Postprint of a Hesitant Correlative Multi-Attribute Decision-Making Method Based on the IVHFWHM Operator

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Abstract

For multi-attribute decision making (MADM) problems where decision attributes are represented as interval-valued hesitant fuzzy numbers (IVHFN) and attributes are interrelated, this paper proposes a novel decision-making method based on the interval-valued hesitant fuzzy weighted Heronian mean (IVHFWHM) operator. Building upon the operational rules of IVHFN and the Heronian mean (HM) operator, we introduce the interval-valued hesitant fuzzy Heronian mean (IVHFHM) operator and the IVHFWHM operator. The properties of the IVHFHM operator are systematically investigated, including permutation invariance, idempotency, monotonicity, boundedness, and parameter symmetry. A multi-attribute decision-making model is then established based on the IVHFWHM operator, and the feasibility and effectiveness of the proposed model are validated through numerical experiments in the MADM context.

Full Text

Hesitant Multi-Attribute Association Decision-Making Method Based on IVHFWHM Operator

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Abstract: This paper proposes a novel decision-making method based on the interval-valued hesitant fuzzy weighted Heronian mean (IVHFWHM) operator to address multiple attribute decision-making (MADM) problems where attribute values are expressed as interval-valued hesitant fuzzy numbers (IVHFN) and attributes are interrelated. Building upon the operational laws of IVHFN and the Heronian mean (HM) operator, we introduce the interval-valued hesitant fuzzy Heronian mean (IVHFHM) operator and the IVHFWHM operator.

We investigate key properties of the IVHFHM operator, including permutation invariance, idempotency, monotonicity, boundedness, and parameter symmetry. A MADM model based on the IVHFWHM operator is constructed, and numerical experiments demonstrate the feasibility and effectiveness of the proposed approach.

Keywords: interval-valued hesitant fuzzy numbers; Heronian mean operator; interval-valued hesitant fuzzy weighted Heronian mean operator; multiple attribute decision-making

0 Introduction

The rapid development of social informatization has increased the complexity of MADM problems. Coupled with the inherent fuzziness and uncertainty in human cognition and problem characteristics, decision attribute values are often represented using fuzzy information. Due to the complexity of decision-maker thinking, the provided decision information exhibits intricate relationships. While intuitionistic fuzzy set (IFS) theory has effectively addressed fuzziness, uncertainty, and attribute correlations in MADM problems, Torra and Narukawa extended IFS to hesitant fuzzy sets (HFS) by representing membership degrees with precise values, and studied the distinctions and connections between HFS and IFS. However, existing HFS approaches assume membership degrees are precise and clear values. In many practical situations, due to attribute complexity and limitations in decision-maker cognition and preferences, decision information is typically uncertain or fuzzy, making precise membership values inadequate for modeling real decision problems. In fact, human decision expressions, including preference information, allow membership degrees to be a set of possible interval numbers. Therefore, research on interval-valued hesitant fuzzy sets (IVHFS), formed by introducing interval numbers into HFS, holds significant theoretical and practical importance.

Most current MADM research assumes attributes are independent, yet in decision applications, attributes are often not mutually independent but exhibit varying degrees of correlation. Thus, investigating information integration methods that account for interrelationships among decision information is crucial. Chen et al. proposed the IVHFS concept by representing HFS membership degrees with interval numbers on [0,1] and studied MADM methods based on IVHF preference relations. Numerous scholars have devoted considerable attention to IVHFS fusion operators. Reference [3] investigated IVHFWA, IVHFWG, GIVHFWA, GIVHFWG, IVHFOWA, IVHFOWG, GIVHFOWA, and GIVHFOWG operators along with their properties. Wei et al. combined HFS with interval numbers, studying hesitant interval-valued fuzzy information aggregation operators such as HIVFWA, HIVFOWA, HIVFWG, HIVFOWG, HIVFFCOA, and HIVFCOG operators, and proved their idempotency, monotonicity, boundedness, and permutation invariance properties. Yu et al. integrated IVHFS with the traditional ELECTRE method to study MADM approaches based on interval-valued hesitant fuzzy ELECTRE.

However, the aforementioned IVHFS aggregation operators all assume attribute independence. Since attributes in MADM problems often exhibit some correlation, research on information fusion operators that capture attribute interrelationships is essential. The Heronian mean (HM) operator can effectively handle information fusion when attributes are correlated. Beliakov et al. demonstrated HM operator properties in inequality theory. Sykora et al. proposed generalized HM operators and investigated their special forms. Liu et al. studied the Heronian OWA operator and compared its handling of attribute correlations with the Bonferroni mean operator. Yu and Hou applied Heronian mean operators to hesitant fuzzy linguistic MADM problems, investigating HFLHM and HFLGHM operators and their properties, thereby addressing limitations in hesitant fuzzy linguistic information aggregation operators and making decision results more realistic. Wang et al. combined hesitant fuzzy sets with HM operators based on Archimedean norms, studying HFHM and HFWHM operators for traffic flow model selection. Zhou and Yao integrated HM operators with interval-valued intuitionistic fuzzy numbers, developing a decision method based on the IVIFGWHM operator. Lin combined hesitant fuzzy linguistic sets with HM operators based on Archimedean norms, studying HLHGM and HLHWGM operators.

Currently, research on IVHFS fusion operators that consider attribute correlations remains scarce. To address this gap, this paper combines IVHFS with HM operators to propose IVHFHM and IVHFWHM operators. We investigate properties of the IVHFHM operator including permutation invariance, idempotency, monotonicity, boundedness, and parameter symmetry. We construct a MADM model based on the IVHFWHM operator that not only effectively captures interrelationships among input variables but also allows decision-makers to select different parameters according to their risk preferences. Finally, we apply the IVHFWHM operator to an ecological environment assessment case study, demonstrating the effectiveness and simplicity of the proposed operators.

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1 Preliminaries

1.1 IVHFS Concepts

Definition 1. Let X be a non-empty set. An interval-valued hesitant fuzzy element (IVHFE) on X is defined as $\tilde{h} = \{[h^L(x), h^U(x)] | x \in X\}$, where $0 \leq h^L(x) \leq h^U(x) \leq 1$.

1.2 IVHFE Operational Laws

Definition 2. For any three IVHFEs $h_1 = \{[h_1^L, h_1^U]\}$, $h_2 = \{[h_2^L, h_2^U]\}$, and $h = \{[h^L, h^U]\}$, with $\lambda > 0$, their operational laws are:

1. $h^\lambda = \{[h^L, h^U]^\lambda | [h^L, h^U] \in h\} = \{[(h^L)^\lambda, (h^U)^\lambda] | [h^L, h^U] \in h\}$

2. $\lambda h = \{\lambda[h^L, h^U] | [h^L, h^U] \in h\} = \{[1-(1-h^L)^\lambda, 1-(1-h^U)^\lambda] | [h^L, h^U] \in h\}$
3. $h_1 \otimes h_2 = \{[h_1^L \cdot h_2^L, h_1^U \cdot h_2^U] | [h_1^L, h_1^U] \in h_1, [h_2^L, h_2^U] \in h_2\}$
4. $h_1 \oplus h_2 = \{[h_1^L + h_2^L - h_1^L h_2^L, h_1^U + h_2^U - h_1^U h_2^U] | [h_1^L, h_1^U] \in h_1, [h_2^L, h_2^U] \in h_2\}$

1.3 IVHFE Possibility Degree

Definition 3. For any two IVHFEs $h_1 = \{[h_1^L, h_1^U]\}$ and $h_2 = \{[h_2^L, h_2^U]\}$, the possibility degree of $h_1 \geq h_2$ is:

$$p(h_1 \geq h_2) = \max\left\{1 - \max\left(\frac{h_2^U - h_1^L}{\operatorname{len}(h_1) + \operatorname{len}(h_2)}, 0\right), 0\right\}$$

where $\operatorname{len}(h_1) = h_1^U - h_1^L$ and $\operatorname{len}(h_2) = h_2^U - h_2^L$. For all elements, we can obtain the possibility degree matrix $P = (p_{ij})_{n\times n}$, where $p_{ij} = p(h_i \geq h_j)$, satisfying $0 \leq p_{ij} \leq 1$, $p_{ij} + p_{ji} = 1$, and $p_{ii} = 0.5$ for $i, j = 1, 2, \dots, n$. IVHFEs can be ranked by calculating each row sum $p_i = \sum_{j=1}^{n} p_{ij}$.

1.4 IVHFE Score Function

Definition 4. For any IVHFE $h = \{[h_i^L, h_i^U]\}$, the score function is:

$$S(h) = \frac{1}{\#h} \sum_{\gamma \in h} \gamma$$

where $\#h$ is the number of elements in h. For any two IVHFEs h_1 and h_2, if $S(h_1) \geq S(h_2)$, then $h_1 \geq h_2$.

1.5 HM Operator

In information integration, attributes often exhibit relationships such as complementarity or redundancy. The HM operator can capture intrinsic relationships among decision attributes while considering each attribute' s importance.

Definition 5. For a set of non-negative real numbers a_i $(i = 1, 2, \dots, n)$ with parameters $p, q \geq 0$, the HM operator is defined as:

$$HM^{p,q}(a_1, a_2, \dots, a_n) = \frac{2}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} (a_i^p a_j^q)^{\frac{1}{p+q}}$$

2 Construction of the New Decision Model

The HM operator can effectively eliminate correlated information among attributes. For MADM problems where attribute values are expressed as IVHFEs and attributes are interrelated, this section presents the IVHFHM and IVHFWHM operators.

2.1 IVHFHM Operator

Definition 6. Let h_i $(i = 1, 2, \dots, n)$ be a collection of IVHFEs, and let $p, q > 0$ be constants. The interval-valued hesitant fuzzy Heronian mean (IVHFHM) operator is defined as:

$$IVHFHM^{p,q}(h_1, h_2, \dots, h_n) = \left(\frac{2}{n(n+1)} \bigoplus_{i=1}^{n} \bigoplus_{j=i}^{n} (h_i^p \otimes h_j^q) \right)^{\frac{1}{p+q}}$$

Theorem 1. Let h_i $(i = 1, 2, \dots, n)$ be a collection of IVHFEs. The aggregated result using the IVHFHM operator (Eq. 8) is still an IVHFE, given by:

$$IVHFHM^{p,q}(h_1, h_2, \dots, h_n) = \bigcup_{\gamma_i \in h_i, \gamma_j \in h_j} \left\{ \left[\left(1 - \prod_{i=1,j=i}^{n} \left(1 - (\gamma_i^L)^p (\gamma_j^L)^q\right)^{\frac{2}{n(n+1)}} \right)^{\frac{1}{p+q}}, \left(1 - \prod_{i=1,j=i}^{n} \left(1 - (\gamma_i^U)^p (\right.\right.\right.\right.$$

Proof: From Eq. (1), we obtain $h_i^p \otimes h_j^q = \bigcup_{\gamma_i \in h_i, \gamma_j \in h_j} \{[\gamma_i^L, \gamma_i^U]^p \otimes [\gamma_j^L, \gamma_j^U]^q\} = \bigcup_{\gamma_i \in h_i, \gamma_j \in h_j} \{[(\gamma_i^L)^p (\gamma_j^L)^q, (\gamma_i^U)^p (\gamma_j^U)^q]\}$. Using Eqs. (3) and (4), we further derive:

$$\bigoplus_{i=1}^{n} \bigoplus_{j=i}^{n} (h_i^p \otimes h_j^q) = \bigcup_{\gamma_i \in h_i, \gamma_j \in h_j} \left\{ \left[1 - \prod_{i=1,j=i}^{n} \left(1 - (\gamma_i^L)^p (\gamma_j^L)^q\right), 1 - \prod_{i=1,j=i}^{n} \left(1 - (\gamma_i^U)^p (\gamma_j^U)^q\right) \right] \right\}$$

Applying Eq. (2), we get:

$$\frac{2}{n(n+1)} \bigoplus_{i=1}^{n} \bigoplus_{j=i}^{n} (h_i^p \otimes h_j^q) = \bigcup_{\gamma_i \in h_i, \gamma_j \in h_j} \left\{ \left[1 - \prod_{i=1,j=i}^{n} \left(1 - (\gamma_i^L)^p (\gamma_j^L)^q\right)^{\frac{2}{n(n+1)}}, 1 - \prod_{i=1,j=i}^{n} \left(1 - (\gamma_i^U)^p (\gamma_j^U)^q\right)^{\frac{2}{n(n+1)}} \right] \right.$$

Therefore, by Eq. (1), we obtain the final result. Since $0 \le \gamma_i^L \le \gamma_i^U \le 1$ and $0 \le \gamma_j^L \le \gamma_j^U \le 1$, after computation using IVHFE operational laws, we get $0 \le \gamma^L \le \gamma^U \le 1$. Thus, Theorem 1 is proved.

We now examine fundamental properties of the IVHFHM operator: idempotency, permutation invariance, monotonicity, boundedness, and parameter symmetry.

Property 1 (Idempotency). Let h_i $(i = 1, 2, \dots, n)$ be a collection of IVHFEs. If all $h_i = h$, then $IVHFHM^{p,q}(h_1, h_2, \dots, h_n) = h$.

Proof: Since $h_i = h = \{[\gamma^L, \gamma^U]\}$ for all i, we have:

$$IVHFHM^{p,q}(h, h, \dots, h) = \bigcup_{\gamma \in h} \left\{ \left[\left(1 - \prod_{i=1,j=i}^{n} (1 - (\gamma^L)^p (\gamma^L)^q)^{\frac{2}{n(n+1)}} \right)^{\frac{1}{p+q}}, \left(1 - \prod_{i=1,j=i}^{n} (1 - (\gamma^U)^p (\gamma^U)^q)^{\frac{2}{n(n+1}} \right. \right. \right.$$

Simplifying yields $IVHFHM^{p,q}(h, h, \dots, h) = h$.

Property 2 (Permutation Invariance). Let h_i $(i = 1, 2, \dots, n)$ be a collection of IVHFEs, and let $(h'_1, h'_2, \dots, h'_n)$ be any permutation of $(h_1, h_2, \dots, h_n)$. Then $IVHFHM^{p,q}(h_1, h_2, \dots, h_n) = IVHFHM^{p,q}(h'_1, h'_2, \dots, h'_n)$.

Proof: Since $\bigoplus_{i=1}^{n} \bigoplus_{j=i}^{n} (h_i^p \otimes h_j^q) = \bigoplus_{i=1}^{n} \bigoplus_{j=i}^{n} (h_i'^p \otimes h_j'^q)$, the result follows directly.

Property 3 (Monotonicity). Let h_i and $\hat{h}_i$ $(i = 1, 2, \dots, n)$ be two collections of IVHFEs. If $\hat{h}_i \geq h_i$ (i.e., $\hat{\gamma}_i^L \geq \gamma_i^L$ and $\hat{\gamma}_i^U \geq \gamma_i^U$) for all i, then $IVHFHM^{p,q}(\hat{h}_1, \hat{h}_2, \dots, \hat{h}_n) \geq IVHFHM^{p,q}(h_1, h_2, \dots, h_n)$.

Proof: Since $\hat{\gamma}_i^L \geq \gamma_i^L$ and $\hat{\gamma}_i^U \geq \gamma_i^U$, and given the monotonicity of functions and IVHFN operational laws with $p, q > 0$, we have:

$$\prod_{i=1,j=i}^{n} \left(1 - (\hat{\gamma}_i^L)^p (\hat{\gamma}_j^L)^q\right) \leq \prod_{i=1,j=i}^{n} \left(1 - (\gamma_i^L)^p (\gamma_j^L)^q\right)$$

and similarly for the upper bounds. Consequently:

$$\left(1 - \prod_{i=1,j=i}^{n} \left(1 - (\hat{\gamma}_i^L)^p (\hat{\gamma}_j^L)^q\right)^{\frac{2}{n(n+1)}} \right)^{\frac{1}{p+q}} \geq \left(1 - \prod_{i=1,j=i}^{n} \left(1 - (\gamma_i^L)^p (\gamma_j^L)^q\right)^{\frac{2}{n(n+1)}} \right)^{\frac{1}{p+q}}$$

The same holds for the upper bounds, proving $IVHFHM^{p,q}(\hat{h}_1, \dots, \hat{h}_n) \geq IVHFHM^{p,q}(h_1, \dots, h_n)$.

Property 4 (Boundedness). Let h_i $(i = 1, 2, \dots, n)$ be any collection of IVHFEs. For any i, we have:

$$\min\{h_1, h_2, \dots, h_n\} \leq IVHFHM^{p,q}(h_1, h_2, \dots, h_n) \leq \max\{h_1, h_2, \dots, h_n\}$$

Proof: This follows directly from the monotonicity property.

Property 5 (Parameter Symmetry). Let h_i $(i = 1, 2, \dots, n)$ be any collection of IVHFEs. Then $IVHFHM^{p,q}(h_1, h_2, \dots, h_n) = IVHFHM^{q,p}(h_1, h_2, \dots, h_n)$.

Proof: Since $\bigoplus_{i=1}^{n} \bigoplus_{j=i}^{n} (h_i^p \otimes h_j^q) = \bigoplus_{i=1}^{n} \bigoplus_{j=i}^{n} (h_i^q \otimes h_j^p)$, the result follows immediately.

2.2 IVHFWHM Operator

While the IVHFHM operator can eliminate attribute correlations, real-world scenarios often involve different attribute weights. To address this, we propose the IVHFWHM operator.

Definition 7. Let h_i $(i = 1, 2, \dots, n)$ be a collection of IVHFEs with weight vector $w = (w_1, w_2, \dots, w_n)^T$, and let $p, q \geq 0$ be constants. The interval-valued hesitant fuzzy weighted Heronian mean (IVHFWHM) operator is defined as:

$$IVHFWHM^{p,q}(h_1, h_2, \dots, h_n) = \left(\frac{2}{n(n+1)} \bigoplus_{i=1}^{n} \bigoplus_{j=i}^{n} ((w_i h_i)^p \otimes (w_j h_j)^q) \right)^{\frac{1}{p+q}}$$

Theorem 2. Let h_i $(i = 1, 2, \dots, n)$ be a collection of IVHFEs with weight vector $w = (w_1, w_2, \dots, w_n)^T$, and let $p, q \geq 0$ be constants. The aggregated result using the IVHFWHM operator (Eq. 15) is still an IVHFE, given by:

$$IVHFWHM^{p,q}(h_1, h_2, \dots, h_n) = \bigcup_{\gamma_i \in h_i, \gamma_j \in h_j} \left\{ \left[\left(1 - \prod_{i=1, j=i}^{n} (1 - (1 - (1 - \gamma_i^L)^{w_i})^p (1 - (1 - \gamma_j^L)^{w_j})^q)^{\frac{2}{n(n+1)}} \right)^{\frac{1}{p+}} \right. \right.$$

The proof is similar to Theorem 1 and is omitted for brevity.

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3 MADM Model Based on IVHFWHM Operator

Consider a MADM problem with alternative set $A = \{A_1, A_2, \dots, A_m\}$ and attribute set $G = \{G_1, G_2, \dots, G_n\}$, where each attribute has weight w_k $(k = 1, 2, \dots, n)$. In the IVHF environment, the evaluation of alternative A_i on attribute G_j is $h_{ij} = [\gamma_{ij}^L, \gamma_{ij}^U]$, forming the interval-valued hesitant fuzzy decision matrix $H = (h_{ij})_{m \times n}$. The decision model proceeds as follows:

Step a) Standardize the decision matrix $H = (h_{ij})_{m \times n}$ to obtain the standardized matrix $H' = (h'_{ij})_{m \times n}$ using:

$$h'_{ij} = \begin{cases} h_{ij} & \text{if } G_j \text{ is a benefit attribute} \\ h^c_{ij} & \text{if } G_j \text{ is a cost attribute} \end{cases}$$

where h^c_{ij} is the complement of h_{ij}.

Step b) Using the attribute weight vector $w = (w_1, w_2, \dots, w_n)^T$ and the IVHFWHM operator (Eq. 16), integrate the evaluation information for each alternative A_i to obtain comprehensive evaluation values h_i $(i = 1, 2, \dots, m)$.

Step c) Calculate the score function $S(h_i)$ for each alternative A_i using Eq. (6).

Step d) Construct the possibility degree matrix $P = (p_{ij})_{n\times n}$ using Eq. (5) based on the score functions from Step c). Compute the ranking values $p_i = \sum_{j=1}^{n} p_{ij}$ to rank the alternatives.

Step e) Rank the alternatives A_i according to p_i values and select the optimal alternative.

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4 Case Study

With rapid modernization, ecological environment protection has become crucial. In 2016, the Hunan Provincial Environmental Protection Bureau launched pilot ecological protection projects across the province, targeting cities including Changsha (A_1), Chenzhou (A_2), Xiangtan (A_3), Yueyang (A_4), and Changde (A_5). The evaluation considers four factors: implementation risk resistance (G_1), air quality (G_2), surface water quality (G_3), and urban functional area noise (G_4), with attribute weights $w = (0.25, 0.15, 0.20, 0.40)^T$. Since surface water quality, air quality, and noise levels are interrelated (good water quality often correlates with good air quality and low noise, reducing implementation risk), the evaluation values are expressed as IVHFEs in the decision matrix $H = (h_{ij})_{5\times 4}$ shown in .

The decision process (with $p = q = 1$) is as follows:

Step a) Standardize the matrix. Since G_2 and G_3 are benefit attributes while G_1 and G_4 are cost attributes, we apply Eq. (17) to obtain the standardized interval-valued hesitant fuzzy matrix.

Step b) Using the IVHFWHM operator (Eq. 16), integrate the evaluation information for each pilot city A_i to obtain comprehensive evaluation values h_i $(i = 1, 2, \dots, 5)$:

$$h_1 = \{[0.7063, 0.7554], [0.8278, 0.9132], [0.9341, 0.9469], [0.9582, 0.9762]\}$$
$$h_2 = \{[0.7413, 0.7819], [0.7933, 0.8239], [0.8623, 0.8861], [0.9721, 0.9802]\}$$
$$h_3 = \{[0.7794, 0.8481], [0.7765, 0.8240], [0.9263, 0.9561], [0.9327, 0.9582]\}$$
$$h_4 = \{[0.7756, 0.8213], [0.7288, 0.8053], [0.9109, 0.9352], [0.9582, 0.9762]\}$$
$$h_5 = \{[0.6933, 0.7471], [0.7871, 0.8344], [0.8750, 0.8963], [0.9678, 0.9762]\}$$

Step c) Calculate the score functions for each alternative:

$$S(h_1) = [0.8304, 0.8710]$$
$$S(h_2) = [0.8423, 0.8680]$$
$$S(h_3) = [0.8537, 0.8966]$$
$$S(h_4) = [0.8434, 0.8845]$$
$$S(h_5) = [0.8308, 0.8635]$$

Step d) Construct the possibility degree matrix $P = (p_{ij})_{5\times 5}$ using Eq. (5):

$$P = \begin{pmatrix} 0.500 & 0.418 & 0.393 & 0.275 & 0.482 \\ 0.582 & 0.500 & 0.302 & 0.804 & 0.518 \\ 0.607 & 0.782 & 0.500 & 0.607 & 0.788 \\ 0.725 & 0.196 & 0.393 & 0.500 & 0.698 \\ 0.518 & 0.482 & 0.212 & 0.302 & 0.500 \end{pmatrix}$$

Compute the ranking values $p_i = \sum_{j=1}^{5} p_{ij}$ to obtain $p_1 = 2.068$, $p_2 = 2.706$, $p_3 = 3.284$, $p_4 = 2.512$, and $p_5 = 2.014$.

Step e) Rank the alternatives: $A_3 > A_4 > A_2 > A_5 > A_1$, with A_3 (Xiangtan) being the optimal pilot city.

Comparative Analysis

Comparing with existing methods yields different rankings ():

The IVHFWA and IVHFWG operators from Reference [3] select A_2 as optimal, while the IVHF ELECTRE method from Reference [5] selects A_4. These methods assume attribute independence and ignore interrelationships. However, in this ecological project evaluation, surface water quality, air quality, and noise levels are clearly correlated. Only by fully considering these relationships can decisions be reasonable. Reference [4]' s HIVFCOA and HIVFCOG operators (which combine HIVFE with fuzzy measure Choquet integrals) also select A_3, but their fuzzy measures are subjective and computationally complex. In contrast, the IVHFWHM operator is more practical and computationally efficient.

Parameter Sensitivity Analysis

We further analyze the influence of parameters p and q on the fusion results. When $q \in [1,2] \cup [8,10]$ and p is fixed, the evaluation scores decrease as p increases ([Figure 1: see original paper]), with the ranking $A_3 > A_4 > A_2 > A_5 > A_1$ remaining stable. When p is fixed and q varies, scores similarly decrease as q increases ([Figure 2: see original paper]). When $q \in [8,10]$, slight ranking changes occur among some cities, but the overall order $A_3 > A_4 > A_2 > A_5 > A_1$ persists, confirming A_3 as optimal. This demonstrates that the IVHFWHM operator robustly handles attribute correlations in real decision-making scenarios.

[Figure 1: see original paper]
[Figure 2: see original paper]

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5 Conclusion

The continuous development of social informatization increasingly complicates MADM problems. Decision-makers, constrained by experience and knowledge, often provide correlated attribute values. For instance, when selecting pilot locations for ecological projects, air quality, surface water quality, and urban noise pollution are interrelated—cities with superior air and water quality typically have lower noise pollution. Existing IVHFS aggregation operators largely assume attribute independence, limiting their practical applicability. This paper addresses this limitation by:

1. Proposing IVHFHM and IVHFWHM operators that effectively capture attribute correlations
2. Investigating key mathematical properties (idempotency, permutation invariance, monotonicity, boundedness, parameter symmetry)
3. Developing a MADM model based on IVHFWHM
4. Validating the approach through an ecological environment assessment case study

The proposed operators eliminate the influence of attribute correlations on decision outcomes, producing more credible results and providing a new approach for solving complex MADM problems. Future research will explore extensions to other fuzzy environments and applications in large-scale group decision-making.

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