Postprint: Delay-Time-Based Joint Optimization of Multi-Type Maintenance and Economic Production Quantity

Authors: Qinming Liu, Lü Wenyuan, Ye Chunming

Date: 2018-05-02T00:00:00+00:00

Abstract

To address the current issue of inadequately considering the joint optimization of multiple maintenance types and economic production lot size, this study first examines the relationships among multiple maintenance types and derives expressions for the number of failures and defects based on delay-time theory. Second, building upon this foundation and comprehensively considering both production and maintenance costs, a joint optimization model for multiple maintenance types and economic production lot size is constructed, with the objective of minimizing total cost per unit time to obtain the optimal inspection interval and economic production lot size. Finally, the validity of the model is verified through numerical example analysis, demonstrating that the number of inspections for the first type of defect has minimal impact on cost and economic production lot size.

Full Text

Preamble

Joint Optimization of Multiple Maintenance Types and Economic Production Quantity with Time Delay

Liu Qinming, Lyu Wenyuan, Ye Chunming
(Business School, University of Shanghai for Science and Technology, Shanghai 200093, China)

Abstract: Current research has not adequately addressed the joint optimization problem of multiple maintenance types and economic production quantity. First, by considering the relationships among multiple maintenance types, expressions for the number of faults and defects are derived based on time delay theory. Second, a joint optimization model for multiple maintenance types and economic production quantity is constructed by comprehensively considering production costs and maintenance costs. The optimization objective is to minimize the total cost per unit time, thereby obtaining the optimal inspection interval and economic production quantity. Finally, the effectiveness of the model is verified through a case study, which demonstrates that the number of inspections for the first type of defect has minimal impact on cost and economic production quantity.

Keywords: multiple maintenance types; economic production quantity; inspection interval; time delay; joint optimization

0 Introduction

Maintenance planning and production planning are two critical components in manufacturing enterprises. Production planning assumes that equipment operates normally during the planning horizon, while maintenance planning ensures this normal operation. However, maintenance consumes actual production time, resulting in effective capacity that is lower than expected. Unreasonable maintenance planning can disrupt production schedules, affect delivery performance, and reduce customer satisfaction.

The economic production quantity (EPQ) model is based on the premise that equipment operates normally during the planning period, which cannot accurately reflect actual equipment conditions, leading to imprecise production batch decisions. To address this limitation, numerous studies have integrated equipment condition into EPQ models. Da et al. [1] integrated predictive maintenance into the EPQ model and used an ARIMA model to predict system health indicators. Chen et al. [2] proposed an improved EPQ model that determines optimal production and product parameters by minimizing total expected social loss, including manufacturing costs and customer usage costs under failure conditions. Chen et al. [3] developed an EPQ model for flexible production systems with Markovian demand, primarily considering constant production rates and unit production costs. Xu et al. [4] addressed an EPQ problem with constraints on maximum production runtime and minimum preventive maintenance time, proposing two heuristic algorithms based on optimal properties of the relaxed problem. Jafari et al. [5,6] introduced new models to determine optimal economic production quantities and preventive maintenance strategies for complex production equipment. Guo and Hu et al. [7,8] considered EPQ models with non-fixed product defect rates and scrap rates. Hu et al. [9] further studied EPQ problems with allowable shortages based on randomly occurring production defects. Zheng et al. [10] established an equipment maintenance interval optimization model under insufficient maintenance inspection data based on time delay theory. Jin et al. [11] conducted joint research on preventive maintenance and EPQ, but only considered equipment reliability without accounting for defect and failure repair times. Chakraborty et al. [12] integrated production, inventory, and maintenance models for equipment subject to defects and random failures. Zhai et al. [13] and Ma et al. [14] investigated mass production and collaborative production scenarios.

In summary, most existing models focus on single-defect-type, single-product production systems. Although some studies have considered multiple defect types [15-18], they have not integrated production planning comprehensively. Since both maintenance and production planning are critical enterprise functions, this paper addresses the joint optimization of multiple defect maintenance types and economic production quantity. By analyzing equipment defects and failures comprehensively and incorporating EPQ, we propose a joint optimization decision model for multiple maintenance types and economic production quantity. The model uses time delay theory to derive expressions for equipment failure frequency and defect frequency. Based on these expressions, we consider production costs (setup and inventory) and maintenance costs (inspection, failure repair, and defect repair). Utilizing the nested property of inspection intervals and enumeration methods, we determine the optimal inspection intervals for production equipment and calculate the corresponding optimal economic production quantity. We further analyze how inspection frequency affects costs and production batch decisions, helping management develop effective and reliable production and maintenance plans.

---

1 Problem Description

This study addresses how to determine optimal inspection cycles for different maintenance types and the corresponding economic production quantity to minimize equipment downtime and total cost per unit time. The relationship between maintenance inspection activities and production batches is illustrated in [Figure 1: see original paper], where P represents production rate, D represents demand rate, T represents equipment operating time, and Tp represents production cycle. Inspection activities for each maintenance type occur during equipment operation, with the inspection cycle for type N exactly equal to the equipment operating time. All inspection cycles are smaller than the production cycle (Tp).

The model employs time delay theory to comprehensively consider defect and failure conditions across all inspection types. Downtime caused by failures is assumed negligible. The primary costs include inspection costs, defect repair costs, failure repair costs, production setup costs, and inventory holding costs. The optimization objective is to minimize the expected total cost per unit time, yielding two optimal decision variables: the optimal inspection interval for each defect type (T*) and the optimal economic batch quantity (Q*).*

---

2 Assumptions

The following assumptions are made in this model:

a) Product production rate P and demand rate D are constant, with P > D.

b) Equipment has N maintenance types, indexed by i = 1, 2, 3, ···, N. If k > i, then type k inspection includes type i inspection, and the included type i inspection has the same expected cost, failure detection rate, and repair cost as a separate type i inspection.

c) Equipment defects are categorized into N+1 types. Type i inspection can identify defects of types 1, 2, ···, i (i   N), while type N inspection can identify defects of types 1 through N. A type N+1 defect is assumed to have zero time delay—once it occurs, it immediately becomes a failure that cannot be identified by any inspection. This defect type only exists in the final inspection category and incurs only repair costs.

d) All maintenance inspection intervals occur within equipment operating time and are smaller than the production cycle.

e) Defect repair inspections for each type do not affect production.

---

3 Model Notation

Basic notation is summarized in . The table includes parameters such as production rate (P), demand rate (D), setup cost (K), operating time (T), unit holding cost (H), mean failure repair costs (Cf), mean inspection costs (Cp), mean defect repair costs (Cr), defect occurrence rate ( ), defect type proportions (i), inspection intervals (Ti), interval ratios (ni), inspection counts, time delay distribution functions (Fi(h) and fi(h)), expected failure counts (ENf), expected defect detection counts (ENr), total expected cost (EC), and expected total cost per unit time (ECT).

---

4 Model Formulation

4.1 Production Setup and Inventory Costs

From [Figure 1: see original paper], the production cycle formula is derived as follows:

$$T_p = \frac{P \times T}{D}$$

The inventory cost expression is:

$$I = \frac{(P - D) \times T^2 \times H}{2}$$

Total production setup cost = K.

4.2 Expected Inspection Cost

Due to the nested relationship between inspection intervals, the process also includes N-1 types of inspections. For example, the number of type N-1 inspections included in this process is nN. By extension, the counts for other inspections can be expressed as:

$$\prod_{j=1}^{i} n_j \quad \text{for } i = 1, 2, ..., N$$

Since the cost per inspection is known, the total expected inspection cost during TN is:

$$E[C_p] = \sum_{i=1}^{N} \left(C_{pi} \times \prod_{j=1}^{i} n_j \right)$$

4.3 Expected Failure Repair Cost

During TN, there are nN type N-1 inspections. The expected number of failures encountered during each type N-1 inspection is ENf(TN-1), and the expected failure repair cost is Cf(N-1) × ENf(TN-1). Therefore, the total expected failure repair cost for type N-1 inspections during TN is:

$$E[C_{f(N-1)}] = n_N \times C_{f(N-1)} \times EN_f(T_{N-1})$$

Similarly, since the number of type N-2 inspections included is nN-1 × nN, the total expected failure repair cost for type N-2 inspections is:

$$E[C_{f(N-2)}] = n_{N-1} \times n_N \times C_{f(N-2)} \times EN_f(T_{N-2})$$

Extending this logic, the total expected failure repair cost during TN is:

$$E[C_f] = \sum_{i=1}^{N} \left(\prod_{j=i}^{N} n_j \times C_{fi} \times EN_f(T_i) \right)$$

The last term in this expression represents failures caused by zero-time-delay defects, indicating the expected repair cost for such failures.

4.4 Expected Defect Repair Cost

Following the same approach as Section 4.3, the expected defect repair cost is:

$$E[C_r] = \sum_{i=1}^{N} \left(\prod_{j=i}^{N} n_j \times C_{ri} \times EN_r(T_i) \right)$$

4.5 Joint Optimization Model

Based on equations (1)-(3), (7), and (8), the total expected cost during TN is:

$$EC(n) = K + \sum_{i=1}^{N} \left(\prod_{j=i}^{N} n_j \times C_{pi} \right) + \sum_{i=1}^{N} \left(\prod_{j=i}^{N} n_j \times C_{fi} \times EN_f(T_i) \right) + \sum_{i=1}^{N} \left(\prod_{j=i}^{N} n_j \times C_{ri} \times EN_r(T_i) \right) + \frac{(P-D) \times}{2}$$

Let h represent the time interval from defect occurrence to failure formation, with probability distribution function F(h). For type i defects, the time delay distribution function is Fi(h). Given the occurrence rate i, the expected number of failures and expected number of detected defects are:

$$EN_f(T_i) = \int_0^{T_i} \lambda \alpha_i [1 - F_i(u)] du$$

$$EN_r(T_i) = \int_0^{T_i} \lambda \alpha_i F_i(u) du$$

The expected total cost per unit time is:

$$ECT(n) = \frac{EC(n)}{T_p} = \frac{D \times EC(n)}{P \times T_N}$$

Substituting the relationships between inspection intervals, the final optimization model becomes:

$$ECT(n) = \frac{D}{P \times T_N} \left[K + \sum_{i=1}^{N} \prod_{j=i}^{N} n_j \times C_{pi} + \sum_{i=1}^{N} \prod_{j=i}^{N} n_j \times \left(C_{fi} \times \int_0^{T_i} \lambda \alpha_i [1 - F_i(u)] du + C_{ri} \times \int_0^{T_i} \lambda \alpha_i F_i(u) d \right.\right.$$

6 Case Study

The model aims to determine optimal inspection intervals for each maintenance type and the corresponding economic production quantity. Due to the complexity of solving for multiple unknown inspection interval variables directly, we demonstrate the model' s validity and applicability by considering a simplified two-type inspection and EPQ model.

Consider a production enterprise with equipment whose defect occurrence follows a homogeneous Poisson process with = 0.5. The time delay distribution is exponential with scale parameter . The equipment has production rate P = 1000, demand rate D = 500, setup cost K = 200, and holding cost H = 0.5. We assume two inspection types, each corresponding to one defect type. Since one defect type follows zero time delay, we solve a two-inspection-type, three-defect-type EPQ decision problem. Other parameter values are shown in .

The solution approach is as follows:

a) n2 is the ratio of type 2 inspection interval to type 1 inspection interval, which must be an integer. Since the two inspection intervals are clearly unequal, we assume n2 ranges from 2 to 10. Using enumeration, we assign successive integer values to n2. Once n2 is determined, equation (13) simplifies to an optimization problem with a single unknown variable T1.

b) By constructing a linear function, we can find the optimal solution for each n2 value. Substituting the data into equation (13) yields:

$$ECT(n) = \frac{500}{1000 \times T_2}\left[200 + 15n_1 + 45n_1n_2 + 80 \times \int_0^{T_1} 0.5 \times 0.6 \times e^{-0.15u} du + 120 \times \int_0^{T_2} 0.5 \times 0.3 \times e\right.$$

c) After obtaining all optimal solutions, we compare and analyze them to determine the optimal inspection intervals and economic production quantity for each maintenance type. Using MATLAB, we calculate the optimal values of T1 and ECT(n) for several fixed n2 values, as shown in [Figure 2: see original paper].

The optimal inspection intervals and economic production quantities are presented in . The optimal solution is: n2* = 2, T1* = 189 days, T2* = 378 days (with 360 days as one year), Q* = 1051, and ECT(n)* = 271.6.

The results show that increasing n2 affects T2, Q, and ECT(n) in the same direction with similar magnitude, while T1 decreases as n2 increases. As n2 grows, the inspection interval for type 1 maintenance shortens, while that for type 2 lengthens. Additionally, the expected total cost per unit time increases with n2, reaching its minimum when n2 = 2. The economic production quantity generally increases with n2, while the expected maintenance cost per unit time increases steadily, indicating that the number of type 1 defect inspections has minimal impact on cost and production quantity. Therefore, the joint optimization model effectively determines optimal inspection intervals for each maintenance type and establishes a reasonable economic production quantity.

7 Conclusion

This paper develops a joint optimization decision model for multiple maintenance types and economic production quantity based on time delay theory. The model integrates production setup costs, inventory holding costs, inspection costs, failure repair costs, and defect repair costs, with the objective of minimizing the expected total cost per unit time. The joint optimization model determines optimal inspection intervals for each maintenance type and the corresponding economic production quantity, simultaneously optimizing maintenance and production plans. This addresses the limitation of production planning models that consider only single defect types.

Compared to single-defect scenarios, the multi-defect approach results in shorter actual effective capacity time, enabling more precise estimation of true equipment capacity and providing more practical guidance for production planning. Future research could extend this work to joint optimization of maintenance types and EPQ for multi-product systems.

References

[1] Da W, Pan E, Wang Y, et al. An economic production quantity model for a deteriorating system integrated with predictive maintenance strategy [J]. Journal of Intelligent Manufacturing, 2014, 27 (6): 1-11.
[2] Chen C H, Chou C Y. The joint determination of specification limits, process mean, and economic manufacturing quantity [J]. Journal of the Chinese Institute of Industrial Engineers, 2017, 34 (4): 283-288.
[3] Chen J, Chen C P, Chen Z X, Xing L B. Optimal (P, Q, T) strategy for EPQ model with Markovian demand under flexible production [J]. Mathematics in Practice and Theory, 2016, 46 (02): 84-93.
[4] Xu Q, Xu J. Heuristics for the economic production quantity problem under restrictions on production and maintenance time [J]. Mathematical Problems in Engineering, 2016, 2016: 1-9.
[5] Jafari L, Makis V. Joint optimization of lot-sizing and maintenance policy for a partially observable two-unit system [J]. International Journal of Advanced Manufacturing Technology, 2016, 87 (5-8): 1-19.
[6] Jafari L, Makis V. Optimal lot-sizing and maintenance policy for a partially observable production system [J]. Computers & Industrial Engineering, 2016, 93: 88-98.
[7] Guo C Y, Hu J S, Wang L. Economic production quantity model with defect rate and allowable shortages under fuzzy environment [J]. Computer Integrated Manufacturing Systems, 2008, (11): 2161-2166.
[8] Hu J S, Guo C Y. Fuzzy production inventory model with allowable shortages and repairable defective products [J]. Computer Integrated Manufacturing Systems, 2009, 15 (5): 932-938.
[9] Hu J S, Ji Y P, Guo C Y, et al. Fuzzy random defective production system

with allowable shortages [J]. Control and Decision, 2011, 26 (2): 213-220.
[10] Zheng R, Lyu W Y. Preventive maintenance model based on time delay theory and case study [J]. Chinese Journal of Management Science, 2010, 18 (2): 48-54.
[11] Jin Y, Pan E S, Wang Y. Joint economic design model based on statistical process control and maintenance strategy [J]. Computer Integrated Manufacturing Systems, 2012, 18 (9): 1964-1971.
[12] Chakraborty T, Giri B C. Lot sizing in a deteriorating production system under inspections, imperfect maintenance and reworks [J]. Operation Research, 2014, 14 (1): 29-50.
[13] Zhai Y H, Liang L, Liu Y X, et al. Remanufacturing intensive production planning model for mass customization [J]. Journal of University of Shanghai for Science and Technology, 2014, 36 (6): 603-613.
[14] Ma H M, Ye J F. Disruption management for collaborative production planning and procurement [J]. Journal of University of Shanghai for Science and Technology, 2015, 37 (3): 301-306.
[15] Nobil A H, Sedigh A H A. A multiproduct single machine economic production quantity (EPQ) inventory model with discrete delivery order, joint production policy and budget constraints [J]. Annals of Operations Research, 2017, (9): 1-37.
[16] Lyu W Y, Zheng R. Optimal combination model of maintenance types based on time delay and case analysis [J]. Systems Engineering—Theory & Practice, 2013, 33 (07): 1654-1660.
[17] Zheng R, Lyu W Y. Joint decision model of production control and maintenance planning considering failure downtime [J]. Chinese Journal of Management Science, 2016, 24 (8): 116-122.
[18] Cui T J, Li S S, Ma Y D, et al. Maintenance rate distribution determination for systems composed of heterogeneous components with constraints [J]. Application Research of Computers, 2017, 34 (11): 1-7.

Note: Figure translations are in progress. See original paper for figures.