EEG Emotion Classification Based on Sparse Representation: Postprint

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Abstract

Research on computer-based recognition of human emotions and affect has become a prominent research focus in the field of brain-computer interfaces. Analyzing various emotional states in human daily life, extracting features from electroencephalogram (EEG) signals, and recognizing and classifying affective states constitute an important direction in affective intelligence. This study investigates the DEAP dataset, which is based on music-video-induced emotions. After extracting frequency-domain features from EEG signals, a sparse representation classification model that employs the Accelerated Proximal Gradient (APG) algorithm and the Orthogonal Matching Pursuit (OMP) algorithm to solve sparse coding is proposed for affective classification, with performance compared against the Support Vector Machine (SVM) algorithm. Experimental results demonstrate that the APG algorithm, through L1-norm regularized approximate solution and its fast convergence speed, achieves favorable classification performance on the affective dataset, while the classification efficacy of the OMP algorithm is comparable to that of the SVM algorithm, thereby accomplishing the classification of affective EEG signals.

Full Text

Emotion Classification of EEG Based on Sparse Representation

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Abstract: Computer recognition of human emotions has become a research hotspot in the field of brain-computer interface (BCI). By analyzing various emotional states in human life, extracting features from EEG signals, and classifying these emotional states represents an important direction in affective intelligence. This study investigates the DEAP dataset, which uses music video induction, extracts frequency-domain features from EEG signals, and proposes a sparse representation classification model solved using the Accelerated Proximal Gradient (APG) and Orthogonal Matching Pursuit (OMP) algorithms for emotion classification, comparing their performance with Support Vector Machine (SVM). Experimental results demonstrate that the APG algorithm, through L1-norm regularization approximation, achieves favorable classification performance on the emotion dataset with rapid convergence speed, while the OMP algorithm yields classification results comparable to SVM, successfully achieving EEG-based emotion classification.

Keywords: EEG; sparse representation; emotion; APG; OMP

0 Introduction

For a long period, research on human emotions was neglected by cognitive scientists. It was not until the late 20th century that emotion, as a crucial component of cognitive processes, began to receive proper attention. Compared with motor function studies, investigating the specific functions of neurons involved in emotional activities presents significantly greater challenges. Advances in neurophysiology and psychology have gradually demystified the origins of emotion. Emotion represents a unique physiological activity in humans, encompassing emotional expression, recognition, and transformation. Computer-based emotion recognition constitutes a vital aspect of artificial intelligence, cognitive science, and natural human-computer interaction.

According to modern cognitive science and psychology, emotion reflects how the human brain responds to external objective 事物, representing an attitudinal experience arising from whether these 事物 meet individual needs. Therefore, studying the relationship between the brain and emotion is essential for future computer-based emotion recognition research. Music, as a combination of human thought, emotion, and artistic creation, has been used by countless musicians throughout history to express diverse emotions, indicating its special significance in human emotion generation and induction. EEG signals can largely reflect changes in psychological and physiological states. Research on bioelectrical signals generated by the central nervous system reveals that EEG signals exhibit specific pattern changes during emotional fluctuations or when receiving different sensory stimuli.

EEG signals are spontaneous, non-stationary, and nonlinear neural electrical activities with frequency variations in low-frequency and ultra-low-frequency rhythms, which can be divided into multiple bands. Common frequency bands include delta (0.5-3 Hz), theta (4-7 Hz), alpha (8-13 Hz), beta (14-30 Hz), and gamma (31–47 Hz), with amplitude ranges of 5–300 V. Karthick et al. [1] collected frontal EEG signals during three states: listening to classical music, listening to rock music, and quiet resting with eyes closed, employing detrended fluctuation analysis (DFA) and multiscale entropy (MSE) to analyze EEG signals, revealing significant differences between music and non-music conditions. Nie et al. [2] designed an experiment using Oscar-winning film clips from different emotional categories as stimuli, extracting frequency-domain features from subjects' EEG signals for positive/negative emotion classification. Sander et al. [3] utilized 40 one-minute music videos as stimulus materials, collecting EEG and physiological signals for emotion recognition research, and established the "DEAP" database for emotion analysis, recording subjects' subjective evaluations and facial expression videos during exposure to different music videos.

1 Emotional Dimensions

Emotions often arise in specific contexts and are constantly changing. The most commonly employed model in current research is the two-dimensional space model based on cognitive theory of emotion: valence and arousal. Valence refers to the pleasantness of emotion, ranging from negative to positive, while arousal refers to the level of excitement, ranging from calm to excited. Danish psychologist Carl Lange proposed a pleasure-excitement two-dimensional space similar to the valence-arousal model, where the horizontal axis represents pleasantness and the vertical axis represents excitement level, with different emotions distributed at various positions in this two-dimensional space, as shown in [Figure 2: see original paper].

Research by Davidson and Petrantonakis et al. [5, 6] demonstrated that the human prefrontal cortex plays a primary role in emotional regulation and conscious experience. Therefore, this study selected four channels from the frontal region: FP1, FP2, F3, and F4 electrodes.

2 DEAP Dataset

The DEAP (database for emotion analysis using physiological signals) database, developed by Sander Koelstra [3], utilizes multimodal physiological signals and music videos for emotion analysis. The physiological signals include multichannel EEG and peripheral physiological signals from subjects. This publicly available emotion EEG dataset enables researchers to test various emotion state classification methods, and this study employs a subset of this database for investigation.

The experiment involved 32 subjects (16 males and 16 females) with an average age of 26.9 years. Subjects were informed of the experimental purpose and details prior to the experiment. The experiment was conducted in a laboratory with controlled lighting and maintained quiet conditions to ensure subjects' emotions remained undisturbed. Subjects sat approximately 1 meter from the screen, with music videos played on a 17-inch LCD display. The music volume was adjusted to an appropriate level through communication with each subject to ensure optimal engagement. After preparation, 40 music video clips were presented following the procedure below.

The experimental procedure consisted of: (a) displaying the music number for 2 seconds to inform subjects of current progress; (b) a 5-second baseline period; (c) playing a 1-minute music video while recording physiological signals at a sampling frequency of 512 Hz; and (d) self-assessment by subjects.

Subjects performed self-assessment using the Self-Assessment Manikin (SAM) system, which recorded ratings for arousal, valence, liking, and dominance for each of the 40 music videos on a scale of 1-9 for each dimension.

According to the international 10–20 electrode system, a 32-channel electrode cap was used for signal acquisition. The experimental data underwent simple preprocessing, with the sampling frequency reduced to 128 Hz. To remove noise, a bandpass filter (4–45 Hz) was first applied to the EEG signals, followed by removal of ocular artifacts using blind source separation. Each music video' s corresponding EEG signal was segmented into 63 seconds, including 60 seconds of music video and 3 seconds of baseline data. The 63-second data comprised 8,064 sampling points; this study removed the initial 3-second baseline data, reducing the sampling points to 7,680.

Given individual differences in music sensitivity, this study analyzed subjects' SAM ratings and selected 10 subjects with higher valence and dominance assessments as test samples: S04, S06, S09, S17, S19, S20, S21, S22, S26, and S27.

In summary, each subject' s EEG data in this study has the form 40×4×7,680, where 40 represents the 40 music videos, 4 represents the selected electrodes (FP1, FP2, F3, F4), and 7,680 represents the sampling points during the 1-minute music video. All 10 subjects' data were processed in MATLAB 2014. [Figure 3: see original paper] shows the original sampling signal for subject S06 at the FP1 channel.

3 Methods

3.1 Algorithm Framework

The algorithmic flowchart of this study is presented in [Figure 4: see original paper].

3.2 z-score Data Standardization

Data standardization is a fundamental preprocessing step. EEG signals typically involve multiple evaluation indicators with different units, necessitating standardization to eliminate dimensional effects on analysis results. Common standardization methods include min-max normalization, log function transformation, and z-score standardization [7]. This study employed standard score (z-score) processing, as shown in Equation (1), where x represents pre-processed data,   denotes the mean,   represents standard deviation, and z indicates standardized data. The processed EEG signal sampling points are scaled to [-1, 1].

$$z = \frac{x - \mu}{\sigma} \quad (1)$$

3.3 Wavelet Transform

EEG signals are typical non-stationary, time-varying, non-Gaussian signals, whereas traditional frequency-domain analysis methods require stationary signals. Consequently, time-frequency analysis methods have been adopted. Wavelet transform (WT), proposed by Grossman and Morlet in 1984, represents a typical time-frequency analysis approach. In 1987, French signal processing expert Mallet introduced multi-scale analysis theory into wavelet analysis, including wavelet function decomposition and signal reconstruction, thereby completing the theoretical framework.

Affective EEG signals result from random combinations of interactions among various neurons in the brain, with this non-linear combination leading to random, non-linear, and non-stationary characteristics. Traditional signal processing methods such as Fourier transform utilize complete time-frequency information, presenting certain limitations. Wavelet transform employs an analytical approach with variable time and frequency windows, making it more suitable for processing non-stationary signals with slow low-frequency changes. For affective EEG signals primarily active in low and ultra-low frequencies, wavelet transform can represent low-frequency information using wide windows at large scales and high-frequency information using narrow windows at small scales, simultaneously presenting detail and approximation coefficients. This enables extraction of different rhythm bands from EEG signals at various scales, demonstrating adaptive characteristics during affective EEG signal processing.

The definition of wavelet transform is given by Equation (2), where a represents the scaling coefficient and   denotes the translation coefficient. Wavelet transform analyzes signals through the mother wavelet function h(t), with representing its complex conjugate. The mother wavelet function selects the analyzed signal through scaling and translation in the time domain, with similar transformations applied in the frequency domain.

$$WT_x(a,\tau) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} x(t) h^* \left(\frac{t-\tau}{a} \right) dt = \int_{-\infty}^{+\infty} x(t) h^*_{a,\tau}(t) dt \quad (2)$$

Wavelet transform decomposes original affective EEG signals into approximation and detail components at different scales by scaling and translating signals within specific subsets. Each layer' s approximation coefficients are further decomposed into new approximation and detail components, iteratively decomposing the signal into n layers, with each layer corresponding to a set of approximation and detail components. For original signal x(t), the wavelet decomposition process is shown in Equation (3), where represents the k-th approximation component at layer n, denotes the k-th detail component at layer j, (t) represents the scaling function, and (t) is the scaling function.

$$x(t) = \sum_{k=-\infty}^{\infty} C_{n,k}\sigma_{n,k}(t) + \sum_{j=1}^{n}\sum_{k=-\infty}^{\infty} D_{j,k}\varphi_{j,k}(t) \quad (3)$$

The DEAP affective EEG data in this study have been preprocessed with a sampling frequency of 128 Hz. According to the Nyquist sampling theorem, the effective frequency is half the sampling frequency, i.e., 64 Hz. Based on Equation (3), EEG signals were decomposed into four layers. From the detail and approximation components of each layer, five bands were extracted: delta (0.5-3 Hz), theta (4-7 Hz), alpha (8-13 Hz), beta (14-30 Hz), and gamma (31-47 Hz). The db4 wavelet was selected as the basis function. [Figure 5: see original paper] shows the corresponding 4-layer wavelet frequency-domain decomposition tree, while [Figure 6: see original paper] and [Figure 7: see original paper] display the approximation and detail coefficients from wavelet decomposition of subject S06’ s original EEG signal.

As shown in [Figure 5: see original paper], the original 64 Hz signal is decomposed into four layers, with each layer comprising approximation components (SRC) and detail components (SRD). The alpha band distributes on detail component SRD2, beta on SRD3, theta on SRD1, gamma on SRD4, and delta on approximation component SRC1.

3.4 Power Spectrum Analysis

Power spectrum analysis represents a primary analytical method for EEG signal research and clinical applications. Frequency-domain analysis transforms amplitude-varying time-domain signals into frequency-domain spectra showing power variation over time. Classical spectrum estimation methods employ periodograms based on Fourier transforms of finite time segments, using windowed truncation to estimate signal power spectra. According to the Wiener-Khinchin theorem, the power spectrum of signal x(n) and its autocorrelation function form a Fourier transform pair, defined by Equation (4). This method treats N observed data points of random signal x(n) as an energy-limited sequence, computes the Fourier transform of x(n), and uses the squared magnitude divided by N as the true power spectrum estimate of sequence x(n).

$$P_{xx}(w) = \frac{1}{N}\left|\sum_{n=0}^{N-1} x(n)e^{-jwn}\right|^2 = \frac{1}{N}|X(w)|^2 \quad (4)$$

The Fourier transform of x(n) is periodic, making I_N(w) periodic as well, and it represents a biased estimate. First, the Fourier transform (spectrum) of N data points is calculated as shown in Equation (5).

$$X(w) = \sum_{n=0}^{N-1} x(n)e^{-jwn} \quad (5)$$

Then, the power spectrum is obtained by multiplying the spectrum with its conjugate function, as shown in Equation (6).

$$P_{xx}(w) = \frac{1}{N}X(w)X^*(w) = \frac{1}{N}|X(w)|^2 \quad (6)$$

Based on classical spectrum estimation methods, this study calculated power spectra for the five extracted rhythmic waveforms and selected them as features, yielding a total of $4{\times}5 = 20$ features across four channels.

3.5 Sparse Representation

Sparse representation was first proposed by Olshausen and Field [9], with the fundamental principle of representing input signals as linear combinations of overcomplete bases selected from a dictionary. For an N-dimensional vector x, if most elements are zero with only a few non-zero elements, x is considered sparse. In 2006, Donoho proposed compressed sensing (CS) theory [10], advancing sparse signal representation. Compressed sensing liberates signal acquisition and measurement from Nyquist sampling theorem limitations, replacing strict bandwidth requirements with data sparsity [11].

Affective EEG signals exhibit small amplitudes and significant noise interference, making useful feature extraction crucial. Consider dataset D as a matrix where each row represents a sample and each column represents a feature. Feature selection must consider these features' "sparsity"—removing features irrelevant to the learning task reduces training difficulty and computational/storage overhead. Conversely, if D contains many zero elements not in entire rows or columns, the dataset remains sparse. When samples possess such sparse representations, most problems become linearly separable while storage burden decreases significantly, as efficient storage methods exist for sparse matrices. Based on compressed sensing theory, signal sampling requires less than half the original samples to reconstruct the original signal with high quality. This study leverages the sparse (or compressible) characteristics of different affective EEG rhythms as prior conditions, enabling high-probability reconstruction of original EEG signals or high-dimensional signals through different decoding algorithms while preserving important rhythmic features highly correlated with emotion.

The sparse representation problem can be realized by solving sparse regularization problems, as shown in Equation (7).

$$\min_{a \in A} g(a) + \lambda\Omega(a) \quad (7)$$

where Ω represents the sparse regularization term,   is a non-negative parameter, A is a convex set, and a is a coefficient in A. For coefficient a to be sparse, Ω can take the L0 norm, as shown in Equation (8).

$$\Omega(a) = |\{k|a_k \neq 0\}| \quad (8)$$

where | | denotes set cardinality. The error term f(a) typically uses squared error, as shown in Equation (9).

$$f(a) = \frac{1}{2}\|x - Da\|^2 \quad (9)$$

The model using sparse representation for classification is called Sparse Representation-based Classification (SRC) [12], which primarily represents input signals as sparse linear combinations of overcomplete basis atoms, as shown in Equation (10).

$$\min \|x\|_0 \quad \text{subject to} \quad Ax = y \quad (10)$$

where |x|_0 denotes the number of non-zero elements in x, i.e., the L0 norm. While theoretically feasible, L0 norm sparsity representation is mathematically challenging. Common practice converts the L0 norm to L1 regularization, as shown in Equation (11).

$$\min \|x\|_1 \quad \text{subject to} \quad Ax = y \quad (11)$$

Equation (11) transforms into an optimal convex approximation problem, typically obtained through L1 regularization approximation, as shown in Equation (12).

$$\min_{a \in \mathbb{R}^p} \frac{1}{2}\|x - Da\|^2 + \lambda\|a\|_1 \quad (12)$$

Equation (12) represents the Lasso problem in statistics, which can be viewed as a regularized least squares problem. This study uses original affective EEG signals as an overcomplete basis, leveraging the redundant characteristics of overcomplete bases to better capture essential features in EEG signals. The SRC classification algorithm is proposed for affective EEG signal classification.

The specific SRC classification framework proceeds as follows:

a) Input training sample matrix A = [A_1, A_2, ···, A_k] ^{m×n} and test sample y ^m;

b) Normalize matrix A columns to have L2 norm;

c) Solve the L1 norm minimization problem: , where is a constant representing reconstruction error constraint;

d) Compute residuals: for i = 1, 2;

e) Determine test sample label based on reconstruction error redundancy between reconstructed and test samples.

3.5.1 Greedy Algorithm Solution Greedy algorithms are widely used in compressed sensing and sparse signal reconstruction due to their efficiency. The fundamental principle involves iteratively selecting dictionary atoms most similar to the signal to approximate the original signal. Greedy algorithms mainly include Matching Pursuit (MP) [13], Orthogonal Matching Pursuit (OMP) [14], and Stagewise Orthogonal Matching Pursuit [15].

MP algorithms select the vector most correlated with the error vector in each iteration, compute residuals, then project onto the atom most matching the residual to form coefficient increments. Specific steps are:

a) Initialize error vector r_0 = x, iteration count t = 1;

b) Search for atom index most correlated with error vector: ;

c) Update reconstruction coefficients and error vector: ;

d) Increment t = t + 1, return to step b) unless termination condition met.

In MP algorithms, the updated error vector is orthogonal to the currently selected atom (i.e.,), but orthogonality with other atoms cannot be guaranteed. Therefore, signals in each iteration within the subspace spanned by selected dictionary atoms may be suboptimal. In other words, MP algorithms cannot guarantee optimal signal approximation [16].

Addressing these issues, Pati proposed the OMP algorithm [14], which adds newly selected atoms to an active set of previously selected atoms, stopping iteration when required sparsity is achieved. The next atom is then selected based on maximum reduction of the objective function value, as shown in Equation (13).

$$l_t = \arg\min_{l \in \Lambda^c} \|x - D_{\Lambda \cup \{l\}} \beta\|^2 \quad (13)$$

where Λ represents the active set of selected atoms. The OMP algorithm solves the least squares problem shown in Equation (14).

$$\min_{\beta \in \mathbb{R}^{|\Lambda|}} \|x - D_\Lambda \beta\|^2 \quad (14)$$

The OMP algorithm requires solving a least squares problem, with updated active set, error term, and reconstruction coefficients.

3.5.2 Convex Optimization Solution Algorithm For unconstrained convex optimization problems, solutions can be obtained through linear programming, gradient descent, and other convex optimization methods. When the objective function comprises a differentiable function f with Lipschitz continuous gradient and a non-differentiable function, the Accelerated Proximal Gradient (APG) method is particularly suitable.

During each iteration, when f is linearized at the current point, the solution transforms to Equation (15), where the quadratic term represents the proximal term that keeps the deviation between updated f and linear approximation within a certain range, and L_f is the Lipschitz constant.

$$a_{t+1} = \arg\min_{a \in \mathbb{R}^p} \left\{ f(a_t) + \langle \nabla f(a_t), a - a_t \rangle + \frac{L_f}{2} \|a - a_t\|^2 + \lambda \Omega(a) \right\} \quad (15)$$

Equation (15) can be rewritten as Equation (16).

$$a_{t+1} = \arg\min_{a \in \mathbb{R}^p} \left\{ \frac{L_f}{2} \left\| a - \left(a_t - \frac{1}{L_f} \nabla f(a_t) \right) \right\|^2 + \lambda \Omega(a) \right\} \quad (16)$$

The APG algorithm steps for solving sparse representation problems are:

a) Compute: ;

b) Solve: ;

c) Update learning rate: ;

d) Update iteration result: ;

Exit if iteration condition satisfied. Beck and Teboulle proved that the APG algorithm converges at rate O(1/t^2), demonstrating its effectiveness for solving sparse problems [18].

4 Emotion Recognition Experimental Results and Analysis

This study performed emotion classification based on the DEAP affective database. Data from 10 subjects were processed into the form 40×4×7,680, where 40 represents the 40 music videos viewed, 4 indicates the four frontal channels, and 7,680 denotes the one-minute sampling data. Based on each subject' s SAM ratings, emotional states were categorized into four classes: high valence-high arousal, high valence-low arousal, low valence-high arousal, and low valence-low arousal. Binary and four-class classification experiments were conducted, with binary classification primarily using the valence dimension to identify positive/negative emotional states.

This study extracted alpha, beta, theta, gamma, and delta rhythms via wavelet transform and calculated each waveform' s power spectrum, using the mean value as features. Post-feature-extraction data became $40\times4\times5$, where 5 represents the average power spectrum values of five rhythms per channel. Different waveforms show distinct power spectrum differences between positive and negative emotions, as illustrated in [Figure 9: see original paper], which reveals higher power spectrum values for theta, alpha, and beta rhythms under negative emotions, while delta and gamma rhythms become prominent during positive emotion transitions.

Since EEG signals are typical non-linear signals, Support Vector Machine (SVM) has become a mainstream algorithm for affective EEG classification due to its advantages in non-linear data processing and good generalization capability. This study compares the proposed sparse representation classification algorithm with classical SVM. Feature data were imported into MATLAB 2014' s SPAMS sparse modeling toolbox and Libsvm toolbox, conducting binary and four-class experiments using SVM, OMP-decoded sparse classification, and APG-decoded sparse classification. The SVM algorithm employed radial basis function kernel [19], and all three classification methods used 10-fold cross-validation. [Figure 10: see original paper] presents the accuracy comparison after cross-validation.

As shown in [Figure 10: see original paper], the accelerated proximal gradient method (APG) in sparse classification demonstrates superior performance. In binary classification, the highest accuracy reaches 85% with an average of approximately 70%; in four-class classification, the highest accuracy reaches 67.25% with an average of approximately 57.5%. The orthogonal matching pursuit algorithm (OMP) and support vector machine (SVM) show inferior average classification accuracies: 64.2% and 64.5% for binary classification, and 48.55% and 44% for four-class classification, respectively.

This study extracted power spectra from five frequency bands as features. [Figure 11: see original paper] and [Figure 12: see original paper] present classification accuracies of APG and OMP algorithms when using individual bands as features. The gamma rhythm demonstrates the best and most stable performance. When using gamma rhythm power spectrum as features, APG achieves 64.2% average accuracy in binary classification and 47% in four-class classification, while OMP achieves 63.25% and 43.3%, respectively. These results confirm gamma rhythm' s important role in emotion recognition [20].

Comparative analysis between using individual rhythm power spectra versus fused five-rhythm power spectra as features reveals that the fused approach yields better classification performance.

5 Conclusion

This paper detailed the DEAP affective dataset induced by music videos, extracted frequency-domain features through wavelet transform and fast Fourier transform after preprocessing, and applied sparse representation classification algorithms solved by OMP and APG for classification, comparing results with SVM. EEG signals are reconstructed via sparse vectors and compared with test samples for classification through redundancy error. Experimental results demonstrate that the accelerated proximal gradient algorithm (APG), through L1-norm regularization approximation, achieves favorable classification performance on the emotion dataset with rapid convergence speed. The greedy algorithm-based orthogonal matching pursuit (OMP) achieves comparable results to SVM, successfully realizing EEG-based emotion classification. Emotion classification and recognition from EEG signals represents a worthwhile research domain, and classification based on sparse representation reconstruction proves effective and warrants further in-depth investigation.

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Note: Figure translations are in progress. See original paper for figures.