Postprint: Wideband Compressed Sensing Method Based on P-Ifourier Measurement Matrix

Authors: Liu Yang, Ren Qinghua, Meng Qingwei, Su Yuze

Date: 2018-05-02T00:00:00+00:00

Abstract

To address the problem of poor reconstruction accuracy when applying compressed sensing theory to broadband spectrum sensing, a broadband compressed spectrum sensing method based on the P-Ifourier (Partial-Inverse Fourier) measurement matrix is proposed, exploiting the sparse characteristics exhibited by stationary signals in the frequency domain. The proposed method first models the spectrum sensing problem as a typical compressed sensing problem, and constructs the measurement matrix by utilizing a standard orthogonal Fourier basis with excellent correlation properties, thereby endowing the measurement matrix with favorable reconstruction performance and accuracy. Simulation results demonstrate that, compared with the Gaussian random measurement matrix and the embedded chaotic sequence-cyclic Toeplitz structure measurement matrix, the proposed method can significantly reduce the mean square error of signal reconstruction in low signal-to-noise ratio environments, and the reconstruction probability is substantially improved under identical conditions.

Full Text

Broadband Compressed Sensing Method Based on P-Ifourier Observation Matrix

Liu Yang[1], Ren Qinghua[1,2], Meng Qingwei[1], Su Yuze[1] [1]College of Information & Navigation, Air Force Engineering University, Xi' an 710077, China [2]CETC Key Laboratory of Aerospace Information Applications, Shijiazhuang 050081, China

Abstract: Aiming at the problem of poor reconstruction precision when applying compressed sensing theory to broadband spectrum sensing, this paper proposes a broadband compressed spectrum sensing method based on a P-Ifourier

(Partial-Inverse Fourier) observation matrix, leveraging the sparse characteristics of stationary signals in the frequency domain. The new method first models the spectrum sensing problem as a typical compressed sensing problem, then constructs the observation matrix using a standard orthogonal Fourier basis with excellent incoherence properties, thereby achieving good reconstruction performance and precision. Simulation results demonstrate that compared with Gaussian random observation matrices and embedded chaotic sequence-cyclic Toeplitz structured observation matrices, the proposed method can significantly reduce the mean square error of signal reconstruction in low SNR environments and substantially improve the reconstruction probability under identical conditions.

Keywords: broadband spectrum sensing; compressed sensing; observation matrix; Fourier basis

0 Introduction

With the continuous development of the information age, demand for radio resources is increasing daily. Since wireless spectrum resources are limited, efficient and rational allocation and management of spectrum resources has become a critical challenge. Spectrum sensing technology based on cognitive radio theory provides a solution by dynamically exploring spectrum holes for resource allocation, but faces numerous challenges. One major challenge is that as radio signal bandwidth continues to increase, hardware devices based on traditional Nyquist sampling theory struggle to handle the burden imposed by high sampling rates. To address this issue, introducing compressed sensing (CS) theory into broadband spectrum sensing technology has become a prominent research focus.

Many scholars domestically and internationally have conducted research on various aspects of this field. Tian et al. [?] first proposed and validated the effectiveness of broadband compressed spectrum sensing. Subsequently, Tian et al. [?] proposed a broadband compressed spectrum sensing algorithm based on cyclostationary detection. Reference [?] introduced a deterministic embedded chaotic sequence-cyclic Toeplitz structured observation matrix, though this matrix depends on the autocorrelation properties of the selected pseudo-random sequence. Reference [?] proposed an optimized adaptive broadband compressed spectrum sensing algorithm that combines observation matrix optimization with adaptive compressed sampling, but the construction process for such matrices is complex and fails to exploit the characteristics of frequency-domain sparse signals, leaving room for improvement in reconstruction performance. Reference [?] introduced cross-validation theory to optimize the observation matrix in compressed sensing, but this matrix is obtained through continuous optimization of Gaussian random matrices, thereby increasing hardware complexity.

The core of compressed sensing lies in observation matrix construction. Selecting or constructing an appropriate observation matrix for the sensing environment is prerequisite for accurate recovery. This paper constructs a P-Ifourier observation matrix tailored to the spectrum sensing scenario under investigation, with detailed discussion of the matrix construction rationale and performance presented in the following sections.

1 Model Description

Consider a cognitive user' s detection bandwidth B uniformly divided into L non-overlapping sub-bands. Primary users can occupy these sub-bands arbitrarily. For unoccupied sub-bands, their power spectrum density (PSD) approaches zero, demonstrating sparse characteristics in the frequency domain [**?**]. Let the wideband signal to be detected be $x(t)$, which can be expressed as

$$x(t) = \sum_{i=1}^{L} \alpha_i \cos(2\pi f_i t) + n(t)$$

where α_i represents the amplitude of the i-th sub-band, f_i denotes the center frequency of the i-th sub-band, and $n(t)$ represents environmental noise. Under certain SNR conditions, the noise can be approximated as zero. Since natural signals generally lack sparsity in the time domain but exhibit sparse characteristics in the frequency domain, we transform $x(t)$ to the frequency domain:

$$s = Fx$$

where F is the discrete Fourier transform basis and s is the N-dimensional sparse coefficient vector of signal x in the frequency domain. Vector s contains only K non-zero coefficients ($K < N$), with remaining coefficients being zero or approximately zero, making s a K-sparse vector.

Sparse representation constitutes the first step in applying compressed sensing theory. Compressive sampling of the frequency-domain sparse coefficient vector s yields the observation vector

$$y = As$$

where y is an $M \times 1$ dimensional observation vector ($M \ll N$) and A is an $M \times N$ dimensional observation matrix. This paper models the traditional spectrum sensing problem as a classical compressed sensing framework, as illustrated in [Figure 1: see original paper]. From [Figure 1: see original paper] and equation (3), the spectrum information can be obtained by reconstructing the sparse vector s from the measurement vector y using recovery algorithms. For equation (3), the signal spectrum s can be recovered by solving an ℓ_0 norm minimization problem:

$$\min \|s\|_0 \quad \text{s.t.} \quad y = As$$

This method yields the optimal solution, but since $M < N$, solving equation (4) constitutes an NP-Hard problem. For such problems, a primary approach in compressed sensing theory involves solving an ℓ_1 norm minimization problem using linear programming [?]. Reference [?] proved that ℓ_0 minimization is equivalent to ℓ_1 minimization under certain conditions. Thus, equation (4) can be transformed to:

$$\min \|s\|_1 \quad \text{s.t.} \quad y = As$$

Two main algorithmic approaches exist for solving such problems: convex optimization algorithms and greedy algorithms. The former achieves global optimal solutions but suffers from high computational complexity and slow speed. The latter, based on greedy iteration, offers lower complexity and faster speed, though with slightly reduced recovery precision. Common greedy algorithms include Matching Pursuit (MP), Orthogonal Matching Pursuit (OMP), and Compressive Sampling Matching Pursuit (CoSaMP). Among these, OMP provides fast convergence and high recovery precision, making it the most commonly used recovery algorithm and the one employed in our simulations.

2 Observation Matrix Construction and Proof

2.1 Observation Matrix Construction Conditions

In compressed sensing theory, successful sparse signal reconstruction depends on whether the observation matrix satisfies the Restricted Isometry Property (RIP). Reference [?] provides a detailed introduction to RIP, which can be expressed as:

$$(1-\delta)\|s\|_2^2 \leq \|A_T s\|_2^2 \leq (1+\delta)\|s\|_2^2$$

where s is any K-sparse vector and δ is a constant. However, equation (6) is difficult to compute directly in practice, so it can be equivalently transformed to:

$$\|A_T^H A_T - I\|_2 \leq \delta$$

where T is an index set satisfying $T \subset \{1, 2, \dots, N\}$ and $|T| \leq K$. A_T is the matrix formed by selecting columns from observation matrix A corresponding to index set T. $\lambda_{\max}(\cdot)$ and $\lambda_{\min}(\cdot)$ represent the maximum and minimum eigenvalues of a matrix, respectively. From equation (7), smaller δ indicates that the maximum and minimum eigenvalues of the observation matrix approach 1 more closely, resulting in better RIP performance and reconstruction quality.

Additionally, references [?]-[?] note that the coherence criterion theory proposed by Donoho et al. can intuitively evaluate observation matrix morphology and performance. It can be simply expressed as:

$$\mu(A) = \max_{1 \le i,j \le N, i \ne j} |a_{ij}|$$

where A is the original square matrix of observation matrix A, and $|a_{ij}|$ represents the magnitude of the element in the i-th row and j-th column of square matrix A. In essence, the maximum magnitude among all elements in the original square matrix defines the matrix coherence. Smaller coherence of the observation matrix yields superior reconstruction performance.

For the observation matrix A in the compressive sampling module, common choices include Gaussian random matrices, partial Fourier matrices, and Toeplitz matrices [?]. Gaussian random matrices exhibit low coherence and are widely applied but suffer from high computational complexity and difficult hardware implementation [?]. Toeplitz matrices heavily depend on the autocorrelation properties of pseudo-random sequences, limiting their application scope. Partial Fourier matrices, obtained by randomly selecting M rows from the Fourier matrix and normalizing columns, possess ideal incoherence [?]. However, partial Fourier matrices only apply to time-domain sparse signals and are unsuitable for the spectrum sensing scenario in this paper. Therefore, we construct a P-Ifourier observation matrix tailored to this scenario.

2.2 Observation Matrix Construction

For frequency-domain sparse signals and combining the proposed spectrum sensing model, this paper constructs a P-Ifourier matrix as the observation matrix using a standard orthogonal Fourier basis with excellent coherence properties. The P-Ifourier matrix construction proceeds as follows:

a) Generate a Fourier transform matrix $F_N \in \mathbb{C}^{N \times N}$:

$$F_N = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & W_N & W_N^2 & \cdots & W_N^{N-1} \\ 1 & W_N^2 & W_N^4 & \cdots & W_N^{2(N-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & W_N^{N-1} & W_N^{2(N-1)} & \cdots & W_N^{(N-1)(N-1)} \end{bmatrix}$$

where $W_N = e^{-2\pi j/N}$. Since $\|F_N\|_2 = N$, each column of matrix F_N is not a unit vector, making F_N non-unitary.

b) Normalize matrix F_N by $1/\sqrt{N}$ to obtain the Fourier basis F:

$$F = \frac{1}{\sqrt{N}} F_N$$

Since $F^H F = I$ where I is the $N \times N$ identity matrix, matrix F is unitary and forms an orthonormal basis in unitary space.

c) Invert the Fourier basis F to obtain the inverse Fourier basis Φ:

$$\Phi = F^{-1} = F^H$$

d) Randomly select M row vectors from the inverse Fourier basis Φ to construct the P-Ifourier matrix A. The P-Ifourier matrix A maintains the orthonormal properties of Fourier basis F, thus possessing ideal incoherence. The following section proves that A satisfies the RIP property.

2.3 RIP Property Proof of Observation Matrix

According to the Gersgorin disc theorem, for $X = AA^H$, we have:

$$D_i = \{z \in \mathbb{C} : |z - x_{ii}| \leq r_i\}$$

where D_i is a disc in the complex plane centered at x_{ii} with radius $r_i = \sum_{j \neq i} |x_{ij}|$. From Section 2.2, we know $X = AA^H = I$, which implies:

$$\begin{cases} x_{ij} = 1, & i = j \\ x_{ij} = 0, & i \neq j \end{cases}$$

Thus, $r_i = 0$ and $\lambda_i \in D_i = 1$. Consequently:

$$\lambda_{\min}(AA^H) = \lambda_{\max}(AA^H) = 1$$

From equations (7) and (15), we conclude that the P-Ifourier matrix A not only satisfies the RIP property but exhibits ideal RIP characteristics.

3 Simulation Analysis

To verify the feasibility of the proposed spectrum sensing model and whether the constructed P-Ifourier observation matrix demonstrates significant improvement in reconstruction performance and precision compared to Gaussian random observation matrices and the deterministic embedded chaotic sequence-cyclic Toeplitz structured observation matrix proposed in [?], this section presents experimental analysis using MATLAB simulation software.

3.1 Model Feasibility Verification Experiment

The time-domain interference signal x selected for this experiment is a multi-tone jamming signal with sampling frequency 512. The frequency-domain sparsity of interference signal x is $K = 7$, the sparse basis F is the standard orthonormal Fourier basis, the observation matrix is the P-Ifourier matrix constructed in this paper, observation dimension $M = 128$, and the recovery algorithm is OMP. Simulation results are shown in [Figure 2: see original paper] through [Figure 4: see original paper].

[Figure 2: see original paper] depicts the time-domain multi-tone interference signal x to be detected, showing that x lacks sparsity in the time domain. Therefore, we use the standard orthonormal Fourier basis F for sparse representation of x. [Figure 3: see original paper] shows the sampled waveform after compressive sampling by the P-Ifourier observation matrix based on the frequency-domain sparse characteristics of x. Comparison of [Figure 2: see original paper] and [Figure 3: see original paper] reveals that the sampling process is not uniform sampling based on signal bandwidth, but rather non-uniform sampling according to the structural characteristics of the interference signal' s frequency domain, reducing the sampling rate by 75%.

[Figure 4: see original paper] compares the original spectrum of x with the spectrum recovered using the P-Ifourier observation matrix, visually demonstrating that the constructed P-Ifourier observation matrix can accurately recover x' s spectrum and validating the feasibility of the proposed model.

3.2 Reconstruction Performance Analysis

While the previous section validated model feasibility intuitively, this section compares the reconstruction performance of the P-Ifourier observation matrix against Gaussian random observation matrices and the deterministic embedded chaotic sequence-cyclic Toeplitz structured observation matrix from [**?**]. The detected signal bandwidth is $N = 512$, sparsity K varies from 5 to 70, noise is not considered, observation dimension $M = 128$, recovery algorithm is OMP, and Monte Carlo simulations run 10,000 times. Results are shown in [Figure 5: see original paper].

[Figure 5: see original paper] illustrates reconstruction probability comparison among different observation matrices under noiseless conditions. Under noiseless conditions, all three matrices achieve near-100% reconstruction probability when signal sparsity K is below 25. When K ranges from 25 to 45, reconstruction probabilities for Gaussian random and chaotic Toeplitz matrices drop sharply, while the P-Ifourier matrix maintains near-100% reconstruction probability. When K exceeds 50, Gaussian random and chaotic Toeplitz matrices fail to reconstruct the original signal, causing distortion, whereas the P-Ifourier matrix still achieves 98% reconstruction probability at $K = 50$ before declining gradually.

Real-world sensing environments typically involve low SNR conditions. The following experiment adds noise at 5 dB SNR, with sparsity K varying from 5 to 45 and other conditions unchanged. Results are shown in [Figure 6: see original paper].

[Figure 6: see original paper] indicates that reconstruction performance degrades under low SNR conditions. When K ranges from 5 to 8, all three matrices achieve near-100% reconstruction probability. When K ranges from 8 to 20, Gaussian random and chaotic Toeplitz matrices exhibit sharp performance degradation: at $K = 20$, chaotic Toeplitz achieves only 19.5% reconstruction probability and Gaussian random achieves 17.1%, failing to meet reconstruction requirements and causing significant distortion. In contrast, the P-Ifourier matrix still maintains 96.1% reconstruction probability at $K = 20$, declining only gradually when K exceeds 20.

3.3 Reconstruction Accuracy Analysis

This experiment varies SNR from -10 dB to -1 dB, with other conditions identical to Section 3.1. The reconstruction accuracy metric is mean square error (MSE):

$$\mathrm{MSE} = \frac{E\{\|s - \hat{s}\|_2^2\}}{E\{\|s\|_2^2\}}$$

Simulation results are shown in [Figure 7: see original paper].

[Figure 7: see original paper] presents MSE for each observation matrix across different SNR values. All matrices show decreasing MSE with increasing SNR. The P-Ifourier matrix's MSE curve is significantly better than those of Gaussian random and chaotic Toeplitz matrices. This advantage stems from the P-Ifourier matrix' s original square matrix being orthonormal in unitary space, providing ideal incoherence, while the other two matrices exhibit relatively poorer coherence properties. Calculations show that the P-Ifourier matrix reduces MSE by an average of 3.1% compared to the chaotic Toeplitz matrix and approximately 4.7% compared to the Gaussian random matrix. Thus, the constructed P-Ifourier observation matrix demonstrates noticeable improvement in reconstruction accuracy.

4 Conclusion

Applying compressed sensing theory to traditional spectrum sensing problems can effectively reduce signal sampling rates, but exhibits certain deficiencies in reconstruction performance and precision. This paper constructs an observation matrix using a standard orthonormal Fourier basis with ideal orthonormality and incoherence properties, and applies this matrix to a broadband compressed spectrum sensing model. The matrix maintains the coherence properties of the standard orthonormal Fourier basis, achieving significant improvements in reconstruction performance and precision, representing an excellent observation matrix for spectrum sensing applications.

References

[1] Tian Z, Giannakis G B. Compressed sensing for wideband cognitive radios[C]//Proc of IEEE International Conference on Acoustics, Speech and Sig-

nal Processing. 2007: IV-1357-IV-1360.

[2] Tian Z, Tafesse Y, Sadler B M. Cyclic feature detection with sub-Nyquist sampling for wideband spectrum sensing[J]. IEEE Journal of Selected Topics in Signal Processing, 2012, 6(1): 58-69.

[3] Li N, Ren Q, Su Y. TDCS compressed sensing observation matrix construction method[J]. Computer Engineering and Design, 2017, 1(1): 7-11.

[4] Wang W, Yang Z, Gu B, et al. Optimized adaptive compressed spectrum sensing based on observation matrix optimization[J]. Journal on Communications, 2014, 35(08): 33-39.

[5] Wang Z, Xu Y, Huang X, et al. Adaptive compressed wideband spectrum sensing based on observation matrix optimization[J]. Communications Technology, 2016, 49(01): 62-67.

[6] Li S, Wei D. Compressed sensing: a survey[J]. Acta Automatica Sinica, 2009, 35(11): 1369-1377.

[7] Ren Y, Zhang Y, Li Y. Compressed sensing and its image processing applications: advances and prospects[J]. Acta Automatica Sinica, 2014, 40(8): 1563-1575.

[8] Dong J S, Yin J Y, Li C F. A gradient-based steering kernel reconstruction strategy for semi-random Fourier measurements in compressed remote sensing[J]. Journal of Infrared & Millimeter Waves, 2015, 34(6): 673-679.

[9] Li Q, Zhao X, et al. Wideband spectrum sensing algorithm based on AOMP reconstruction[J]. Journal of Jilin University: Information Science Edition, 2015, 2(3): 113-119.

[10] Madni A M. A systems perspective on compressed sensing and its use in reconstructing sparse networks[J]. IEEE Systems Journal, 2014, 8(1): 23-34.

[11] Meng Q, Huang J, He C, et al. Compressed sensing sparse channel estimation method using time-domain measurement matrix[J]. Journal of Xi' an Jiaotong University, 2012, 46(8): 94-99.

[12] Jiao L, Yang S, Liu F, et al. Compressed sensing: review and prospect[J]. Acta Electronica Sinica, 2011, 39(7): 1651-1662.

[13] Huang T, Liu Y, Meng H, et al. Adaptive compressed sensing via minimizing Cramér-Rao bound[J]. IEEE Signal Processing Letters, 2014, 21(3): 270-274.

[14] Huang C C, Wang L C. Dynamic sampling rate adjustment for compressive spectrum sensing over cognitive radio network[J]. IEEE Wireless Communications Letters, 2012, 1(2): 57-60.

[15] Zhang J, Chen L, Boufounos P T, et al. On the theoretical analysis of cross validation in compressive sensing[C]//Proc of IEEE International Conference on Acoustics, Speech and Signal Processing. 2014: 3370-3374.