

Postprint: Decision-theoretic Rough Set Attribute Reduction Based on Cost-sensitive and Approximate Classification Quality

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Abstract

To address the problem of low classification accuracy in decision-theoretic rough set attribute reduction after introducing cost, this study investigates the balance between cost sensitivity and classification accuracy. By employing total classification cost and approximate classification quality as constraint conditions in the attribute reduction process, in conjunction with the simulated annealing method, a decision-theoretic rough set attribute reduction algorithm based on cost-sensitive and approximate classification quality (ARACOQ) is proposed. Simulation experiments were conducted on the algorithm using UCI datasets, and the experimental results verify the effectiveness of the ARACOQ algorithm, which can find an attribute reduction set with the highest classification accuracy within an affordable cost range.

Full Text

Preamble

Study on Decision-Theoretic Rough Set Attribute Reduction Constrained by Cost-Sensitive and Classification Quality

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Abstract: Addressing the problem of low classification accuracy in decision-theoretic rough set attribute reduction when cost is introduced, this paper investigates the balance between cost-sensitive considerations and classification precision. By employing total classification cost and approximate classification quality as constraints in the attribute reduction process and integrating simulated annealing methods, we propose a Decision-Theoretic Rough Set Attribute Reduction algorithm based on Cost-sensitive and Approximate Classification

Quality (ARACQ). Simulation experiments conducted on UCI datasets validate the effectiveness of the ARACQ algorithm, demonstrating its ability to identify an attribute reduction set with the highest classification accuracy within an affordable cost range.

Keywords: decision-theoretic rough set; attribute reduction; cost sensitive; approximate classification quality; classification precision

0 Introduction

Rough set theory, introduced by Pawlak in 1982, is a computational tool primarily used for analyzing and processing imprecise and fuzzy data [2]. Classical rough set theory is established based on strict algebraic inclusion relationships, which are often difficult to satisfy in practical applications, resulting in limited fault tolerance when handling real-world classification problems. To address this limitation, Yao et al. combined Bayesian risk theory with rough sets to propose the decision-theoretic rough set model, which possesses fault-tolerant capabilities [3].

As research on decision-theoretic rough sets has deepened, the attribute reduction problem has garnered widespread attention from scholars. Attribute reduction aims to eliminate redundant attributes while preserving certain key characteristics of the information system. The introduction of decision semantics in decision-theoretic rough sets renders attribute reduction non-monotonic. Consequently, Yao et al. first proposed the attribute reduction problem for decision-theoretic rough sets and introduced an attribute reduction method based on attribute α -positive region significance [4]. Jia Xiuyi et al. proposed an attribute reduction approach based on decision risk minimization, using minimized decision risk as the optimization objective [5]. Bi et al. presented attribute reduction methods from both algebraic and information-theoretic perspectives [18]. Zhang et al. developed a minimum-cost attribute reduction method for incomplete systems under the decision-theoretic rough set model [19]. Song et al. integrated decision-theoretic rough sets with fuzzy sets to propose two attribute reduction methods: global reduction and local reduction. Global reduction maintains or reduces the cost for all decision classes, while local reduction maintains or reduces the cost for individual decision classes [20].

Typically, different test attribute sets yield different classification results. Within a certain range, as the number of attributes in the test set increases, misclassification results decrease, leading to lower misclassification costs and higher classification accuracy [6]. However, in real-life scenarios, data acquisition incurs economic or time costs, known as test costs [7]. For instance, various medical tests in diagnostic processes involve certain expenses [8]. As the number of test attributes increases, while misclassification costs decrease, test costs simultaneously increase. Therefore, practical problems require simultaneous consideration of both test costs and misclassification costs to identify an optimal balance. Building on this insight, Min et al. [7]

first introduced test costs as a constraint in rough set attribute reduction problems. Li Huaxiong et al. incorporated cost-sensitive considerations into decision-theoretic rough sets, proposing a cost-sensitive attribute reduction method for decision risk minimization [6]. Liu Cai et al. employed simulated annealing algorithms combined with traditional decision-theoretic rough set positive region reduction algorithms to search for attribute sets with minimal total test costs [9], achieving favorable results.

In the aforementioned studies, some approaches do not consider costs and yield attribute subsets that satisfy certain conditions (such as attribute reduction based on attribute α -positive region significance [4]), while others incorporate costs—including misclassification costs, test costs, or total costs—to obtain attribute subsets with minimal cost. However, the classification accuracy of such attribute sets is often unsatisfactory. In practical applications, classification accuracy should be the primary concern. For example, in diagnosing critical diseases, diagnostic precision is far more important than test costs. Therefore, in decision-theoretic rough set attribute reduction, both classification accuracy and classification costs should be comprehensively considered, striving to maximize classification accuracy within an affordable total classification cost range.

In rough set theory, approximate classification quality indicates the percentage of objects that can be definitively classified into known categories using knowledge R [10]. In recent years, numerous attribute reduction algorithms based on approximate classification quality have been proposed [14-17]. Seeking reductions under the premise of unchanged approximate classification quality ensures that the classification decision capability of the reduced set is not diminished [14]. Consequently, this paper adopts approximate quality as one of the constraints in attribute reduction to ensure that classification decision capability is not substantially reduced. We propose a decision-theoretic rough set attribute reduction method that employs both total classification cost and approximate classification quality as iterative criteria, aiming to identify test attribute subsets with high classification decision capability within an affordable total classification cost range.

1 Basic Concepts of Decision-Theoretic Rough Sets

Let $\Omega = \{w_1, w_2, w_3, \dots, w_s\}$ denote a set of s states, and $A = \{a_1, a_2, a_3, \dots, a_m\}$ represent m possible decisions. Let x be an object in the universe, denote the attribute feature description of object x , and represent the conditional probability that object x has state w under description f . Let $\lambda(a|w)$ denote the risk cost of making decision a when the state is w , where λ is typically derived from experience. The decision-theoretic rough set model typically considers three classification decisions: positive region (belongs to class w), negative region (does not belong to class w), and boundary region (pending result). Therefore, for an object x with description f , the expected decision risk when taking decision a can be expressed as:

For convenience of exposition, this paper only considers a set Ω containing two complementary states, $\Omega = \{X, \neg X\}$. Let the decision set be $A = \{a_+, a_-, a_B\}$, where a_+ , a_- , and a_B represent decisions for positive region $\text{POS}(X)$, negative region $\text{NEG}(X)$, and boundary region $\text{BND}(X)$, respectively. When object $x \in X$, the loss functions for assigning the object to regions $\text{POS}(X)$, $\text{NEG}(X)$, and $\text{BND}(X)$ are λ_+ , λ_- , and λ_B , respectively. Conversely, when object $x \in \neg X$, the loss functions for assigning the object to regions $\text{POS}(X)$, $\text{NEG}(X)$, and $\text{BND}(X)$ are λ_+ , λ_- , and λ_B , respectively.

In rough set theory, the equivalence class $[x]$ represents objects x with identical feature descriptions. Therefore, this paper uses equivalence class $[x]$ to denote objects with feature description x . The expected risks for the three decisions can be expressed as:

For the values of decision cost functions, the following relationships obviously hold: $\lambda_+ \leq \lambda_B < \lambda_-$ and $\lambda_+ \leq \lambda_B < \lambda_-$.

Considering that the risk of correct classification is 0 (i.e., $\lambda_+ = \lambda_- = 0$), the expected risks for the three classifications can be expressed as:

According to the Bayesian minimum risk decision principle, the decision rules are as follows: - If $R(a_+|[x]) \leq R(a_-|[x])$ and $R(a_+|[x]) \leq R(a_B|[x])$, then select $x \in \text{POS}(X)$. - If $R(a_-|[x]) \leq R(a_+|[x])$ and $R(a_-|[x]) \leq R(a_B|[x])$, then select $x \in \text{NEG}(X)$. - If $R(a_B|[x]) \leq R(a_+|[x])$ and $R(a_B|[x]) \leq R(a_-|[x])$, then select $x \in \text{BND}(X)$.

2 Overview of Simulated Annealing Algorithm

The simulated annealing algorithm is a stochastic optimization algorithm proposed by Metropolis et al. in 1953. The algorithm solves optimization problems by simulating the thermodynamic process of an object cooling slowly from a high temperature (a process known as annealing) and eventually reaching thermal equilibrium at a certain temperature, thereby obtaining the minimum value. It has been proven that although the temperature decreases slowly, the simulated annealing algorithm can ultimately achieve global optimality [12,13].

Starting from an initial temperature T_0 and initial state $x(0)$, the simulated annealing algorithm randomly and continuously performs an iterative process of “generate new solution—evaluate—accept/reject” from feasible solutions, producing a state sequence $x(0), x(1), \dots, x(i)$. The new state $x(i+1)$ depends only on the previous state $x(i)$ and is independent of earlier states $x(0), x(1), \dots, x(i-1)$; therefore, this state sequence constitutes a Markov chain. The simulated annealing algorithm essentially approaches the optimal solution through the evolution of the Markov chain. After reaching the stopping criterion, the control parameter value is reduced using a decay function. Repeating the above steps until the control parameter reaches its termination yields the optimal solution.

3.1 Test Cost

Assuming that the test cost for the same attribute value is identical across all samples, the test cost for sample x computed on test attribute set B equals the sum of test costs for each attribute $c \in B$ in B , where the test cost is set as a non-negative real number. Therefore, the test cost function can be calculated as [6]:

where $TC(c)$ is the test cost of a single attribute in B .

3.2 Misclassification Cost

Let the decision table information system be $S = (U, At = B_{\text{Origin}} \cup D, V, f)$, where $U = \{x_1, x_2, \dots, x_n\}$ is a non-empty subset of the universe, B_{Origin} and D are condition attribute set and decision attribute set respectively. Given a condition attribute subset B , the equivalence relation R_B determined by B and the equivalence class of sample on attribute set B are [6]:

According to the decision rules of decision-theoretic rough sets, the misclassification cost values for different attribute subsets can be calculated as follows:

When object x is assigned to the positive region:

When object x is assigned to the negative region:

When object x is assigned to the boundary region:

3.3 Cost Sensitivity

Let $\text{Sumcost}(x, B)$ denote the total classification cost of sample x on test attribute set B . The total classification cost is the sum of error cost and test cost, i.e.,

3.4 Approximate Classification Quality

Let $\gamma(x, B)$ denote the approximate classification quality of sample x on test attribute set B [10]:

3.5 Attribute Reduction

Literature [6] employs a heuristic algorithm that always adds the attribute with the smallest total classification cost to the optimal attribute set. Typically, the optimal attribute set obtained under the same sample is consistent. However, attributes with minimal classification total cost may not have high classification accuracy. Therefore, the algorithm presented in [6] has limitations. This paper introduces the simulated annealing method [11] to the attribute reduction problem of decision-theoretic rough sets, comprehensively considering the relationship between total classification cost and classification accuracy. Specifically, within an affordable total classification cost range (this paper uses 10%

of the total classification cost of the full attribute set as the maximum affordable classification cost), approximate classification quality is used to maximally ensure the improvement of classification decision capability. The maximum affordable classification cost is defined as:

The simulated annealing algorithm randomly finds a subset of attributes and determines whether this attribute subset satisfies the following conditions:

where x is the sample object, $\text{Sumcost}(x, B)$ is the total classification cost of object x under attribute subset B , and $\text{isMax}(\gamma(x, B))$ determines whether the approximate classification quality of attribute subset B is the highest among all found attribute subsets.

When these conditions are satisfied, the attribute subset is considered optimal. Based on this principle, we present the following algorithm.

Algorithm 1: Decision-Theoretic Rough Set Attribute Reduction Algorithm Based on Cost-Sensitive (hereinafter referred to as ARACQ Algorithm)

Input: A decision table $S = (U, At = B_{\{\text{Origin}\}} \cup D, V, f)$, sample to be classified x , full attribute set of the sample $B_{\{\text{Origin}\}}$, error cost matrix, test cost matrix, simulated annealing algorithm parameters.

Output: Optimal attribute subset BestB , total classification cost BestSC , approximate classification quality BestQuality (initial value is 0).

Procedure: a) Calculate the total classification cost of the full attribute set $\text{Sumcost}(x, B_{\{\text{Origin}\}})$ and the affordable classification total cost AffordSC . b) Initialize $t_{\{\min\}}$, t , $B = B_{\{\text{Origin}\}}$, $\text{Num} = 0$. c) Generate a new attribute subset B^+ from attribute set B through random addition, replacement, or deletion of attributes. Then set $B = B^+$, $\text{Num} = \text{Num} + 1$. d) If $\text{Num} < 5$, return to step c). e) If $\text{Sumcost}(x, B) < \text{MaxAffordSC}$ && $\text{Sumcost}(x, B) \leq \text{AffordSC}$ && $\gamma(x, B) > \text{BestQuality}$, then $\text{BestB} = B$, $\text{BestSC} = \text{Sumcost}(x, B)$, $\text{BestQuality} = \gamma(x, B)$. f) If $t > t_{\{\min\}}$ and the result has not converged, return to step c) while setting $t = \text{DecayScale} \times t$. g) Output BestB as the optimal test attribute set with the highest classification accuracy within the affordable cost range.

The ARACQ algorithm has two termination conditions: first, when the initial temperature t decays to the specified value; second, when experimental results show no change during K iterations of the Markov chain, indicating convergence. If the result has not converged, the algorithm terminates when temperature t decays to the specified value. Relevant literature has proven that the simulated annealing algorithm is convergent, albeit with relatively slow convergence speed [12].

In terms of computational time, heuristic algorithms have a clear advantage over the ARACQ algorithm. However, in classification accuracy, the ARACQ algorithm demonstrates absolute superiority over heuristic algorithms. Moreover,

in practical problems, improving classification accuracy within a certain classification cost range is of great importance.

4 Experimental Results and Analysis

To validate the effectiveness of the ARACOQ algorithm, we conducted simulation experiments using UCI datasets and performed comparative analysis with the decision-theoretic rough set cost-sensitive attribute selection and classification algorithm proposed by Li Huaxiong.

Since test costs are not provided in UCI datasets, this paper assumes that attribute test costs in each dataset follow a normal distribution $N(\mu, \sigma)$. Specifically, test costs for attributes in the Car and Spambase datasets follow $N(0.2, 0.1)$, while test costs for attributes in the WPBC dataset follow $N(0.02, 0.01)$. The `normrnd` function in Matlab was used to generate test costs for attributes in each dataset.

As this paper employs classification approximate quality as one of its metrics, training samples must be selected to cover every decision category in the sample data. In the Car dataset, data entries 1028 to 1227 were selected as training samples. In the Spambase dataset, entries 1714 to 1913 were used as training samples. In the WPBC dataset, the first 100 data entries served as training samples [6]. All remaining samples were used as test samples. On these three datasets, the optimal test attribute sets were calculated using both the ARACOQ algorithm and Li Huaxiong's heuristic algorithm, respectively. WEKA was then used to validate the optimal test attribute sets on the test samples, employing risk-minimization decision-theoretic rough set decision classification to obtain the average classification accuracy.

The experimental results will be compared using cost reduction rate, classification accuracy, and attribute reduction rate [6], where:

In the experiments, misclassification costs are assumed to satisfy $\lambda_{11} \leq \lambda_{12} < \lambda_{21}$ and $\lambda_{22} \leq \lambda_{21} < \lambda_{12}$, with $\lambda_{11} = \lambda_{22} = 0$. For experimental comparability, different misclassification cost matrices were adopted for the first dataset (Car) and the latter two datasets (WPBC and Spambase), as shown in Table 2 and Table 3.

The parameter settings for the simulated annealing algorithm are as follows: Markov chain iteration count $K = 1000$, Markov chain length $\text{MarkovLength} = 1000$, decay factor $\text{DecayScale} = 0.95$, step size $\text{StepFactor} = 0.02$, initial temperature $t = 30$, and tolerance $t_{\min} = 10^{-8}$.

The basic information of the preprocessed experimental data is shown in Table 1.

Table 1: Basic Information of Experimental Data

Dataset	Number of Classes	Number of Attributes	Number of Attributes After Processing	Number of Samples
Car				
WPBC				
Spambase				

Table 2: Misclassification Cost Matrix for Car

	POS	BND	NEG
POS			
BND			
NEG			

Table 3: Misclassification Cost Matrices for WPBC and Spambase

	POS	BND	NEG
POS			
BND			
NEG			

As shown in Table 4 , both algorithms achieve cost reduction rates exceeding 90%. The cost reduction rates of the ARACOQ algorithm and the heuristic algorithm differ by no more than 7% on the same dataset, demonstrating that the ARACOQ algorithm can significantly reduce the total classification cost. In practical applications, this effectively addresses issues such as delayed treatment due to excessive costs.

Table 4: Average Total Classification Cost

Dataset	Total Cost of Full Attribute Set	Total Cost of Heuristic Algorithm	Total Cost of ARACOQ Algorithm	Cost Reduction Rate of Heuristic Algorithm (%)	Cost Reduction Rate of ARACOQ Algorithm (%)
Car					
WPBC					
Spambase					

Table 5 presents the average classification accuracy obtained using WEKA for risk-minimization decision-theoretic rough set decision classification on the full

attribute set, the optimal attribute set obtained by the heuristic algorithm, and the optimal attribute set obtained by the ARACOQ algorithm.

Table 5: Average Classification Accuracy

Dataset	Full Attribute Set	Heuristic Algorithm	ARACOQ Algorithm
	Classification Accuracy (%)	Optimal Attribute Set Classification Accuracy (%)	Optimal Attribute Set Classification Accuracy (%)
Car			
WPBC			
Spambase			

A comprehensive analysis of the results in Tables 4 and 5 reveals that blindly pursuing total cost minimization leads to a substantial decline in classification accuracy. Therefore, it is necessary to consider both factors in combination. Compared with the heuristic algorithm, when the total classification cost does not exceed the minimum cost by more than 7%, the ARACOQ algorithm achieves a significant improvement in classification accuracy on the same dataset. This effect is particularly pronounced on datasets with large numbers of test samples, such as Car and Spambase. These results prove that the ARACOQ algorithm can substantially improve classification accuracy within an affordable total classification cost range. Compared with the full attribute set, the ARACOQ algorithm achieves cost reduction rates exceeding 90% while maintaining classification accuracy within a 10% decrease range. In practical problems requiring comprehensive consideration of the relationship between cost and classification accuracy, the ARACOQ algorithm's approach and experimental results are more realistic.

Table 6: Average Attribute Reduction Rate

Dataset	Heuristic Algorithm	ARACOQ Algorithm	Heuristic Algorithm	ARACOQ Algorithm
	Reduced Attribute Set (average count)	Reduced Attribute Set (average count)	Reduction Rate (%) (average)	Reduction Rate (%) (average)
Car				
WPBC				
Spambase				

As shown in Table 6, when the full attribute set is relatively small, the reduction rates between the ARACOQ algorithm and the heuristic algorithm differ significantly. When the full attribute set is moderately sized, the ARACOQ

algorithm achieves higher reduction rates while substantially improving classification accuracy. When the full attribute set is large, the ARACOQ algorithm achieves a reduction rate of 65.44% compared with the full attribute set. This demonstrates that the ARACOQ algorithm possesses strong attribute reduction capability, capable of significantly reducing the number of attributes to obtain an optimal test attribute set.

Figures 1 [Figure 1: see original paper] through 3 [Figure 3: see original paper] graphically present the normalized average total test cost (due to different orders of magnitude, all three datasets are displayed using the same scale), average classification accuracy, and normalized attribute set size.

Figure 1: Normalized Average Total Classification Cost

Figure 2: Average Classification Accuracy

Figure 3: Normalized Attribute Set Size

A comprehensive analysis of Figures 1 and 2 clearly shows that on the same dataset, the ARACOQ algorithm achieves higher classification accuracy than the heuristic algorithm with comparable total classification cost. The ARACOQ algorithm's total classification cost is controlled within an affordable range, which is manually set (e.g., 10% of total classification cost). Therefore, it demonstrates stronger applicability than the heuristic algorithm in practical problems.

As can be observed from Figures 2 and 3, the optimal attribute set obtained by the ARACOQ algorithm significantly reduces the number of attributes compared with the full attribute set, substantially decreasing the total classification cost while maintaining comparable classification accuracy.

In summary, the ARACOQ algorithm can provide an optimal test attribute set with high classification accuracy within an affordable cost range. Compared with the full attribute set, both the number of attributes and total classification cost obtained by the ARACOQ algorithm are significantly reduced. Compared with the heuristic algorithm, the optimal attribute set obtained by the ARACOQ algorithm achieves substantially improved classification accuracy, which holds significant practical importance.

5 Conclusion

In decision-theoretic rough set attribute reduction, traditional methods use approximate classification quality as the sole criterion without considering classification costs, which may lead to excessively high costs. Some algorithms optimize classification cost as the objective, pursuing cost minimization but resulting in unsatisfactory classification accuracy.

This paper proposes a decision-theoretic rough set attribute reduction method that balances classification accuracy and total classification cost by considering both total classification cost and approximate classification quality. The

method uses a simulated annealing algorithm to randomly find attribute sets within an affordable total classification cost range, then identifies the optimal attribute set by comparing approximate classification quality, ensuring that the classification results feature moderate total cost and high classification accuracy. Experimental results demonstrate the effectiveness of the proposed algorithm.

In practical applications requiring high accuracy, such as clinical diagnosis, the ARACOQ algorithm can effectively reduce the number of examination items, save diagnostic costs, and ensure relatively high diagnostic accuracy. However, compared with heuristic algorithms, the ARACOQ algorithm has higher time complexity and is therefore significantly slower in terms of runtime. Additionally, for different initial values, the optimal attribute set obtained by the ARACOQ algorithm may vary slightly, and both total classification cost and classification accuracy may change accordingly. Therefore, the variation patterns of different initial values on the same dataset and how to determine initial values for different datasets will be the focus of future discussions.

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