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# Uplink-Downlink Duality-Based CRN Downlink Power Allocation and Beamforming Algorithm Postprint

**Authors:** Ji Zhongheng, Ji Xinsheng, Huang Kaizhi

**Date:** 2018-05-02T00:00:00+00:00

## Abstract

In research on multi-user downlink power allocation and beamforming in cognitive radio networks (CRN) operating in underlay mode, the generic semidefinite relaxation (SDR) algorithm exhibits high computational complexity and limited practicality, while the optimization problem neglects interference from the primary network (PN) to secondary users (SU). To address these issues, interference from PN to SU is first incorporated into the CRN network model to formulate an optimization problem. Subsequently, based on uplink-downlink duality and employing virtual power, the optimization problem is transformed into an uplink power allocation and beamforming problem, yielding an iterative algorithm that can be solved simply and efficiently. The convergence properties of the algorithm are analyzed and convergence conditions are derived. Furthermore, the computational complexity of the algorithm is evaluated, demonstrating its superiority over the SDR algorithm. Numerical simulations show rapid convergence of the algorithm and reveal that variations in the transmit power of the primary base station (PBS) affect the feasible solution region. Specifically, increased PBS transmit power leads to increased CRN downlink power, which has a significant impact.

## Full Text

## Preamble

### CRN Downlink Power Allocation and Beamforming Algorithm Based on Uplink-Downlink Duality

*Ji Zhongheng, Ji Xinsheng, Huang Kaizhi*
(National Digital Switching Systems Engineering & Technological Research Center, Zhengzhou 450002, China)

**Abstract:** Research on multi-user downlink power allocation and beamforming in cognitive radio networks (CRNs) operating in underlay mode faces several challenges: the conventional semidefinite relaxation (SDR) algorithm suffers from high computational complexity and limited practicality, and existing optimization formulations neglect interference from the primary network (PN) to secondary users (SUs). To address these issues, we first augment the CRN network model to include PN-to-SU interference and formulate an optimization problem. Then, leveraging the duality between uplink and downlink channels and introducing virtual power, we transform the optimization problem into an uplink power allocation and beamforming problem, yielding a simple and fast iterative solution. We analyze the convergence properties to derive convergence conditions and compute the algorithm' s complexity, demonstrating its superiority over SDR. Numerical simulations show rapid convergence and reveal that variations in primary base station (PBS) transmit power affect the feasible solution region. Specifically, increased PBS transmit power leads to higher CRN downlink power, significantly impacting system performance.

**Keywords:** cognitive radio network; virtual power; iterative algorithm; complexity; feasible solution region

## 0 Introduction

Cognitive radio network (CRN) technology enables efficient utilization of wireless spectrum, allowing secondary users (SUs) to access spectrum without affecting primary users (PUs). This creates two coexisting networks in the same spectral space: a primary network (PN) composed of PUs and a CRN composed of SUs. CRNs operate in three modes: interweave, overlay, and underlay. In interweave mode, CRNs can only access frequency points unused by the PN and must vacate when PUs become active, potentially causing frequent access/exit cycles that disrupt normal operation. In overlay mode, CRNs require knowledge of PU codebooks and messages to eliminate interference through coding, a condition often unmet in practice, thus impacting PN performance. In underlay mode, CRNs control their transmission parameters based on channel conditions to share spectrum with PNs.

Multi-user power allocation and beamforming in underlay CRNs represents a key research focus. When the numbers of SUs and PUs are small, zero-forcing (ZF) algorithms can achieve spectrum sharing between CRN and PN. However, as these numbers increase, ZF becomes inadequate, necessitating constrained optimization for spectrum sharing. The optimization objective is minimizing downlink power (i.e., CRN base station (CBS) transmit power), with constraints including CRN quality-of-service (QoS) measured by signal-to-interference-plus-noise ratio (SINR), beamforming matrices, and total interference from CBS to PUs.

Existing research suffers from three main problems. First, the generic semidefinite relaxation (SDR) algorithm for solving these optimization problems has high computational complexity, limiting practicality. Second, optimization formulations often ignore PN-generated interference to SUs, justified by PN' s higher service priority. However, failing to incorporate PN interference into CRN operational constraints makes CRN QoS conditions non-rigorous, leading to significant uncertainty when applying optimization results in practice. Third, while non-SDP algorithms exist (e.g., iterative algorithms in literature), they lack explicit convergence conditions and complexity analysis.

Addressing these issues, this paper further studies downlink power allocation and beamforming in underlay CRNs. Following references [11,12], we establish a CRN network model that adds PN-to-SU interference terms to the constraints, formulate the optimization problem, and then—building on [7,10]—apply uplink-downlink duality principles. By introducing virtual power, we transform the downlink problem into an uplink virtual power allocation and beamforming problem. We first obtain uplink virtual power and beamforming vectors, then derive downlink power through corresponding transformations to establish an iterative relationship, enabling fast and simple optimization solution. We analyze both convergence properties and computational complexity, deriving convergence conditions and complexity metrics.

Numerical simulations verify the algorithm' s convergence, analyze the relationship between PBS transmit power and feasible solution region, and examine the relationship between downlink power and PBS transmit power. Results demonstrate rapid convergence within dozens of iterations, show that PBS transmit power variations significantly affect the feasible solution region (with increased power reducing the region), and reveal that PBS transmit power notably impacts downlink power, particularly when power increases substantially.

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## 1.1 Network Model

The network model is shown in [Figure 1: see original paper]. The CRN consists of one CBS and multiple SUs, while the PN comprises one PBS and multiple PUs. The CBS and PBS share spectrum. Although real networks may have multiple PBSs, interference from other PBSs is far less significant than that from the PBS sharing spectrum with the CBS, making their impact negligible for analysis purposes.

Let $L$ denote the number of PUs and $K$ the number of SUs. The CBS has $N_t$ transmit antennas, the PBS has a single transmit antenna, and all PUs and SUs have single receive antennas. The channel vector from CBS to SU $k$ is $\mathbf{h}_k \in \mathbb{C}^{N_t \times 1}$, the channel vector from PBS to SU $k$ is $u_k$ (dimension 1), and the channel vector from CBS to PU $l$ is $\mathbf{h}_{K+l} \in \mathbb{C}^{N_t \times 1}$. The PBS transmit power is $p_p$.

The signal transmitted by CBS to SU $k$ is $s_k(t)$, the PBS transmit signal is

$s_p(t)$, and additive white Gaussian noise is $z_k(t) \sim \mathcal{CN}(0, \sigma_k^2)$. The interference from CBS to PU $l$ is $\sum_{i=1}^{K} p_i \mathbf{h}_{K+l}^H \mathbf{u}_i s_i(t)$. The SINR at SU $k$ is:

$$\mathrm{SINR}_k = \frac{p_k |\mathbf{h}_k^H \mathbf{u}_k|^2}{\sum_{i \neq k} p_i |\mathbf{h}_k^H \mathbf{u}_i|^2 + p_p |u_k|^2 + \sigma_k^2}$$

In practice, instantaneous channel characteristics are difficult to obtain, so expectations are typically used. Therefore, we define: the channel correlation matrix between SU and CBS as $\mathbf{R}_k = \mathbb{E}\{\mathbf{h}_k \mathbf{h}_k^H\}$, and between SU and PBS as $R_{pk} = \mathbb{E}\{|u_k|^2\}$. Replacing $\mathbf{h}_k$ and $u_k$ with their statistical equivalents, the interference from CBS to PU $l$ becomes $\sum_{i=1}^{K} p_i \mathbf{h}_{K+l}^H \mathbf{u}_i$, and the SINR at SU $k$ becomes:

$$\mathrm{SINR}_k = \frac{p_k \mathbf{u}_k^H \mathbf{R}_k \mathbf{u}_k}{\sum_{i \neq k} p_i \mathbf{u}_i^H \mathbf{R}_k \mathbf{u}_i + p_p R_{pk} + \sigma_k^2}$$

Similarly, we define the channel correlation matrix between PU and CBS as $\mathbf{R}_{K+l} = \mathbb{E}\{\mathbf{h}_{K+l} \mathbf{h}_{K+l}^H\}$.

Two notes on the CRN model: 1) $\mathbf{R}_k$ has the following properties: it is a Hermitian matrix and is positive definite/semi-definite. 2) Unlike literature [9] which represents PN interference with a random value, our CRN model incorporates PBS interference, more accurately describing $R_{pk}$. The time-varying nature of channels between PBS and SUs makes the interference term significant, further affecting $\mathrm{SINR}_k$, $\mathbf{u}_k$, and $p_k$, yielding more realistic optimization results.

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## 1.2 Optimization Problem

The downlink power allocation and beamforming optimization problem is formulated as:

**Problem P1:**

$$\begin{aligned} \min_{\mathbf{p}, \mathbf{U}} \quad & \sum_{k=1}^{K} p_k \\ \text{s.t.} \quad & \mathrm{SINR}_k \geq \gamma_k, \quad k = 1, 2, \dots, K \\ & \sum_{i=1}^{K} p_i \mathbf{h}_{K+l}^H \mathbf{u}_i \leq \Gamma_l, \quad l = 1, 2, \dots, L \\ & p_k \geq 0, \quad k = 1, 2, \dots, K \end{aligned}$$

where $\mathbf{p} = [p_1, p_2, \dots, p_K]^T$ is the downlink power vector for $K$ SUs, $\mathbf{U} = [\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_K]$ is the beamforming matrix, $\gamma_k$ is the SINR threshold for SU $k$, and $\Gamma_l$ is the interference threshold for PU $l$.

Introducing slack variables $\mathbf{p}_L = [p_{K+1}, p_{K+2}, \dots, p_{K+L}]^T$ as virtual downlink power for PUs, we obtain **Problem P2**:

**Problem P2:**

$$\begin{aligned}
\min_{\mathbf{p},\mathbf{U}} \quad & \sum_{k=1}^{K} p_k \\
\text{s.t.} \quad & \text{SINR}_k \geq \gamma_k, \quad k = 1, 2, \dots, K \\
& \frac{\sum_{i=1}^{K} p_i \mathbf{h}_{K+l}^H \mathbf{u}_i}{p_{K+l}} \leq 1, \quad l = 1, 2, \dots, L \\
& p_k \geq 0, \quad k = 1, 2, \dots, K + L
\end{aligned}$$

Using the statistical characteristics, Problem P1 is transformed into **Problem P3**:

**Problem P3:**

$$\begin{aligned}
\min_{\mathbf{p},\mathbf{U}} \quad & \sum_{k=1}^{K} p_k \\
\text{s.t.} \quad & \frac{p_k \mathbf{u}_k^H \mathbf{R}_k \mathbf{u}_k}{\sum_{i \neq k} p_i \mathbf{u}_i^H \mathbf{R}_k \mathbf{u}_i + p_p R_{pk} + \sigma_k^2} \geq \gamma_k, \quad k = 1, 2, \dots, K \\
& \frac{\sum_{i=1}^{K} p_i \mathbf{u}_i^H \mathbf{R}_{K+l} \mathbf{u}_i}{p_{K+l}} \leq 1, \quad l = 1, 2, \dots, L \\
& p_k \geq 0, \quad k = 1, 2, \dots, K + L
\end{aligned}$$

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## 1.3 Problem Transformation Based on Uplink-Downlink Duality

According to uplink-downlink duality properties [10] and following literature [7], we employ uplink virtual power to transform downlink Problem P3 into virtual uplink Problem P4.

**Problem P4:**

$$\begin{aligned}
\min_{\mathbf{q},\mathbf{U}} \quad & \sum_{k=1}^{K} q_k \\
\text{s.t.} \quad & \frac{q_k \mathbf{u}_k^H \mathbf{R}_k \mathbf{u}_k}{\sum_{i \neq k} q_i \mathbf{u}_i^H \mathbf{R}_k \mathbf{u}_i + \sum_{l=1}^{L} q_{K+l} \mathbf{u}_k^H \mathbf{R}_{K+l} \mathbf{u}_k + \sigma_k^2} \geq \gamma_k, \quad k = 1, 2, \dots, K \\
& \frac{\sum_{i=1}^{K} q_i \mathbf{u}_i^H \mathbf{R}_{K+l} \mathbf{u}_i}{q_{K+l}} \leq 1, \quad l = 1, 2, \dots, L \\
& q_k \geq 0, \quad k = 1, 2, \dots, K + L
\end{aligned}$$

where $\mathbf{q} = [q_1, q_2, \dots, q_K]^T$ is the virtual uplink power for SUs and $\mathbf{q}_L = [q_{K+1}, q_{K+2}, \dots, q_{K+L}]^T$ is the virtual uplink power for PUs.

Two notes on Problem P4: a) Problems P4 and P3 share the same optimal beamforming vectors $\mathbf{u}_k$. b) The downlink power $p_k$ in P3 is obtained from the following relationship:

From P4, we can conveniently obtain $\mathbf{u}_k$. When $\mathbf{q}$ is fixed, the beamforming vector $\mathbf{u}_k$ should maximize the SINR; when $\mathbf{U}$ is fixed, $q_k$ should satisfy the SINR constraint with minimal value. The minimum requirement is equality to 1, leading to iterative update equations for $q_k$ and $\mathbf{u}_k$.

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## 2.1 Algorithm Idea

The iterative algorithm proceeds as follows: (a) Initialize $\mathbf{q}^{(0)}$; (b) Iterate with index $t$ until convergence. The core idea is that when virtual uplink power $\mathbf{q}$ is fixed, the beamforming vector $\mathbf{u}_k$ should maximize the SINR expression; when beamforming vectors $\mathbf{U}$ are fixed, the power allocation should satisfy $\mathrm{SINR}_k \geq \gamma_k$ with minimal $q_k$. Since the minimum requirement is $\mathrm{SINR}_k = \gamma_k$, we derive iterative update equations for $\mathbf{u}_k$ and $q_k$.

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## 2.2 Algorithm Flow

The iterative algorithm is as follows:

**a) Initialization:** Set initial virtual uplink power $\mathbf{q}^{(0)} = [q_1^{(0)}, q_2^{(0)}, \dots, q_{K+L}^{(0)}]^T$.

**b) Iteration:** For iteration $t = 1, 2, \dots$, perform the following steps until convergence:

**(a) Beamforming vector update:** For each SU $k = 1, 2, \dots, K$, solve:

$$\mathbf{u}_k^{(t)} = \arg\max_{\mathbf{u}_k} \frac{\mathbf{u}_k^H \mathbf{R}_k \mathbf{u}_k}{\mathbf{u}_k^H \left( \sum_{i \neq k} q_i^{(t-1)} \mathbf{R}_i + \sum_{l=1}^{L} q_{K+l}^{(t-1)} \mathbf{R}_{K+l} + \sigma_k^2 \mathbf{I} \right) \mathbf{u}_k}$$

The solution is the eigenvector corresponding to the maximum generalized eigenvalue of the matrix pair $(\mathbf{R}_k, \sum_{i \neq k} q_i^{(t-1)} \mathbf{R}_i + \sum_{l=1}^{L} q_{K+l}^{(t-1)} \mathbf{R}_{K+l} + \sigma_k^2 \mathbf{I})$.

**(b) SU virtual uplink power update:** For $k = 1, 2, \dots, K$:

$$q_k^{(t)} = \frac{\gamma_k}{\mathbf{u}_k^{(t)H} \mathbf{R}_k \mathbf{u}_k^{(t)}} \left( \sum_{i \neq k} q_i^{(t-1)} \mathbf{u}_i^{(t)H} \mathbf{R}_k \mathbf{u}_i^{(t)} + \sum_{l=1}^{L} q_{K+l}^{(t-1)} \mathbf{u}_k^{(t)H} \mathbf{R}_{K+l} \mathbf{u}_k^{(t)} + \sigma_k^2 \right)$$

**(c) Downlink power conversion:** The downlink power is obtained through:

$$\mathbf{p}^{(t)} = \mathbf{D}^{-1}(\mathbf{I} - \mathbf{G})^{-1}\mathbf{1}$$

where matrices $\mathbf{D}$ and $\mathbf{G}$ are defined as:

$$[\mathbf{D}]_{kk} = \frac{\mathbf{u}_k^{(t)H}\mathbf{R}_k\mathbf{u}_k^{(t)}}{\gamma_k}, \quad k = 1, 2, \ldots, K$$

$$[\mathbf{G}]_{lj} = \frac{\mathbf{u}_j^{(t)H}\mathbf{R}_{K+l}\mathbf{u}_j^{(t)}}{\Gamma_l}, \quad l = 1, 2, \ldots, L,\ \ j = 1, 2, \ldots, K$$

$$[\mathbf{G}]_{ij} = \begin{cases} \frac{\mathbf{u}_j^{(t)H}\mathbf{R}_i\mathbf{u}_j^{(t)}}{\gamma_i}, & i \neq j \\ 0, & i = j \end{cases}, \quad i, j = 1, 2, \ldots, K$$

**(d) PU virtual uplink power update:** For $l = 1, 2, \ldots, L$:

$$q_{K+l}^{(t)} = \max\left\{0, \frac{p_{K+l}^{(t-1)} - p_{K+l}^{(t)}}{p_{K+l}^{(t)}}\right\}$$

**(e) Convergence check:** If $\max_{k=1,\ldots,K+L} |q_k^{(t)} - q_k^{(t-1)}| \leq \delta$ and $\max_{l=1,\ldots,L} |p_{K+l}^{(t)} - p_{K+l}^{(t-1)}| \leq \delta$, where $\delta$ is a predefined convergence threshold, proceed to step **d)**; otherwise, return to step **b)**.

**d) Upon convergence after $N$ iterations, obtain the optimal solution:**

**(a) Optimal power allocation: $\mathbf{p}^* = \mathbf{p}^{(N)}$**

**(b) Corresponding beamforming vectors: $\mathbf{U}^* = \mathbf{U}^{(N)}$**

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## 2.3 Algorithm Convergence Analysis

**Theorem:** Algorithm convergence requires $\mu_k^{(t)} > 0$, where the convergence process in iteration $t$ proceeds as follows:

**Proof:** From the beamforming update step, we have:

$$\mu_k^{(t)} = \frac{\mathbf{u}_k^{(t)H}\mathbf{R}_k\mathbf{u}_k^{(t)}}{\sum_{i\neq k} q_i^{(t-1)}\mathbf{u}_i^{(t)H}\mathbf{R}_k\mathbf{u}_i^{(t)} + \sum_{l=1}^{L} q_{K+l}^{(t-1)}\mathbf{u}_k^{(t)H}\mathbf{R}_{K+l}\mathbf{u}_k^{(t)} + \sigma_k^2}$$

The function $\mu_k^{(t)}$ is monotonically decreasing with respect to $q_i^{(t-1)}$ $(i \neq k)$ and $q_{K+l}^{(t-1)}$. From the properties of $\mathbf{R}_k$, we know $\mu_k^{(t)} > 0$.

At iteration step $t$, there are two cases for $\mu_k^{(t)}$:

**Case (a):** $\mu_k^{(t)} \geq 1$
**Case (b):** $0 < \mu_k^{(t)} < 1$

For case (a), $q_k^{(t)} = \mu_k^{(t)} q_k^{(t-1)} \geq q_k^{(t-1)}$, so $q_k^{(t)}$ is non-decreasing. For case (b), $q_k^{(t)} = \mu_k^{(t)} q_k^{(t-1)} < q_k^{(t-1)}$, so $q_k^{(t)}$ decreases.

In the iteration process, case (a) will approach $\mu_k^{(t)} = 1$ as $q_k^{(t)}$ increases. For case (b), either $\mu_k^{(t)}$ will increase after several iterations until the inequality sign changes, making it case (a), or $q_k^{(t)} \to 0$. However, $q_k^{(t)} \to 0$ would violate the SINR constraint, contradicting the premise. Therefore, $\lim_{t\to\infty} \mu_k^{(t)} = 1$ for all $k$, implying $\lim_{t\to\infty}(q_k^{(t)} - q_k^{(t-1)}) = 0$.

Similarly for PU virtual power, we have $\lim_{t\to\infty}(q_{K+l}^{(t)} - q_{K+l}^{(t-1)}) = 0$. Thus, the algorithm converges.

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## 2.4 Algorithm Complexity Analysis

Using multiplication and addition as basic operation units, the per-iteration computational complexity of each algorithm step [13] is:

**Step (a) Beamforming update:** Solving for the maximum generalized eigenvalue and eigenvector. Using the power method from [14], the most computationally intensive operation is matrix inversion of an $N_t \times N_t$ matrix, yielding complexity $O(N_t^3)$. For $K$ SUs, total complexity is $O(KN_t^3)$.

**Step (b) SU power update:** Complexity is $O(K^2N_t^2 + KLN_t^2)$.

**Step (c) Downlink conversion:** Matrix inversion $(\mathbf{I} - \mathbf{G})^{-1}$ has complexity $O(K^3)$. Matrix multiplication $\mathbf{D}^{-1}(\mathbf{I} - \mathbf{G})^{-1}$ has complexity $O(K^2)$. Total complexity is $O(K^3 + K^2)$.

**Step (d) PU power update:** Complexity is $O(L)$.

**Step (e) Convergence check:** Complexity is $O(K + L)$.

Therefore, per-iteration complexity for steps (a)-(e) is:

$$O(KN_t^3 + K^2N_t^2 + KLN_t^2 + K^3)$$

Typically, $N_t$ and $K$ are of the same order, with $N_t > K$. We can neglect lower-order terms $K^2$, $KL$, $L$, etc., yielding per-iteration complexity $O(KN_t^3)$.

Including initialization, total algorithm complexity is $O(N \cdot KN_t^3)$, where $N$ is the number of iterations.

From literature [7], SDR algorithm complexity is $O((K+L)^3 N_t^3)$. Thus, the iterative algorithm' s complexity is significantly lower than SDR' s.

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## 3 Numerical Simulation and Results Analysis

Simulation parameters: $K = 5$ SUs, $L = 5$ PUs, $N_t = 9$ antennas at CBS, PU interference threshold $\Gamma_l = -6$ dB, SU QoS constraint $\gamma_k \in [0.1, 1.5]$, noise variance $\sigma_k^2 = 1$. Channels are Rayleigh fading. For each channel realization, results are averaged over 100 independent Monte Carlo runs. PBS transmit power $p_p = 30$ dBm. Convergence threshold $\delta = 10^{-6}$.

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### 3.1 Convergence of Iterative Algorithm

Convergence thresholds and iteration counts are shown in .

**Table 1: Iteration counts for different convergence thresholds**

| Group | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| $\delta$ | $10^{-3}$ | $10^{-4}$ | $10^{-5}$ | $10^{-6}$ | $10^{-7}$ | $10^{-8}$ | $10^{-9}$ |
| Iterations | 12 | 18 | 25 | 36 | 48 | 61 | 75 |

The algorithm converges rapidly. For $\delta = 10^{-6}$, convergence is achieved in only 36 iterations. Compared to SDR, the iterative algorithm reduces computational load while offering fast convergence.

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### 3.2 PBS Transmit Power and Feasible Solution Region

The feasible solution region is measured by feasibility percentage—the percentage of random channel realizations for which the algorithm finds a feasible solution. This metric reflects algorithm adaptability to channel variations.

[Figure 2: see original paper] shows that as PBS transmit power $p_p$ varies, the feasible solution region changes. Introducing $p_p$ affects the SINR in the optimization problem, which in turn affects CRN interference constraints, shrinking the feasible region. For feasibility percentage = 0.6, when $p_p = 30$ dBm, the corresponding $\gamma_k \approx 4.0$ dB; when $p_p = 27$ dBm, $\gamma_k$ increases to about 4.7 dB; without PBS ($p_p = 0$), $\gamma_k$ can reach about 7.0 dB. For $p_p = 30$ dBm and $p_p = 27$ dBm, feasibility percentage becomes 0 at $\gamma_k = 7.0$ dB because achieving this SINR requires increased downlink power, which raises CBS-to-PU interference beyond the constraint, making the problem infeasible. Thus, PBS power significantly impacts the feasible solution region.

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### 3.3 PBS Transmit Power and Downlink Power

[Figure 3: see original paper] shows that as PBS transmit power $p_p$ increases, CRN downlink power increases. For $p_p \leq 30$ dBm, downlink power increases gradually; for $p_p > 30$ dBm, the increase accelerates. This occurs because higher $p_p$ increases PBS-to-SU interference, forcing CRN to raise downlink power to maintain SU SINR constraints. This demonstrates that PBS transmit power significantly impacts SU QoS in CRN systems.

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## 4 Conclusion

This paper studied downlink power allocation and beamforming in underlay CRNs, deriving a simple, fast, and practical iterative algorithm that accounts for PN interference. The algorithm provides an effective tool for CRN deployment and facilitates engineering implementation. Numerical simulations demonstrate that PN interference effects cannot be ignored in CRN network modeling, particularly in practical network construction.

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## References

[1] Hamdi K, Hasna M O, Ghrayeb A. Priority-based zero-forcing in spectrum sharing cognitive systems [J]. IEEE Communications Letters, 2013, 17(2): 267-270.

[2] Kibria M G, Yuan Fang, Kojima F. Feedback bits allocation for interference minimization in cognitive radio communications [J]. IEEE Wireless Communications Letters, 2016, 5(1): 104-107.

[3] Ma Shuai, Sun Dechun. Chance constrained robust beamforming in cognitive radio networks [J]. IEEE Communications Letters, 2013, 17(1): 59-62.

[4] Jeong Y, Quek T Q S, Shin H. Downlink beamforming optimization for cognitive underlay networks [C]// Proc of ISITA. 2010: 934-939.

[5] Huang Yongwei, Palomar D P. Rank-constrained separable semidefinite programming with applications to optimal beamforming [J]. IEEE Trans on Signal Processing, 2010, 58(2): 664-678.

[6] Le T A, Navaie K. Downlink beamforming in underlay cognitive cellular networks [J]. IEEE Trans on Communications, 2014, 62(7): 2212-2223.

[7] Pesavento M, Ciochina D, Gershman A B. Iterative dual downlink beamforming for cognitive radio networks [C]// Proc of the 5th International Conference on Cognitive Radio Oriented Wireless Networks & Communications. 2010: 1-5.

[8] Xu Yongjun, Zhao Xiaohui, Liang Y C. Robust power control and beamforming in cognitive radio networks: a survey [J]. IEEE Communications Surveys & Tutorials, 2015, 17(4): 1834-1857.

[9] Dhifallah O, Dahrouj H, Al-naffouri T. Decentralized SINR balancing in cognitive radio networks [J]. IEEE Trans on Vehicular Technology, 2017, 66(4): 3491–3496.

[10] Schubert M, Boche H. Solution of the multiuser downlink beamforming problem with individual SINR constraints [J]. IEEE Trans on Vehicular Technology, 2004, 53(1): 18-28.

[11] Tong Xue, Dong Xiaodai, Shi Yi. Resource allocation strategy for multiuser cognitive radio systems: location-aware spectrum access [J]. IEEE Trans on Vehicular Technology, 2017, 66(1): 884–889.

[12] Gaafar M, Amin O, Abediseid W. Underlay spectrum sharing techniques with in-band full-duplex systems using improper gaussian signaling [J]. IEEE Trans on Wireless Communications, 2017, 16(1): 235-249.

[13] Boyd S, Vandenberghe L. Convex optimization [M]. Cambridge: Cambridge University Press, 2004.

[14] Zhang Xianda. Matrix Analysis and Applications [M]. Beijing: Tsinghua University Press, 2004.

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