurate Mueller matrix measurement of beam splitters serves two purposes: first, to correct for polarization effects (dichroism and crosstalk) in solar telescope measurements, and second, to select beam splitter designs with minimal polarization effects to avoid degrading the signal-to-noise ratio of Stokes Q, U, and V parameters.

For coated beam splitters, the Mueller matrix can be parameterized by the P and S transmission ratio t and phase retardation difference δ:

$$M = \frac{1}{2}\begin{pmatrix} 1+t & 1-t & 0 & 0 \\ 1-t & 1+t & 0 & 0 \\ 0 & 0 & 2\sqrt{t}\cos\delta & 2\sqrt{t}\sin\delta \\ 0 & 0 & -2\sqrt{t}\sin\delta & 2\sqrt{t}\cos\delta \end{pmatrix}$$

When the beam splitter’ s S direction has a rotation angle θ relative to the measurement system’ s Q axis, a coordinate rotation matrix $R(\theta)$ is applied.

Beam Splitter A (1:9 splitting ratio, unoptimized):

$$M_A = \begin{pmatrix} 1.0000 & 0.0720 & 0.0037 & 0.0007 \\ 0.0729 & 1.0012 & 0.0033 & 0.0034 \\ 0.0039 & 0.9898 & 0.0020 & 0.0021 \\ 0.1278 & 0.0017 & 0.0027 & 0.1272 \end{pmatrix}$$

Corresponding parameters: $t = 1.156$, $\delta = -7.35°$, $\theta = -0.64°$.

Beam Splitter B (polarization-optimized):

$$M_B = \begin{pmatrix} 1.0000 & 0.0018 & 0.0035 & 0.0012 \\ 0.0015 & 0.9993 & 0.0102 & 0.0114 \\ 0.0058 & 0.0081 & 0.9904 & 0.1329 \\ 0.0009 & 0.0151 & 0.1325 & 0.9915 \end{pmatrix}$$

Corresponding parameters: $t = 1.004$, $\delta = -7.67°$, $\theta = 2.87°$.

Comparison shows the optimized beam splitter B effectively suppresses dichroism, reducing the polarization effect from 0.5% to 0.1%. Since solar polarization signals Q, U, and V are relatively weak compared to intensity I, minimizing instrumental polarization and crosstalk is crucial.

4. Error Analysis

Measurement system errors originate from formula approximation errors and light intensity fluctuations. The approximation accuracy depends on how small Δ_x is. From the air matrix measurement, the maximum deviation p is 0.0196. For weak polarization samples where Δ_x is small, the maximum approximation error is less than 2×10^{-4}.

Random errors from light intensity fluctuations can be calculated using error propagation formulas. Light intensity stability measurements show temporal variation of less than 3×10^{-4} per minute after stabilization. The RMS value of light intensity random error in one measurement cycle (2 minutes) is 6×10^{-5}. Using error propagation, the random error transferred to Mueller matrix elements is $\delta M < 2 \times 10^{-4}$.

Experimental results from 10 repeated air matrix measurements yield standard deviations consistent with these calculations. Both systematic approximation errors and random intensity errors contribute less than 2×10^{-4} to the final measurement uncertainty. The total measurement error for weak polarization samples is less than 5×10^{-3}.

5. Conclusion

This paper addresses Mueller matrix measurement of beam splitters for solar telescopes by employing air Mueller matrix calibration. The measurement system' s accuracy reaches 5×10^{-3} after calibration. The system successfully measured standard flat glass samples, with deviations from theoretical values within 5×10^{-3}, and accurately characterized NVST beam splitters. The comparison between polarization-optimized and unoptimized beam splitters demonstrates the necessity of such measurements for solar telescope polarization calibration and beam splitter design optimization.

For higher precision measurements, improvements should include more stable light sources and higher-quality polarizers and wave plates.

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