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Abstract

In broadband beamforming, interference signals entering from the mainlobe direction cause mainlobe distortion and elevated sidelobe levels, thereby severely degrading beamforming performance. To address these issues, a broadband beamforming algorithm based on covariance matrix reconstruction and eigen-projection preprocessing (EMP) is proposed. The algorithm first employs the EMP algorithm to obtain a blocking matrix, which performs interference cancellation preprocessing on the received signal to block mainlobe interference. Subsequently, a reasonable covariance matrix is obtained through the coherent signal subspace (CSM) method and covariance matrix reconstruction. Finally, beamforming is performed. In scenarios with simultaneous mainlobe and sidelobe interference, the algorithm can adaptively block mainlobe interference while suppressing sidelobe interference, thereby resolving the waveform distortion problem in broadband beamforming under mainlobe interference conditions. Computer simulations validate the effectiveness of the proposed algorithm.

Full Text

A Broadband Beamforming Algorithm Based on Covariance Matrix Reconstruction and EMP

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Abstract: In broadband beamforming, when interference signals enter from the mainlobe direction, they cause mainlobe distortion and elevated sidelobe levels, severely degrading beam performance. To address these issues, this paper proposes a broadband beamforming algorithm based on eigen-projection preprocessing (EMP) and covariance matrix reconstruction. The algorithm first employs EMP to construct a blocking matrix that cancels mainlobe interference through preprocessing. It then obtains a proper covariance matrix using the coherent signal-subspace method (CSM) and covariance matrix reconstruction. Finally, beamforming is performed. In scenarios with both mainlobe and sidelobe interference, the algorithm adaptively blocks mainlobe interference while suppressing sidelobe interference, thereby solving the waveform distortion problem in broadband beamforming under mainlobe interference conditions. Computer simulations verify the effectiveness of the proposed algorithm.

Keywords: broadband beamforming; mainlobe interference; eigen-projection preprocessing; covariance matrix reconstruction

0 Introduction

Broadband beamforming algorithms are widely used in radar, communications, and electronic countermeasures [1-3]. In recent years, many scholars have proposed numerous algorithms for suppressing sidelobe interference. Reference [4] discussed sidelobe interference suppression algorithms for sparse arrays, while reference [5] studied Capon beamforming based on interference-noise matrices to achieve higher output signal-to-interference-plus-noise ratio (SINR). References [6] and [7] respectively applied compressive sensing and robust Capon algorithms to improve output SINR with enhanced robustness.

When interference signals enter from the mainlobe direction in beamforming, two serious problems occur: (a) elevated sidelobe levels, and (b) severe distortion of the main beam. Reference [8] proposed two methods for blocking mainlobe interference: (a) beamforming based on blocking matrix preprocessing (BMP), and (b) beamforming based on eigen-projection preprocessing (EMP). The BMP algorithm suffers from several drawbacks: high sidelobes and strict requirements for the estimated angle of mainlobe interference. The EMP algorithm also exhibits peak shift problems. Reference [9] improved upon the EMP algorithm from reference [8] by addressing the peak shift issue. For broadband beamforming under mainlobe interference, reference [10] presented a processing method, but it suffers from high sidelobe levels, which leads to a sharp increase in false alarm probability.

This paper addresses the peak shift problem in the EMP algorithm from reference [8] through covariance matrix reconstruction and extends the EMP algorithm to broadband signals.

1 Wideband Signal Model

Assume a uniform linear array with N elements. The total number of signal sources is D, all sharing the same bandwidth B. The noise is Gaussian white noise with the same bandwidth as the source signals. The array output data vector $x(t)$ is given by:

$$x(t) = [x_1(t), \cdots, x_N(t)]^T$$

Through fast discrete Fourier transform (DFT) decomposition, this is divided into J sub-bands. The data vector in the sub-band with center frequency f_j can be expressed as:

$$X(f_j) = A(f_j) \cdot S(f_j) + N(f_j) \quad j = 1, 2, \cdots, J$$

where f_j is the center frequency of the j-th sub-band signal; $X(f_j)$ is the $N \times 1$ sub-band received signal vector; $S(f_j)$ is the spectral component of the signal in the j-th sub-band; $N(f_j)$ is an $N \times 1$ vector representing the noise spectral vector at f_j; and $A(f_j) = [a(\theta_1, f_j), a(\theta_2, f_j), \cdots, a(\theta_D, f_j)]$ is the direction matrix of signals in the j-th sub-band, with $a(\theta_i, f_i) = [1, e^{-j2\pi(d/\lambda)\sin\theta_i}, \cdots, e^{-j2\pi(d/\lambda)(N-1)\sin\theta_i}]^T$, where θ_i is the incident angle of the i-th signal relative to the array elements, $i = 1, 2, \ldots, D$, and D is the number of signal sources.

2 Sub-band Interference Cancellation Preprocessing

Assume there are P sidelobe interference signals and one mainlobe interference signal in space. The covariance matrix containing only interference and noise in the j-th sub-band is $R_X(f_j)$, with eigenvalue decomposition given by:

$$R_X(f_j) = \sum_{i=1}^{N} \lambda_i u_i u_i^H = U_s \Lambda_s U_s^H + U_m \Lambda_m U_m^H$$

where $\lambda_i (i = 1, 2, \ldots, N)$ are the eigenvalues of $R_X(f_j)$ with $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_P \geq \lambda_{P+1} \geq \cdots \lambda_N$; u_i is the eigenvector corresponding to eigenvalue λ_i; $u_1, u_2, \cdots, u_{P+1}$ span the interference subspace; $u_{P+2}, u_{P+3}, \cdots, u_N$ span the noise subspace; $U_s = [u_1, u_2, \cdots, u_{P+1}]$, $\Lambda_s = \mathrm{diag}(\lambda_1, \lambda_2, \cdots, \lambda_{P+1})$, $U_n = [u_{P+2}, u_{P+3}, \cdots, u_N]$, and $\Lambda_n = \mathrm{diag}(\lambda_{P+2}, \lambda_{P+3}, \cdots, \lambda_N)$. In practice, $R_X(f_j)$ can be replaced by the actual received signal covariance matrix $\hat{R}_X(f_j)$, i.e.,

$$\hat{R}_X(f_j) = \frac{1}{K} X(f_j) X^H(f_j)$$

where K represents the number of training snapshots.

The blocking matrix $B(f_j)$ for the j-th channel can be expressed as:

$$B(f_j) = I - u_m (u_m^H u_m)^{-1} u_m^H$$

where u_m represents the eigenvector corresponding to mainlobe interference, and I is the $N \times N$ identity matrix. u_m is determined by:

$$|\rho(u_m, a(\theta_s, f_j))| = \max_i |\rho(u_i, a(\theta_s, f_j))|$$

where $\rho(a_1, a_2)$ denotes the correlation coefficient between vectors a_1 and a_2, defined as:

$$\rho(a_1, a_2) \triangleq \frac{|a_1^H a_2|}{\|a_1\|\|a_2\|}$$

and $a(\theta_s, f_j)$ is the steering vector of the desired signal at direction θ_s and frequency f_j.

The signal after interference cancellation preprocessing in the sub-band with center frequency f_j is:

$$Y_b(f_j) = B(f_j)X(f_j)$$

where $X(f_j)$ can be viewed as the combination of multiple spatial signals:

$$X(f_j) = a(\theta_s, f_j)s(\theta_s, f_j) + \sum_{i=1}^{P} a(\theta_i, f_j)s(\theta_i, f_j) + N(f_j)$$

where $s(\theta_s, f_j)$ represents the complex envelope of the desired signal in the j-th channel, $s(\theta_{P+1}, f_j)$ represents the complex envelope of the mainlobe interference in the j-th channel, and $s(\theta_i, f_j)(j = 1, 2, \cdots, P)$ represents the complex envelope of sidelobe interference.

The preprocessed signal is given by:

$$Y_b(f_j) = \sum_{i=1}^{P} [a(\theta_i, f_j) - u_m(u_m^H u_m)^{-1} u_m^H a(\theta_i, f_j)] s(\theta_i, f_j) + [a(\theta_{P+1}, f_j) - u_m(u_m^H u_m)^{-1} u_m^H a(\theta_{P+1}, f_j)] s(\theta_{P+1}, f_j) +$$

where $\bar{N}(f_j) = B(f_j)N(f_j)$ is the noise after preprocessing. Since u_m is the eigenvector corresponding to mainlobe interference, we have:

$$u_m^H a(\theta_i, f_j) \approx 0 \quad i = 1, 2, \cdots, P$$

and

$$u_m^H a(\theta_{P+1}, f_j) \approx a(\theta_{P+1}, f_j)$$

Therefore, the preprocessed sub-band signal vector becomes:

$$Y_b(f_j) = a(\theta_s, f_j)s(\theta_s, f_j) + \sum_{i=1}^{P} a(\theta_i, f_j)s(\theta_i, f_j) + \bar{N}(f_j)$$

It can be seen that after sub-band interference cancellation preprocessing, the mainlobe interference component in the j-th sub-band array received data is blocked while sidelobe interference components are preserved, facilitating subsequent beamforming.

3 Focusing Processing

This algorithm employs the RSS (rotational signal subspace) algorithm from reference [11] to solve for the focusing matrix. In the RSS algorithm:

$$T(f_j) = V(f_j)U^H(f_j)$$

where $V(f_j)$ is the left singular vector of $A(f_0, \theta_s)A^H(f_j, \theta_s)$, and $U(f_j)$ is the right singular vector of $A(f_0, \theta_s)A^H(f_j, \theta_s)$, with θ_s being the direction of the desired signal. Here:

$$A(f_j, \theta_s) = [1, e^{-j2\pi(\frac{d}{\lambda_j})\sin\theta_s}, \cdots, e^{-j2\pi((M-1)\frac{d}{\lambda_j})\sin\theta_s}]^T$$

$$A(f_0, \theta_s) = [1, e^{-j2\pi(\frac{d}{\lambda_0})\sin\theta_s}, \cdots, e^{-j2\pi((M-1)\frac{d}{\lambda_0})\sin\theta_s}]^T$$

The final output of the j-th channel after focusing is:

$$Y(f_j) = T(f_j)(B(f_j)X(f_j))$$

The signal covariance matrix at this point is:

$$R_Y(f_j) = Y(f_j)Y^H(f_j)$$

4 Covariance Matrix Reconstruction

If the classical SMI (sample matrix inversion) algorithm from reference [12] is used to obtain the weight vector, the adaptive weight vector for the j-th sub-band is:

$$w_j = \frac{R_Y^{-1}(f_j)a(\theta_s, f_j)}{a^H(\theta_s, f_j)R_Y^{-1}(f_j)a(\theta_s, f_j)}$$

Let $\mu = 1/a^H(\theta_s, f_j)R_Y^{-1}(f_j)a(\theta_s, f_j)$. Through eigenvalue decomposition of the covariance matrix, this can be written as:

$$w_j = \mu \sum_{i=1}^{N} \frac{1}{\lambda_i} u_i u_i^H a(\theta_s, f_j) = \mu \left[\frac{1}{\lambda_m} u_m u_m^H a(\theta_s, f_j) + \sum_{i=1, i\neq m}^{P+1} \frac{1}{\lambda_i} u_i u_i^H a(\theta_s, f_j) + \sum_{i=P+2}^{N} \frac{1}{\lambda_i} u_i u_i^H a(\theta_s, f_j) \right]$$

The adaptive pattern is obtained as:

$$F_a(\theta, f_j) = w_j^H a(\theta, f_j) = \mu \left[F_q(\theta, f_j) - \sum_{i=1}^{P+1} \frac{\lambda_i - \lambda_{\min}}{\lambda_i} [a^H(\theta_s, f_j) u_i] F_i(\theta, f_j) \right]$$

where $F_a(\theta, f_j)$ is the adaptive pattern in the j-th sub-band; $F_q(\theta, f_j)$ is the quiescent pattern; $F_i(\theta, f_j)$ are the eigen-beams with $F_q(\theta, f_j) = a^H(\theta_s, f_j)a(\theta, f_j)$ and $F_i(\theta, f_j) = u_i^H a(\theta, f_j)$. The $F_i(\theta, f_j)(i = 1, 2, \cdots, P+1)$ are the eigen-beams corresponding to $P+1$ interference signals, where $F_i(\theta, f_j)$ has maximum gain in the direction of the i-th interference signal and zero gain in other interference directions; and $\lambda_{\min}$ is the minimum eigenvalue of $R_Y(f_j)$.

When λ_i corresponds to interference signals ($i = 1, 2, \cdots, P+1$), λ_i is much larger than $\lambda_{\min}$, therefore:

$$\frac{\lambda_i - \lambda_{\min}}{\lambda_i} \approx 1 \quad i = 1, \cdots, P+1$$

This shows that eigenvector λ_i forms a null in the direction of its corresponding interference. However, if λ_i is set as the eigenvalue corresponding to noise ($i = P+2, P+3, \cdots, N$), then λ_i is approximately equal to $\lambda_{\min}$:

$$\frac{\lambda_i - \lambda_{\min}}{\lambda_i} \approx 0 \quad i = P+2, \cdots, N$$

Therefore, if the mainlobe interference eigenvalue is changed to a noise eigenvalue, the resulting weight vector will be affected by mainlobe interference, thus avoiding the peak shift problem. In the proposed algorithm, as shown in equation (23), the eigenvalue λ_m corresponding to mainlobe interference is set to the mean value $\tilde{\lambda}_m$ of the noise eigenvalues, thereby reducing the mainlobe interference eigenvalue to the level of background noise and eliminating its influence on the weight vector:

$$\tilde{\lambda}_m = \tilde{\sigma}_n^2 = \frac{\lambda_{P+2} + \cdots + \lambda_N}{N - P - 1}$$

where $\tilde{\sigma}_n^2$ represents the average of the noise eigenvalues. Since the eigenvalues corresponding to noise exhibit fluctuations that can raise the sidelobe levels of the antenna pattern and affect beamforming performance, to eliminate noise jitter, the noise eigenvalues of the covariance matrix are modified as:

$$\tilde{\lambda}_{P+2} = \cdots = \tilde{\lambda}_N = \tilde{\sigma}_n^2$$

The reconstructed covariance matrix for the j-th sub-band can be expressed as:

$$\tilde{R}_Y(f_j) = U\tilde{\Lambda}U^H$$

where $\tilde{\Lambda}$ is the reconstructed eigenvalue matrix:

$$\tilde{\Lambda} = \mathrm{diag}(\lambda_1, \cdots, \tilde{\lambda}_m, \cdots, \lambda_{P+1}, \tilde{\sigma}_n^2, \cdots, \tilde{\sigma}_n^2)$$

In this algorithm, the eigenvectors remain unchanged. After focusing processing, the covariance matrix becomes:

$$\tilde{R}_Y = \sum_{j=1}^{J} T(f_j)\tilde{R}_Y(f_j)T^H(f_j)$$

According to the MVDR (minimum variance distortionless response) algorithm, the weight vector for broadband beamforming is:

$$\tilde{w}(f_0) = \tilde{\mu}\tilde{R}_Y^{-1}a(\theta_s, f_0)$$

where $\tilde{\mu} = 1/a^H(\theta_s, f_0)U\tilde{\Lambda}^{-1}U^H a(\theta_s, f_0)$ is a scalar. It can be seen that when calculating the weight vector, the influence of the mainlobe interference eigenvalue has been eliminated, so the weight vector will not cause peak shift in the antenna pattern. Since $\tilde{\Lambda}$ is a diagonal matrix, its inversion only requires taking the reciprocal of the diagonal elements, avoiding complex matrix inversion operations and significantly reducing computational complexity compared to matrix inversion. The final frequency-domain output of the array is $Z(f) = [\tilde{w}^H(f_0)Y(f_1), \cdots, \tilde{w}^H(f_0)Y(f_j), \ldots, \tilde{w}^H(f_0)Y(f_J)]$, and the time-domain output is $z(t) = \mathrm{IDFT}[Z(f)]$.

[Figure 1: see original paper] shows the signal processing flow of the proposed algorithm.

5 Simulation Experiments and Conclusions

The experiment employs a 16-element uniform linear array with element spacing equal to half the wavelength corresponding to the highest frequency of the operating band. Both the desired signal and interference signals are assumed to be linear frequency-modulated signals. The simulation parameters are: center frequency $f_0 = 2$GHz, bandwidth $B = 200$MHz. The noise is Gaussian white noise with a signal-to-noise ratio (SNR) of 0 dB. There are three interference signals: one mainlobe interference and two sidelobe interference signals. The mainlobe interference arrives from 1° with an interference-to-noise ratio (INR) of 40 dB. The two sidelobe interference signals arrive from 40° and -30°, both with INRs of 20 dB. The array received signals are divided into $J = 128$ subbands through DFT. Figures 2-4 show three-dimensional pattern cross-sections at the highest frequency $f_H = 2.1$GHz, center frequency $f_0 = 2$GHz, and lowest frequency $f_L = 1.9$GHz.

Figure 2 shows the simulation result of direct MVDR without interference cancellation preprocessing. The antenna pattern exhibits a deep null near the mainlobe interference direction at 2°, which severely affects the peak position and results in high sidelobes.

[Figure 2: see original paper] MVDR algorithm antenna pattern

Figure 3 shows the simulation result using EMP interference cancellation preprocessing but without covariance matrix reconstruction, followed by MVDR. The blocking matrix constructed by the EMP algorithm successfully blocks the mainlobe interference, and the antenna pattern does not form a null in the mainlobe interference direction. However, the peak position shifts by 2°.

[Figure 3: see original paper] EMP algorithm antenna pattern

[Figure 4: see original paper] shows the broadband beamforming simulation based on covariance matrix reconstruction and EMP. The proposed algorithm successfully solves the peak shift problem of the EMP algorithm.

[Figure 4: see original paper] Antenna pattern of the proposed algorithm

provides a detailed numerical comparison of beam pointing, mainlobe width, and sidelobe level for the three algorithms.

Performance comparison of various algorithms

Algorithm	Beam Pointing	Mainlobe Width (-3dB) at f_0	Mainlobe Width (-3dB) at f_L	Mainlobe Width (-3dB) at f_H	Sidelobe Level
MVDR	2°	-	-	-	-3.13 dB
EMP	-3.7°	-	-	-	-12.9 dB

Algorithm	Beam Pointing	Mainlobe Width (-3dB) at f_0	Mainlobe Width (-3dB) at f_L	Mainlobe Width (-3dB) at f_H	Sidelobe Level
Proposed	0°	-	-	-	-13.9 dB

Experimental results demonstrate that the classical MVDR algorithm cannot block mainlobe interference. The simulation shows that the mainlobe interference null severely affects desired signal reception and results in high sidelobe levels and severe waveform distortion. The EMP algorithm can block mainlobe interference, but exhibits peak position shift. The proposed algorithm solves the peak shift problem through covariance matrix reconstruction. Moreover, it only requires eigenvalue decomposition to calculate the sub-band blocking matrix $B(f_j)$ and weight vector $\tilde{w}(f_0)$, and both $B(f_j)$ and $\tilde{w}(f_0)$ can be computed in parallel, improving algorithm execution efficiency. The classical MVDR algorithm has complexity of $\mathcal{O}(J \cdot K^3)$, while the proposed algorithm and EMP algorithm have complexity of $\mathcal{O}(2 \cdot J \cdot K^3)$. The proposed algorithm solves the waveform distortion problem of the classical MVDR algorithm and, compared with the EMP algorithm, resolves the peak shift issue without changing algorithm complexity.

6 Conclusion

This paper investigates broadband beamforming algorithms in the presence of mainlobe interference signals using a method based on covariance matrix reconstruction and EMP. The proposed algorithm consists of three processing stages: (a) calculating blocking matrices for each sub-band to block mainlobe interference; (b) performing focusing processing on received signals using the classical RSS algorithm; and (c) reconstructing the focused covariance matrix, which solves the peak shift problem. Computer simulations demonstrate that the proposed algorithm can handle broadband beamforming under mainlobe interference conditions, particularly when mainlobe interference severely affects the antenna pattern, enabling pattern preservation.

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