A Survey on Multi-Agent Cooperative Control Based on Switching Topology: Postprint

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Date: 2018-05-02T00:00:00+00:00

Abstract

Multi-agent systems have consistently been a primary research subject across numerous disciplines, and theoretical research on multi-agent cooperative control based on switching topology, as an important component of multi-agent systems research, has been an active area of research in recent years. To advance this research area, based on extensive investigation of existing literature and latest achievements, this work summarizes the current development status from three aspects: consensus problems, distributed optimization problems, and distributed estimation problems. It discusses issues such as the design of consensus protocols, performance analysis methods for consensus protocols and their advantages and disadvantages, implementation approaches for distributed optimization, and practical applications of distributed estimation. Finally, it identifies unresolved problems in the current field and future research directions.

Full Text

Preamble

Title: A Review of Research on Multi-Agent Cooperative Control Based on Switching Topology

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Abstract: Multi-agent systems have consistently been a primary research subject across numerous disciplines. As an important component of multi-agent system research, cooperative control theory based on switching topology has remained a hot topic in recent years. To advance research in this area, this paper extensively surveys existing literature and recent achievements, summarizing the current state of development from three perspectives: consensus problems, distributed optimization problems, and distributed estimation problems. It discusses topics such as consensus protocol design, performance analysis methods for consensus protocols and their advantages and disadvantages, implementation approaches for distributed optimization, and practical applications of distributed estimation. Finally, it identifies outstanding issues in the field and future research directions.

Keywords: multi-agent; switching topology; cooperative control

0 Introduction

Multi-agent systems are formed by multiple autonomous agents that collaborate locally and interact to accomplish complex tasks. Due to their robustness, reliability, and efficiency, multi-agent systems have attracted widespread attention in control and artificial intelligence. Researchers [1,2] pioneered the connection between consensus protocol performance and network algebraic connectivity, establishing stability analysis methods [3] for multi-agent systems using control theory tools, and optimizing overall system performance through a "divide-and-conquer" [4] strategy.

The study of multi-agent systems primarily focuses on accomplishing complex tasks through distributed control of multiple simple agents in collaborative cooperation. In practical applications, the complexity of collaborative cooperation among multiple agents [5] is closely related to the network topology of multi-agent systems. The network topology of multi-agent systems often changes over time, which is generally referred to as switching topology [6]. Switching topology arises mainly due to changes in agent positions and the establishment or disconnection of communication links between agents. Switching topology can be represented by a dynamic graph $\mathcal{G}_{s(t)}$, where $s(t)$ is the switching signal.

Due to the complexity and variability of real-world environments, cooperative control based on switching topology has extensive applications. In industrial manufacturing, multi-agent systems can be used for transporting large objects or hazardous materials. In aerospace, formations of satellites and spacecraft for space exploration not only improve system reliability but also accomplish tasks impossible for single spacecraft. In harsh environments, agents can replace humans in exploring unknown territories. In military applications, armored units can be modeled as agents to construct hierarchical organizational structures for coordinated combat operations.

Therefore, research on coordinated control of multi-agent systems with switching topology has remained a hot research area, with main issues including consensus problems [7-21], distributed optimization problems [37-48], and estimation problems [49-60].

Funding: This work was supported by the NSFC-Guangdong Joint Fund (U1501254), the National Natural Science Foundation of China (61472089), and the Guangdong Provincial Science and Technology Plan (2014B010103005, 2016A040403078).

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1 Multi-Agent System Consensus Problems

Multi-agent system consensus problems constitute an important research aspect in coordinated control. Consensus refers to the process where, within finite time, a certain state variable of all agents in the system evolves according to a distributed control protocol and eventually converges to the same value, causing all state information to approach identical values and the system to ultimately converge. Consensus problems mainly include gathering [7], consensus tracking [8], bipartite consensus [9,10], and other issues. However, in practical scenarios, disturbances or limited communication ranges can cause communication link failures, meaning the system topology changes over time. Therefore, consensus problems based on switching topology are more realistic and have become a current research focus.

1.1 Consensus Protocols

The continuous-time consensus protocol based on switching topology [1] is as follows:

$$\dot{z}_i(t) = -\sum_{j\in N_i(t)} a_{ij}(t)(z_i(t) - z_j(t))$$

where $N_i(t)$ represents the time-varying neighborhood, and $a_{ij}(t)$ is the corresponding element in the adjacency matrix of the time-varying topology graph $G(t)$ at time t. This protocol can be expressed in matrix form as:

$$\dot{Z} = -L(t)Z$$

where L denotes the Laplacian matrix of the multi-agent system network graph. The necessary and sufficient condition for the multi-agent system to achieve consensus by processing neighborhood information is:

$$\lim_{t\to\infty}(z_i(t) - z_j(t)) = 0$$

Jadbabaie et al. [11] studied first-order linear multi-agent systems based on undirected topology graphs, proving that as long as the switching topology of the multi-agent system remains connected, the agents'state values will eventually converge. They proposed a distributed consensus control protocol:

$$x_i(t+1) = x_i(t) + \sum_{j \in N_i(t)} a_{ij}(t)(x_j(t-\tau_{ij}) - x_i(t))$$

where $x_i(t)$ is the state of agent i at time t, $a_{ij}(t)$ is the gain coefficient, and τ_{ij} represents the transmission delay from agent i to agent j.

Ren et al. [12] extended Jadbabaie' s results to directed graphs, demonstrating that if the set of directed switching topology graphs contains sufficiently many spanning trees, the multi-agent system can achieve convergence.

Consensus research has expanded to linear second-order multi-agent systems. Lin et al. [13] utilized linear transformation and Lyapunov function construction methods based on connected undirected switching topology graphs to derive standard convergence conditions for multi-agent systems, though this method was not extended to directed graphs or higher-order systems.

Hajar A et al. [14] studied consensus problems in second-order directed switching topology multi-agent systems, providing a sufficient condition under which all agents' states converge.

Considering higher-order linear multi-agent systems, Su Y et al. [15] transformed the consensus problem for a class of linear multi-agent systems based on undirected switching topology into a matrix stability problem, ultimately obtaining stable convergence values. Since Su Y' s method cannot be applied to directed topology graphs, reference [16] designed a new distributed consensus control protocol: if the directed switching topology graph contains enough spanning trees, the system can converge under this protocol.

Numerous nonlinear factors exist in practical scenarios, so researchers have gradually begun to emphasize consensus problems for nonlinear continuous-time multi-agent systems while studying linear systems. Reference [17] designed a dynamic consensus protocol based on nonlinear multi-agent systems, which ensures convergence to a stable equilibrium in the sense of Lyapunov. Reference [18] studied consensus problems for multi-agent systems with inherent nonlinear terms under directed switching topology graphs; if the inherent nonlinear terms satisfy Lipschitz conditions, the consensus problem can be transformed into a local stability problem using variable transformation methods. Reference [19] investigated consensus problems for nonlinear systems based on undirected switching topology, proposing a non-smooth consensus algorithm and proving its finite-time convergence.

1.2 Performance Analysis Methods for Consensus

Traditional analysis methods for multi-agent systems with switching topology involve constructing Lyapunov functions. However, reference [21] proved that no universal quadratic Lyapunov function exists for discrete-time switching topology consensus algorithms, prompting researchers to develop various novel approaches.

1.2.1 Infinite Product of Stochastic Matrices Method Jadbabaie and Ren et al. [26,27] constructed the following ergodic coefficient:

$$\chi(A) = \min_{i,j} \sum_{k=1}^{N} \min(a_{ik}, a_{jk})$$

where A is a stochastic matrix. When considering time delays [28,29], as long as the topology graph of stochastic matrix A contains a spanning tree and has a self-loop at the root node, the infinite product method of stochastic matrices can be applied to linear Vicsek models with time delays.

1.2.2 Common Lyapunov Function Construction Reference [30] employed finite-time Lyapunov stability theory to design a continuous-time nonlinear consensus algorithm based on undirected switching topology graphs, solving finite-time average consensus problems. Reference [31] used polynomial Lyapunov function construction to study consensus problems in nonlinear multi-agent systems, obtaining more relaxed system consensus constraints. Reference [32] proposed standard consensus conditions for second-order linear multi-agent systems based on initially connected undirected switching topology graphs using linear transformation and Lyapunov function construction.

1.2.3 Proof by Contradiction The proof by contradiction method was proposed in reference [33]. The main idea is: when the graph sequence $\{G_i\}_{i=1}^{\infty}$ is connected, then $M(t) = m(t)$, where $M(t) = \max_i x_i(t)$ and $m(t) = \min_i x_i(t)$. All agents' state values can eventually converge. This method can be effectively applied to various Vicsek models in both leaderless and leader-following cooperation scenarios.

1.2.4 Lyapunov Exponent Calculation Method Chen et al. [34] proposed defining the Lyapunov exponent for matrix set $\mathcal{B}$ as:

$$\rho(\mathcal{B}) = \limsup_{t\to\infty} \sup_{A_i\in\mathcal{B}} \frac{1}{t} \log \|A_t \cdots A_1\|$$

By transforming the convergence performance calculation of general linear multi-agent systems into Lyapunov exponent calculation for given matrix sets, the convergence capability of multi-agent systems can be determined.

1.2.5 Set Construction Method The set construction method was proposed in reference [35]. The main idea involves constructing the set:

$$\mathcal{B} = \{V_i : V_i(x) \geq m(t_k), \forall i\}$$

where V_i are Lyapunov functions and $m(t_k) = \min_i V_i(x_i(t_k))$. For linear Vicsek models with time delays, this method can achieve exponential convergence rates by repeating the process.

1.2.6 Set-Valued Analysis Method Moreau [3] redefined multi-agent stability, proposing to use set-valued functions to analyze and measure system stability. Let χ denote a finite-dimensional Euclidean space and $f : \chi \times \chi \to \chi$ be a mapping. Then there exists a semi-continuous set-valued function $V : \chi \to \chi$ such that for all $x \in \chi$, $V(x) \subseteq \chi$, and for all $t \in \mathbb{R}$ and $x \in \chi$, $V(f(t,x)) \subseteq V(x)$. This method is used for convergence and stability analysis of nonlinear multi-agent systems but requires application constraints: the switching multi-agent system must satisfy convexity assumptions.

1.2.7 M-Matrix Method Chapman et al. [36] conducted asymptotic stability analysis for nonlinear dynamic systems based on M-matrix theory. Define the set of real matrices with non-positive off-diagonal elements as $\mathcal{Z}_N = \{A \in \mathbb{R}^{N\times N} : a_{ij} \leq 0, \forall i \neq j\}$. If $A \in \mathcal{Z}_N$ and is non-singular, then A is called an M-matrix. This method uses the left and right null spaces of M-matrices to characterize system inherent properties and model stability, applicable to nonlinear system stability analysis.

一致性性能分析方法

2 Distributed Optimization Problems

Distributed optimization [4] effectively solves large-scale complex optimization problems through coordination among multiple agents. Each agent controls its own local subproblem without knowledge of information held by other agents, and must obtain an optimal solution through local information exchange.

With the rapid development of cloud computing and big data technologies, distributed optimization methods [37] that improve multi-agent system performance, convergence rate, robustness, and stability have received increasing attention. In cooperative control based on switching topology, traditional research on distributed optimization employs objective cost functions. To further reduce computation and communication, event-based control methods are used.

2.1 Objective Cost Functions

Reference [4] first studied distributed multi-agent optimization problems under a consensus framework: minimizing $\sum_{i=1}^{N} f_i(x)$ where $f_i : \mathbb{R}^n \to \mathbb{R}$ are convex functions. By combining the average consensus algorithm with standard subgradient methods, they proposed a consensus-like algorithm:

$$x_i(k+1) = x_i(k) + \alpha(k) \sum_{j\in N_i} a_{ij}(x_j(k) - x_i(k)) - \alpha(k) g_i(x_i(k))$$

where $\alpha(k)$ is the step size and $g_i(x_i(k)) \in \partial f_i(x_i(k))$ is a subgradient of f_i at $x_i(k)$. However, this distributed solution requires the subgradient step size to be square-summable. Wang et al. [38] further relaxed the step size constraints, proving that if the subgradient step size follows a non-summable diminishing principle, better convergence rates can be achieved.

For a class of local objective convex functions, Lin et al. [39] proposed a scheme combining distributed tracking algorithms with distributed estimation methods, simultaneously designing an adaptive distributed algorithm for handling general locally differentiable objective convex functions. This ensures that the derivatives of local objective functions converge to 0, and multi-agent states converge within finite time.

Paolo D L et al. [40] constructed non-convex smoothing functions that propagate and compute across the entire agent network based on dynamic consensus principles, thereby obtaining solutions to non-convex optimization problems through asymptotic convergence.

Reference [41] proposed a distributed approximate dual subgradient algorithm using non-convex cost functions, enabling agents to asymptotically converge to the dual solution of the approximate problem.

2.2 Event-Based Control Methods

Dimarogonas et al. [42,43] first introduced event-triggered mechanisms for multi-agent systems, defining the measurement error for agent i as:

$$e_i(t) = x_i(t_k^i) - x_i(t), \quad t \in [t_k^i, t_{k+1}^i)$$

They established the event-triggering condition:

$$\|e_i(t)\|^2 \leq \sigma_i \sum_{j \in N_i} \|x_j(t) - x_i(t)\|^2$$

where $\sigma_i \in (0, 1)$ is an adjustment factor, and the summation represents the sum of squared errors with parameter $z_{a,N} = \frac{1}{N} \sum_{j \in N_i} (x_j - x_i)$.

Due to dynamic changes in communication connections and disconnections between agents, network topology is time-varying, and the lack of global information makes distributed control increasingly complex. In real-world scenarios, using distributed estimation strategies [50] to estimate some global information within finite time, thereby designing local controllers to stabilize multi-agent systems, has become a current research focus.

Wu et al. [44] designed corresponding event-triggering functions using pre-estimated neighbor agent state information and proposed a distributed event-triggered communication protocol. In event-triggered approaches, if the next event time is precomputed within the control update interval based on previously received data and information, it is called self-triggering, which can save more energy.

Reference [45] combined event-triggered systems with data sampling control to design a distributed self-triggered consensus algorithm, enabling effective analysis of event-triggered consensus under general network topologies.

Reference [46] relaxed the requirement for high network topology connectivity in multi-agent systems, using self-triggered strategies in sensor networks to reduce power consumption frequency of individual agents and thereby lower overall system energy consumption.

Event triggering causes two agents to connect through an undirected edge, and controller updates based on the agent' s own state are called edge event-driven control methods.

Xiao et al. [47] considered each information connection as an independently defined edge event, establishing an edge event-driven data sampling framework for achieving distributed state consensus requirements.

Meng et al. [48] studied edge event-driven data sampling consensus problems for multiple integrators in switching networks, analyzing agent state consensus under both bidirectional interaction and leader-following scenarios.

3 Distributed Estimation Problems

Distributed estimators estimate the global system [49], while communicating with each other to transmit their respective estimates of the global system state, ultimately achieving consistency between estimated and actual states.

Due to dynamic changes in communication connections and disconnections between agents, network topology is time-varying, and the lack of global information makes distributed control increasingly complex. In real-world scenarios, using distributed estimation strategies [50] to estimate some global information within finite time, thereby designing local controllers to stabilize multi-agent systems, has become a current research focus.

Zhang et al. [51] studied joint estimation and control problems under disturbances, designing an estimator using Kalman filtering as follows:

$$\hat{z}_{k+1} = (I + G_k W_k R_k^{-1} W_k^T G_k^T P_k)^{-1} \hat{z}_k + G_k W_k R_k^{-1} y_{k+1}$$

where k represents the time sequence, y_k is the measurement vector, $\hat{z}_k$ is the prediction vector, W_k is the weight matrix, and P_k is the error covariance matrix.

Federico et al. [52] conducted in-depth research on distributed estimation problems using a diffusion least mean squares strategy, establishing a framework for diffusion LMS algorithms. This distributed estimation algorithm exhibits good robustness against node and link failures in the overall system.

Reference [53] employed distributed synchronous estimation and control methods, using distributed estimators in control algorithm design while proposing an algorithm design framework for cooperation among agents, which significantly advances solutions to joint estimation and control problems.

Reference [54] used optimal filtering methods to integrate local sensor data into a global environmental model, enabling information fusion from each mobile sensor through the communication network.

References [55,56] proposed a gradient-based control strategy based on decentralized estimation properties of undirected topology graphs and algebraic connectivity, designing control algorithms that guarantee global connectivity of the communication graph without maintaining local connectivity between systems.

Reference [57] proposed an iterative hierarchical estimation process implemented in a distributed manner based on directed graphs, designing a PI average consensus estimator:

$$\begin{bmatrix} \hat{x}_{k+1} \\ z_{k+1} \end{bmatrix} = \begin{bmatrix} I - \gamma L_k & 0 \\ -\gamma L_k & I \end{bmatrix} \begin{bmatrix} \hat{x}_k \\ z_k \end{bmatrix} + \begin{bmatrix} \gamma I \\ \gamma I \end{bmatrix} w_k$$

where $\gamma > 0$ is the design parameter, L_k is the Laplacian matrix, I and 0 are identity and zero matrices of appropriate dimensions, and B is a balanced matrix.

To apply distributed estimation algorithms in wireless sensor networks, environmental monitoring, and other scenarios requiring high scalability, strong robustness, and low power consumption, researchers have made numerous attempts.

Yan et al. [50,58] designed an information consensus-based estimation algorithm by partitioning nodes in wireless sensor networks, enabling two types of nodes to cooperate and achieving accurate estimation of temperature distributions. Subsequently, Yan further considered environmental noise and proposed a new information consensus-based tracking algorithm to improve target tracking accuracy.

Liu et al. [59] used distributed estimation schemes in hierarchical wireless cognitive networks to predict radio environment maps, which display radio signal strength over geographical areas.

Wang et al. [49,60] studied distributed dynamic state estimation problems, proposing distributed complex information fusion algorithms that not only guarantee estimation accuracy but also utilize information resources more effectively. These algorithms were applied to scenarios where targets can have multiple hidden models and switching Markov chains, achieving good results.

4 Conclusion

Cooperative control of multi-agent systems based on switching topology has remained a research hotspot in the control field. Through long-term research, consensus problems for linear systems have formed a theoretical framework, and researchers have begun to emphasize studies more aligned with real-world environments, such as consensus problems for nonlinear systems and consensus problems based on signed graphs. Meanwhile, significant progress has been made in efficiently achieving multi-agent system cooperation using distributed optimization methods like event-triggered approaches and distributed estimation methods. Although current theoretical research and experiments have solved many technical challenges in multi-agent cooperation under switching topology, numerous important and challenging research problems remain to be deeply investigated:

a) Most current research focuses on mutual cooperation among agents, with little mention of competition. However, in reality, competition not only exists but also drives structural changes in the entire population according to the "survival of the fittest" principle, playing a positive role in group cooperation. How to introduce competition mechanisms into cooperative control to achieve better overall system performance is a worthwhile future research direction.

b) Under distributed protocols, some agents cannot predict group behavior based on limited local information, leading to uncontrolled behavior in some agents. The impact of agent loss of control is further amplified in dynamically changing network topologies, causing instability in the entire multi-agent system. Therefore, how to balance distributed and centralized approaches in control protocol design to ultimately ensure system stability remains an open research question.

c) The convergence of multi-agent systems is closely related to the connectivity of network topology graphs, but researchers have found that more connections among agents do not always promote faster system convergence. Therefore, having individual agents select a subset of "important" agents within their neighborhood for cooperative communication can not only guarantee the same convergence rate but also reduce overall system energy consumption. How to design agent selection mechanisms that achieve fast convergence with low energy consumption is an important research problem.

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Note: Figure translations are in progress. See original paper for figures.