

## Speed Estimation Method Based on IGSO-Optimized EKF for Sensorless PMSM (Post-print)

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### Abstract

To improve the speed control performance in sensorless permanent magnet synchronous motor (PMSM) control systems, an extended Kalman filter (EKF) speed estimation scheme based on the improved group search optimization (IGSO) algorithm is proposed. First, the PMSM field-oriented control (FOC) system model is analyzed; then, the d-q axis voltage, current, and rotor speed of the motor are taken as state variables to construct the state equation in the EKF for estimating the rotational speed and load. Simultaneously, to improve the estimation performance of the EKF, the integral of squared error (ISE) between the estimated and actual values is used as the fitness function, and the IGSO algorithm is employed to optimize the noise covariance matrices  $Q$  and  $R$  in the EKF, thereby obtaining optimal parameters. Simulation results demonstrate that the proposed control system can accurately estimate the motor speed and achieve effective control.

### Full Text

## Method of EKF Speed Estimation for Sensorless PMSM Based on IGSO Optimization

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### Abstract

To improve speed control performance in sensorless permanent magnet synchronous motor (PMSM) control systems, this paper proposes an extended Kalman filter (EKF) speed estimation scheme based on an improved group

search optimization (IGSO) algorithm. First, the PMSM field-oriented control (FOC) system model is analyzed. Then, the motor's d-q axis voltage, current, and rotor speed are used as state variables to construct the state equation for EKF-based speed and load estimation. Simultaneously, to improve EKF estimation performance, the integral square error (ISE) between estimated and actual values is employed as the fitness function, and the IGSO algorithm is used to optimize the noise covariance matrices  $Q$  and  $R$  in the EKF. Simulation results demonstrate that the proposed control system can accurately estimate motor speed and achieve effective control.

**Keywords:** permanent magnet synchronous motor; speed estimation; extended Kalman filter; noise covariance matrix; group search optimization

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## 0 Introduction

Permanent magnet synchronous motors (PMSM) offer high power density, high efficiency, and good robustness, making them widely used in automation, transportation, aerospace, and other fields [1]. PMSM control is primarily divided into field-oriented control (FOC) and direct torque control (DTC) [2]. Both methods regulate current and flux through output signals from a speed controller at the front stage, making speed control the key to the entire closed-loop system. The foundation of speed control is accurately obtaining the motor's actual speed [3].

Traditional motor control requires speed sensors or optical encoders to measure rotor speed. Currently, to reduce costs and improve device robustness in PMSM systems, sensorless PMSM control is typically employed [4]. In sensorless control, motor speed is predicted through estimation models, which commonly include model reference adaptive systems (MRAS), observers, extended Kalman filters (EKF), fuzzy logic, and artificial neural networks [5]. Among these, EKF is widely used. However, the values of the noise covariance matrices  $Q$  and  $R$  in EKF directly affect estimation performance [6], requiring optimal parameter values.

Therefore, this paper proposes an EKF speed estimation scheme based on an improved group search optimization (IGSO) algorithm, with simulation experiments demonstrating the effectiveness of the proposed method. The main innovations are: (a) using the motor's d-q axis voltage, current, and rotor speed as state variables to construct the state equation in EKF, and (b) improving the traditional GSO algorithm by introducing the global best and personal best fusion technique from particle swarm optimization (PSO) to enhance the searcher's position update mechanism, thereby improving convergence speed.

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## 2 EKF-Based Sensorless PMSM Control Model

### 2.1 PMSM Sensorless Control Model

PMSM exhibits different d-axis and q-axis inductances depending on the effective air gap length. The pole portion in the rotor is defined as the d-axis, which is the direction of flux generation for the excitation winding [7]. The q-axis is defined as the direction at 90° electrical angle from the d-axis. [Figure 1: see original paper] shows the structure of PMSM.

PMSM is typically equivalent to a rotor rotating coordinate system (d-q axis) circuit, as shown in [Figure 2: see original paper]. The stator voltages of the d-axis and q-axis are expressed as:

$$v_d = R_s i_d + L_d \frac{di_d}{dt} - \omega_r \lambda_q$$

$$v_q = R_s i_q + L_q \frac{di_q}{dt} + \omega_r \lambda_d$$

where  $v_d$ ,  $v_q$ ,  $i_d$ ,  $i_q$  are the d-q axis voltages and currents respectively;  $\omega_r$  is the motor speed and  $R_s$  is the stator resistance;  $\lambda_d$  and  $\lambda_q$  are the d-q axis flux linkages, which can be expressed using stator current, inductance, and constant flux  $\lambda_m$  as:

$$\lambda_d = L_d i_d + \lambda_m$$

$$\lambda_q = L_q i_q$$

where  $L_d$  and  $L_q$  are the d-q axis inductances. Substituting these into the voltage equations yields the modified stator voltage equations:

$$v_d = R_s i_d + L_d \frac{di_d}{dt} - \omega_r \lambda_m$$

$$v_q = R_s i_q + L_q \frac{di_q}{dt} + \omega_r L_d i_d$$

The electromagnetic torque is:

$$T_e = \frac{3}{2} \lambda_m i_q$$

The motion equation can be expressed as:

$$J \frac{d\theta_r}{dt} = T_e - T_l - B \omega_r$$

where  $T_e$  is electromagnetic torque;  $T_l$  is load torque;  $P$  represents the number of poles;  $B$  is the damping coefficient;  $\theta_r$  is rotor position; and  $J$  is moment of inertia.

Therefore, the discrete-time system equation can be expressed as:

$$x(k+1) = A x(k) + B u(k) + w(k)$$

$$y(k) = C x(k) + v(k)$$

where  $x$  is the state vector,  $u$  is the input vector,  $w$  is random state noise,  $y$  is the observed or measured variable, and  $v$  is measurement noise.

For PMSM control, this paper adopts the commonly used field-oriented control (FOC) system, which converts PMSM into an equivalent separately excited DC motor. [Figure 3: see original paper] shows the block diagram of the PMSM FOC system, which includes a PWM inverter, PWM generator, PI speed controller, current controller, and EKF speed/position estimator [8].

## 2.2 Extended Kalman Filter (EKF)

Kalman filtering can estimate the states of linear systems, but its application to nonlinear systems is computationally intensive [9]. This is because in nonlinear systems, the state variable equations change at each step, preventing precomputation during iteration. This limitation can be addressed through the EKF method [10].

In the PMSM mathematical model, selecting the current d-q axis voltages, currents, and rotor speed as EKF state variables allows the input variable  $u$  and output variable  $y$  to be defined as:

$x = [i_d, i_q, \omega_r]^T$

The EKF algorithm proceeds as follows:

**a) Initialization:** Initialize the state vector and covariance matrices:

$x(0), P(0), Q, R$

**b) Jacobian matrices:** Determine the Jacobian matrices between  $f$  and  $h$ :

$k_f, k_h$

where:

$k_f = \frac{\partial f}{\partial x}$

**c) Prediction:** Predict the state matrix and error covariance matrix:

$x(1|0), P(1|0)$

$T, F, P, Q, P$

**d) State correction:** Calculate the Kalman gain matrix:

$T, k, P, H, R, K, P, H$

**e) Update state prediction:**

$x(1), P(1)$

**f) Estimate error covariance matrix:**

$P(1)$

## 2.4 Speed Estimation Based on EKF

The dynamic state equations of PMSM can be written in state-space form as:

$\dot{x} = A x + B u + w$      $y = C x + v$      $w \sim N(0, Q)$      $v \sim N(0, R)$

The discrete-time representation of these equations is:

$x(k+1) = A_d x(k) + B_d u(k) + w(k)$      $y(k) = C_d x(k) + v(k)$

The gradient matrix is:

111013(3122nrqssddnfKnrdskskqqqnqsdqnfssPLRTTLLPPLRTTTxLLLPLLiPBTTTJFJJ

The measurement matrix is:

100010KkkhxH

### 3 IGSO Optimization of EKF Parameters

#### 3.1 EKF Parameter Optimization Model

The key step in EKF design is obtaining filter parameters, particularly the initial state and covariance matrices  $Q$  and  $R$ . Matrix  $Q$  reflects system uncertainty and disturbances, while matrix  $R$  reflects measurement noise introduced by current sensors and other components [11]. This paper obtains these coefficient values through iterative search to achieve optimal estimation performance. Modifying covariance matrices  $Q$  and  $R$  simultaneously affects both transient and steady-state filtering behavior. Increasing  $Q$  values implies greater system model-driven noise or uncertainty, which enlarges state covariance terms and filter gain [12]. This requires increasing weight values to accelerate the filter's transient response. Similarly, increasing covariance  $R$  implies stronger fading noise, and thus the filter weights should be reduced, causing the gain matrix  $K$  to decrease and slowing transient performance.

For the initial state covariance matrix  $P$ , the diagonal terms represent variance or mean square error under initial conditions. Varying  $P$  produces different amplitude transient characteristics, but the transient duration remains unchanged and steady-state conditions are unaffected. Due to insufficient statistical information to evaluate off-diagonal terms of covariance matrices  $Q$ ,  $R$ , and  $P$ , they are assumed to be diagonal matrices. Based on experience, this paper sets:

$$(0)000x \quad (0)111Pdiag$$

The primary objective is selecting optimal values for  $Q$  and  $R$ . These could be manually chosen through trial and error, but this is time-consuming. To overcome this limitation, this paper employs a fast IGSO algorithm to adjust the covariance matrices. The system block diagram of IGSO optimizing EKF parameters is shown in [Figure 4: see original paper].

#### 3.2 Traditional GSO Algorithm

The traditional GSO algorithm comprises three operations: producer operation, scrounger operation, and ranger operation. During iteration, the member with the best fitness value is selected as the producer. Multiple members with fitness values above a threshold are selected as scroungers, while those with fitness values below the threshold are selected as rangers [13].

### 3.3 Producer Operation

During producer operation, animals rotate sensory receptors to capture information from the environment. In an  $s$ -dimensional search space, the position of the  $i$ th member at the  $z$ th iteration (search round) is represented as  $iz$ , and the search direction is represented as  $iz$ , which can be calculated from  $iz$  through polar coordinate transformation as:

$$11(1)(,)zzzsiisR$$

$$11(1)(,)zzzsiisFffR$$

where  $iz$  represents the search angle. The position update mechanism is:

$$11(1)1(1)\cos(\sin(\sin())szzippzzzijjzzisijffff$$

Assuming the producer's position at the  $z$ th iteration is  $z_p$ , the producer scans three different angles at its current position: first at zero degrees, then to the right, and finally to the left. The maximum search angle is  $\max$ , and the maximum visual scanning distance is  $\max_d$ , expressed as:

$$2\max_1siiiiidULUL$$

where  $i_U$  and  $i_L$  are the upper and lower bounds of the design variable range, respectively.

The three different positions discovered by the producer through scanning are:

$$1\max_1\max_2\max_1\max_2(0)2(2)zzzeroppzzrightppzzleftpppyrdFyyrdFryrdFr$$

where  $z_{er0}$  represents zero-degree scanning,  $z_{right}$  represents right-side scanning, and  $z_{left}$  represents left-side scanning.  $1_r$  and  $2_r$  are normally distributed random numbers with mean 0 and variance 1, and  $s_r$  is a random sequence within (0,1).

The producer then calculates the fitness of these three new positions and moves to the position with optimal fitness. If none of the new positions are better than the current position, it turns its head to a new angle:

$$12\max_zziir$$

where  $\max$  is the maximum turning angle and  $a_r$  is a random variable. If the producer fails to find a better position after a iterations, it stops searching and remains stationary:

$$azaz$$

### 3.5 Scrounger Operation

The scrounger operation involves following the producer and searching in its surrounding area. At the  $z$ th iteration, the  $i$ th scrounger performs regional search based on position information shared by the producer [14], with position update:

13()zzzziipiyryy

where 3sr is a random number in (0,1).

### 3.6 Improved GSO Algorithm (IGSO)

The traditional GSO algorithm suffers from slow convergence, limiting its applicability in real-time scenarios. To enable effective application of GSO to parameter optimization in motor EKF speed estimation systems, this paper proposes improvements to create a fast GSO algorithm with enhanced convergence speed.

In particle swarm optimization (PSO), another intelligent optimization algorithm, particles update their movement direction based on both the globally best position found by the swarm and their own personal best position, enabling rapid convergence toward the final global optimum [15]. This concept can be introduced into GSO to propose an improved GSO algorithm (IGSO) that enhances convergence speed by improving the searcher's position update mechanism.

At the zth iteration, the member with the highest fitness is designated as the producer, and its position is marked as the global best zGbesty. Scroungers in the group follow the producer and search in its vicinity, with the historical best position of the ith scrounger at the zth iteration marked as its personal best ,ziPbesty. The searchers can then consider both the producer's position and their own discoveries when following the producer, improving their ability to find better positions and accelerating producer formation for faster algorithm convergence. The improved searcher position update mechanism is:

112,()()zzzzziiGbestiiPbestiykrykry

where 1k and 2k are weight factors, both set to 2 in this work.

### 3.7 Objective Function

The integral square error (ISE) between the estimated d-axis current  $i_d$ , q-axis current  $i_q$ , and speed  $\omega_r$  from the EKF module and their actual values, weighted and combined, serves as the objective function F:

112233eweFwwe

where:

212223()()dactdestqactqestrectresteieiei

Proper weight selection is critical to avoid large errors. Through extensive experimentation and result analysis, this paper sets the optimal weights as:

10.225w 20.3w 30.475w

## 4 Experiments and Analysis

### 4.1 Experimental Setup

A PMSM simulation experiment was constructed using MATLAB/Simulink to implement the EKF parameter estimation algorithm. The PMSM parameters in the simulation are listed in . In the simulation, ,,dqdqiiiv are set as EKF algorithm inputs, and ,dqrii are the estimated state variables. To simulate actual system conditions, Gaussian white noise with intensity 6310 is added to ,dqii, with white noise sampling time set to 5210 seconds. For the IGSO algorithm, the maximum iteration number is 200, population size is 20, initial search angle  $z$  is set to 4, maximum search angle  $\max$  is 2a, constant  $a$  is 22a, and weighting parameters are 1s.

**TABLE:1** PMSM Simulation Parameters

Parameter	Value
$R_s$ ( $\Omega$ )	1.23
$L_d$ (mH)	6.5
$L_q$ (mH)	6.5
$\psi_f$ (Wb)	0.175
$J$ ( $\text{kg} \cdot \text{m}^2$ )	0.0008
$B$ ( $\text{N} \cdot \text{m} \cdot \text{s}$ )	0.001
$P_n$	4
$\omega_{max}$ (rpm)	3000

### 4.2 Parameter Optimization Experiment

First, a verification experiment compares the IGSO algorithm with traditional GSO, PSO, and classical genetic algorithm (GA). Using ISE as the objective function, various optimization algorithms are employed to optimize the Q and R matrices of EKF. The ISE curves under different iteration numbers are shown in [Figure 5: see original paper], and lists the Q and R matrices and ISE values obtained by IGSO at three iteration counts.

As shown in [Figure 5: see original paper], GA converges slowest due to its complex genetic, crossover, and mutation operations that are prone to local optima. PSO converges faster than GA and GSO because its individual position update mechanism facilitates rapid optimal solution finding, which motivated incorporating this mechanism into GSO to create IGSO. IGSO achieves the fastest convergence, reaching the optimal solution in approximately 100 iterations compared to about 160 iterations required by traditional GSO. This improvement stems from IGSO's introduction of PSO's global best and personal best fusion method, enhancing the searcher's position update mechanism and making IGSO suitable for real-time optimization tasks.

For the optimized solution at 100 iterations, the ISE value for speed estimation

using EKF with the optimal Q and R matrices is approximately 0.0353, meeting speed estimation accuracy requirements.

**FIGURE:5** Convergence curves of various optimization algorithms

**TABLE:2** EKF Matrix Values Optimized by IGSO

Iteration	Q Matrix	R Matrix	ISE Value
50	diag([17.533, 6.747, 97.589])	diag([1268.176, 1273.562])	0.0421
100	diag([30.637, 6.632, 118.563])	diag([1551.068, 2334.437])	0.0353
150	diag([32.384, 7.538, 126.594])	diag([1582.433, 2246.436])	0.0349

### 4.3 Speed Estimation Experiment

A motor startup experiment was conducted with a rated speed of 500 rpm after startup. Based on the EKF speed estimator with optimal Q and R matrices, the estimated and actual measured waveforms of  $i_d$  and  $i_q$  are shown in [Figure 6: see original paper]. Due to state covariance matrix convergence issues, the estimated d-q axis currents exhibit oscillations up to 0.03 seconds. After 0.03 seconds, when the matrix converges, the  $i_d$  and  $i_q$  trajectories closely match actual values.

[Figure 7: see original paper] shows the EKF speed estimation curve during startup, demonstrating that the estimated speed matches the actual speed within only 0.05 seconds, after which it remains consistent with the actual speed. This validates the effectiveness of IGSO-optimized EKF.

To further verify the robustness of the EKF estimator to rated speed changes, the startup rated speed was set to 200 rpm and then increased to 500 rpm at 0.2 seconds. The experimental results in [Figure 8: see original paper] show that the EKF-estimated speed quickly converges to actual values under rated speed variations.

## 5 Conclusion

This paper presents a sensorless speed control method for PMSM based on EKF to predict speed and dq-axis stator currents. Since EKF performance primarily depends on the error covariance matrices Q and R, a fast IGSO algorithm is proposed to optimize Q and R, thereby improving system estimation accuracy and convergence. Simulation results demonstrate that the proposed method exhibits excellent performance in settling time, noise immunity, and overall stability.

In this work, motor load and parameters are fixed. In reality, environmental temperature variations cause motor parameter changes. Future work will further verify the robustness of the proposed method to load and parameter variations.

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