Nonlinear Dynamics of Thermoacoustic Instability in Rijke Tubes: A Bifurcation Analysis Postprint

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Abstract

A horizontal Rijke tube thermoacoustic model was established, and the Galerkin method was employed to expand the governing equations, enabling numerical solution. The convergence order of Galerkin modes was determined to be 10, and nonlinear dynamical theory was utilized to conduct bifurcation analysis of the system. The bifurcation behavior of the Rijke tube thermoacoustic system belongs to the subcritical Hopf bifurcation. The stability regions of the system are divided into globally stable, globally unstable, and bistable regions. Bifurcation diagrams were obtained for parameters including the dimensionless heating power K, heater position xf, damping coefficient c1, and time delay , revealing that the bifurcation diagram for heater position xf exhibits two Hopf bifurcation points. Within the linearly unstable region, the oscillation amplitude exhibits a trend of first increasing and then decreasing with the increase of time delay .

Full Text

Preamble

Rijke Tube Thermoacoustic Instability Nonlinear Dynamics Study–Bifurcation Analysis

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Abstract: A thermoacoustic model of a horizontal Rijke tube is established. The governing equations are expanded and solved numerically using the Galerkin method. The convergence order of Galerkin modes is determined to be 10, and bifurcation analysis of the system is performed using nonlinear dynamics theory. The bifurcation behavior of the Rijke tube thermoacoustic system is found to be subcritical Hopf bifurcation. The system stability region is divided into globally stable, globally unstable, and bistable regions. Bifurcation diagrams are obtained for parameters including non-dimensional heater power K, heater position xf, damping coefficient c1, and time delay . The bifurcation diagram for heater position xf exhibits two Hopf bifurcation points. Within the linearly unstable region, the oscillation amplitude shows a trend of first increasing and then decreasing with increasing time delay .

Keywords: thermoacoustic instability; Rijke tube; Galerkin method; bifurcation analysis; nonlinear dynamics

0 Introduction

Combustion instability refers to large-amplitude self-excited oscillations of pressure and velocity in combustors, which can cause excessive structural vibration and heat transfer to combustion chamber walls, leading to catastrophic damage to structural components. For decades, combustion instability has been a major challenge in high-performance combustion devices such as rocket engines, gas turbines, aero-engines, and power station boilers. Thermoacoustic instability is the most predominant form of combustion instability, generally believed to arise from coupling between unsteady heat release and acoustic field oscillations, triggering large-amplitude pressure oscillations near the natural acoustic frequencies of the system. The underlying mechanism is a positive feedback between heat source release and the acoustic environment.

The mechanisms behind combustion instability phenomena are highly complex, involving coupled interactions among combustion, flow, and acoustic fields. Flame surface variations, equivalence ratio fluctuations, vortex shedding due to hydrodynamic instabilities, and oscillations in fuel atomization and evaporation processes can all drive combustion instability. Combustion chamber geometry, fuel/air mixing degree, fuel type and composition, and operating conditions (injection air temperature, combustion chamber pressure, equivalence ratio, swirl intensity, lean premixing, etc.) all influence combustion instability. Nonlinear dynamic characteristics of flow and flame represent one of the most important phenomena in combustion instability. Flow and flame characteristics in combustion chambers can change dramatically with control parameters, particularly when parameters cross bifurcation points. The combustion process itself may or may not exhibit bifurcation phenomena, but once bifurcation occurs, nonlinear behavior dominates in many combustion devices. Small perturbations in system parameters may trigger bifurcation, causing the system to transition from a stable state (small or no oscillations) to an unstable state (large-amplitude limit cycles). Conversely, transition from an unstable state to a stable state at the same critical parameter value does not necessarily occur due to hysteresis phenomena.

Bifurcation, hysteresis, limit cycles, and triggering phenomena in combustion instability can all be explained and described using dynamical systems theory, which represents an emerging research focus. Nonlinear dynamics theory enables deep characterization, analysis, and diagnosis of nonlinear system features. Most existing research focuses on thermoacoustic instability in gas turbines and rocket engines, with study objects typically being various simplified combustors or actual gas turbines and engines. The Rijke tube is the most convenient and typical experimental system for studying thermoacoustic phenomena, and many studies have been conducted on Rijke tubes to investigate the mechanisms and control of combustion instability caused by unstable heat release. Heat release in thermoacoustic systems can take various forms, such as exothermic chemical reactions in combustors, heating wires, and hot gauze. Jahnke and Culick first introduced modern dynamical systems theory to combustion instability research to analyze nonlinear combustion instability. They proposed a continuation algorithm for systematic and efficient computation of steady-state and limit cycle solutions, enabling stability analysis under steady-state and limit cycle conditions, with bifurcation analysis used to determine instability onset points. The nature of acoustic waves at instability onset depends on the bifurcation type and limit cycle characteristics at the bifurcation point.

Ananthkrishnan et al. established reduced-order models for unstable motions in combustion chambers and analyzed triggering and limit cycle characteristics. Analysis shows that for axial oscillation modes, first-order instability requires at least fourth-order Galerkin series expansion, while second-order instability requires at least eighth-order series expansion to ensure computational accuracy. Balasubramanian investigated the role and influence of non-normality in simple thermoacoustic systems by deriving acoustic field control equations for horizontal Rijke tubes, focusing on results brought by non-normality and nonlinear effects. Subramanian analyzed the nature of subcritical bifurcation in thermoacoustic systems and the resulting bistability issues. Waugh et al. studied triggering in thermoacoustic systems and drew analogies with bypass transition to turbulence in fluid mechanics—both mechanisms involve small perturbations causing the system to transition to large-amplitude self-sustained oscillations despite being linearly stable. Their research demonstrated that certain types of noise are more effective at triggering oscillations.

Existing research has provided some understanding of the nonlinear dynamics of Rijke tube thermoacoustic instability, but understanding of system bifurcation characteristics remains unclear, and knowledge is lacking regarding how system parameters such as non-dimensional heater power, heater position, and damping coefficient affect bifurcation. This paper establishes a simplified model of Rijke tube thermoacoustic instability, derives control equations for its one-dimensional acoustic field, performs numerical solutions, and conducts bifurcation analysis of Rijke tube thermoacoustic instability to deepen understanding of the generation mechanisms and oscillation phenomena.

1 Thermoacoustic Instability Model and Solution

1.1 Rijke Tube Model

The horizontal Rijke tube model studied in this paper is shown in [Figure 1: see original paper]. In this model, the tube length L is fixed, the heater position xf can be varied by changing the heater location within the duct, the heat source is an electric wire heating device, and under experimental conditions its heating intensity can be changed by varying the current passing through it.

1.2 Governing Equations

Ignoring the effects of mean flow and temperature gradients, the momentum and energy equations governing the acoustic field in the duct are as follows [7]:

In equations (1)-(5), x is the distance along the axial direction, t is time, p̄ is the mean pressure in the tube, u is the acoustic velocity, p' is the acoustic pressure, is the fluid density, is the specific heat ratio of the medium, the speed of sound is c, and M̄ is the mean flow Mach number. Additionally, Q '(t) is the fluctuation in heat release rate per unit area generated by the electric heater, is the standard Dirac distribution, and xf is the heat source location. The symbols'˜' and ¯" denote dimensional and mean quantities, respectively.

The heat source model adopts the Heckl empirical formula [16]:

where l and d represent the length and diameter of the heating wire, T̃w is the wire temperature, T̃0 represents the mean temperature of surrounding air, S is the duct cross-sectional area, k̃ represents the thermal conductivity of air, c̃v denotes the specific heat capacity per unit mass of air at constant volume, and ˜0 is the mean gas density. Due to thermal inertia, there exists a time delay between heat transfer and flow velocity. Note that this heat source model is only suitable for describing "compact" heat sources where the heat source thickness is negligible relative to the wavelength.

Substituting equation (7) into the energy equation (4) yields a modified energy equation:

To simplify and facilitate analysis, equations (1) and (2) are non-dimensionalized to obtain the dimensionless system control equations:

The dimensionless scales are as follows:

1.3 Galerkin Expansion

The conventional method for solving thermoacoustic problems in Rijke tube devices uses Galerkin projection and modal expansion. The principle of the Galerkin method is that any function in a domain can be expressed as a superposition of expansion functions in that domain. The basis functions must satisfy boundary conditions, and the choice of basis functions is not unique. In this approach, pressure and velocity signals varying in space and time are expanded using spatial basis functions that satisfy boundary conditions. The choice of these basis functions is not unique. This paper selects basis functions that are arbitrary, not the system' s eigenfunctions, but rather eigenfunctions of the self-adjoint part of the linearized system. Therefore, the velocity and pressure fields can be expressed in the form of duct natural modes:

Damping can be expressed as:

where parameters c1 and c2 remain constant, kn is the dimensionless wavenumber of the nth acoustic mode in the duct, and n is the dimensionless wavelength of the nth duct mode. Under the limit $N \to \infty$, these basis functions form a complete basis set. Obviously, the expansion functions selected in this paper satisfy boundary conditions.

Substituting equations (9) and (10) into equations (3) and (8) and projecting along the basis function set yields:

where the dimensionless heating power is:

The fully coupled Galerkin model can be further simplified through linearized delay, which is only applicable under the Galerkin modal assumption where $<$ jT, with T being the time period of the jth Galerkin mode. Under this assumption, the delay term can be written as:

Through linearized delay, the final Galerkin-expanded ordinary differential equation system is obtained:

1.4 Galerkin Modal Convergence

The number of Galerkin modes required to accurately capture the system' s linear and nonlinear behavior is called modal convergence. Matlab programming was implemented for numerical solution of equations (15) and (16), and system evolution under different numbers of Galerkin modes was compared. For the linearly unstable region, a comparison of the system' s time evolution with different numbers of acoustic Galerkin modes is shown in [Figure 2: see original paper]. It can be observed that the amplitude and phase of limit cycles under first-order Galerkin acoustic modes differ significantly from those under tenth-order Galerkin acoustic modes, but as the number of Galerkin modes increases, the differences in limit cycle amplitude and phase gradually decrease. According to [Figure 2: see original paper], the differences between numerical solutions obtained with ninth-order and tenth-order acoustic modes are almost negligible, again demonstrating the convergence of the tenth-order acoustic mode Galerkin projection. For tenth-order acoustic modes, adding one Galerkin mode changes the limit cycle amplitude of acoustic velocity by less than 1.4% [12]. Therefore, in all subsequent calculations in this paper, we adopt a 10th-order Galerkin modal model for both linear and nonlinear stability analysis to ensure convergence.

2 Linear Stability Analysis

To investigate the effect of infinitesimal perturbations on system stability, linear stability analysis is performed first. If the system gradually moves away from the stable state, it is ultimately unstable; if it gradually approaches the stable state, it ultimately reaches stability. This refers to local analysis of stability changes near the equilibrium state. On the other hand, nonlinear stability analysis studies the effect of finite-amplitude perturbations on the system and characterizes the resulting asymptotic states. The critical point where system behavior changes due to loss of stability of the equilibrium solution is called a bifurcation point. Once a bifurcation point is determined based on one parameter, the bifurcation point itself continues to vary with another related system parameter. Bifurcation point branches provide linear stability boundaries that separate linearly stable and linearly unstable regions in the relevant parameter space. This stability boundary is a hypersurface in the space of all varying free parameters, but it is convenient to represent several appropriate two-dimensional projection curves.

In the Rijke tube, system parameters include the heater’ s non-dimensional heating power, heater position, damping coefficient, and time delay. However, under actual experimental operating conditions, the parameters that can be precisely varied are only the non-dimensional heating power and the specific heater position; time delay and damping coefficient are changed according to other conditions. Taking the linear stability boundary between heater position and time delay as an example, we analyze the linearly stable and linearly unstable regions when heater position and time delay vary. A typical linear stability boundary variation between heater position and time delay is shown in [Figure 3: see original paper], which shows whether the system is linearly unstable depending on time delay ( ) for a selected range of heater positions (xf), and vice versa, for fixed system parameter values of damping and heater power. For small or reasonably large , such as < 0.05 or > 0.45, the Rijke tube system is linearly stable for any heater position, meaning the system ultimately tends to a stable state for small perturbations. Only within the range $0.05 < \; < 0.45$ does the stability of the system’ s equilibrium solution depend on heater position.

3 Bifurcation Analysis

Bifurcation analysis studies the characteristics and generation mechanisms of bifurcation phenomena. Bifurcation refers to situations where, for certain well-defined nonlinear systems, when a system parameter varies continuously to a critical value c, the global behavior (qualitative properties, topological properties, etc.) of the system undergoes sudden changes. The nature of nonlinear bifurcations for source terms of the form $X = \; X + \; X^3$ can be determined by expanding the nonlinear term into a series and discarding higher-order terms. The two-term expansion of this expression yields:

In the above expression, when < 0, the signs of the first-order term and the third-order term are the same. The sign difference between these two terms determines the nature of the bifurcation point. Whenever these two terms have the same sign, the bifurcation is subcritical; when they have different signs, the bifurcation is supercritical. For the heat release rate fluctuation Q' (t) in the Rijke tube model, this means the model will exhibit subcritical Hopf bifurcation.

In the Rijke tube thermoacoustic system, variable system parameters include the heater' s non-dimensional heating power K, heater position xf, damping coefficient c1, and time delay . Bifurcation analysis is performed for variations of these parameters within the system.

3.1 Effect of Heater Power

The heater in the Rijke tube thermoacoustic system is an electric wire heater. The effect of heater power on the system is investigated by varying the K value. Non-dimensional heater power can be increased by providing more power to the heater, representing an increase in system driving force. Increased driving force makes the system more unstable. Therefore, for small K values, the system equilibrium is stable and decays asymptotically close to zero under all initial perturbations. Increasing K reduces the flow stability margin; near the bifurcation point, the system evolves to linear instability, resulting in oscillatory flow patterns in the tube. [Figure 5: see original paper] shows the computed effect of varying non-dimensional heater power (K) on system evolution. Hollow circles indicate unstable solutions, solid circles represent stable solutions. The figure shows that small-amplitude limit cycles near the Hopf bifurcation point are unstable, coexisting with stable equilibrium solutions. These unstable limit cycle branches further undergo a fold bifurcation to gain stability. This determines that the bifurcation belongs to the subcritical type, confirming previous conclusions. Under fixed other system parameters, the system' s time evolution behavior depends on K value changes. When $K < 0.65$, the system is stable for perturbations of any amplitude. As K increases continuously, when $0.65 < K < 0.72$, the system has three possible states: stable state, unstable limit cycle, and stable limit cycle. Depending on initial conditions, the system may ultimately enter either a stable state or periodic oscillations; this region is called the bistable region. When $K > 0.72$, the system enters the globally unstable region, where for any small initial perturbation, the system ultimately reaches a limit cycle state. The bifurcation diagram for non-dimensional heating power also demonstrates that increasing non-dimensional heating power is the main cause of thermoacoustic instability in the system.

3.2 Effect of Damping Coefficient

To investigate the effect of damping variation on system response, one damping parameter (c1) in the damping model can be varied. Under actual experimental conditions, system damping magnitude is changed by altering the duct end conditions. According to nonlinear theory, increasing damping can enhance system stability. [Figure 6: see original paper] shows the bifurcation diagram of system behavior evolution with damping coefficient (c1) variation. As expected, increasing damping indeed has a stabilizing effect because the equilibrium solution is stable for larger damping coefficients at any time delay . Reducing damping causes the system to lose stability. For the illustrated time delay value, non-dimensional heating power, and heater position parameters, there exists a critical value $c1 = 0.12$. Below this value, initial perturbations of any amplitude ultimately develop into limit cycles with stable amplitude; this critical value is the subcritical Hopf bifurcation point. When $c1 < 0.08$, for initial perturbations of any amplitude, the system ultimately develops into limit cycles with stable amplitude. When $0.08 < c1 < 0.12$, this region is the bistable region, where large-amplitude initial perturbations develop into stable limit cycles while small-amplitude perturbations ultimately tend to equilibrium. As the damping coefficient c1 increases further, when $c1 > 0.12$, initial perturbations of any amplitude ultimately tend to stability, which is the fold point.

3.3 Effect of Heater Position

Heater position (xf) also significantly affects system dynamics, with changes in heater position achieved by placing the heater at different locations along the duct length. System stability changes depend on heater position within the duct in a particular manner. The bifurcation diagram for heater position (xf) variation is shown in [Figure 7: see original paper]. As heater position varies from the upstream open end, the system is initially linearly stable. At the critical value xf1 of heater position, the system evolves to linear instability. With further changes in heater position, the system remains linearly unstable until heater position reaches xf2, where another Hopf bifurcation point forms and the system becomes stable again. The bifurcation diagram for heater position (xf) shows that Hopf bifurcation points at both ends of the tube are subcritical Hopf bifurcations. Between the two Hopf bifurcation points, there exists a globally unstable region. In the globally unstable region, the system asymptotically approaches limit cycles with corresponding stable amplitude for any initial condition. From the bifurcation diagram for heater position variation, we can obtain the specific values of the two subcritical Hopf bifurcation points. For the selected other system parameters, when $xf < 0.18$, the system ultimately tends to stability for perturbations of any amplitude. When $0.18 < xf < 0.25$, the system is in the bistable region, where different evolution behaviors occur depending on initial conditions, giving the first subcritical Hopf bifurcation point position $xf1 = 0.18$. When $0.25 < xf < 0.42$, the system is in the globally unstable region, where the system asymptotically approaches limit cycles with corresponding stable amplitude for any initial condition. When $0.42 < xf < 0.48$, the system enters the bistable region again, giving the second subcritical Hopf bifurcation point position $xf2 = 0.48$. With further changes in heater position, when $xf > 0.48$, the system re-enters the globally stable region, ultimately tending to stability for perturbations of any amplitude.

3.4 Effect of Time Delay

In the Rijke tube, due to thermal inertia between the heater and flowing medium, there exists a time delay between heat transfer and flow rate changes, whose magnitude is related to the diameter of the heating wire in the heater and the incoming flow velocity. Under actual experimental conditions, the time delay term ( ) is difficult to change precisely. The bifurcation diagram corresponding to time delay ( ) variation is shown in [Figure 8: see original paper]. The subcritical Hopf bifurcation point corresponds to the time delay value $= 0.05$. When < 0.05, the system is in a linearly stable state, ultimately tending to a stable state for initial perturbations of any amplitude. When $0.05 < \; < 0.12$, the system is in the bistable region, where small-amplitude perturbations ultimately tend to equilibrium while large-amplitude perturbations develop into limit cycles. When > 0.12, the system is in the linearly unstable region, where arbitrarily small initial perturbations ultimately develop into limit cycles. Notably, within the linearly unstable region of $0.12 < \; < 0.45$, as time delay ( ) gradually increases, the amplitude value first increases and then decreases. This phenomenon can be explained by Rayleigh' s criterion. The existence of time delay is due to thermal inertia between heat release from the source and flow. When heater power increases, time delay increases accordingly. Increasing time delay changes the phase difference between acoustic field oscillations and heat release pulsations. As this phase difference gradually increases and moves away from 0°, the oscillation amplitude gradually decreases, forming this phenomenon. It can also be inferred that when 0.3, the phase difference between acoustic field oscillations and heat release pulsations is exactly 0°, resulting in maximum oscillation amplitude.

4 Conclusions

This paper models the Rijke tube thermoacoustic system, numerically solves the control equations using the Galerkin method, and analyzes relevant dynamic characteristics, leading to the following conclusions:

(1) When solving the Rijke tube thermoacoustic instability model, using 10th-order Galerkin modes is sufficient to ensure convergence.

(2) The bifurcation behavior of the Rijke tube thermoacoustic system belongs to subcritical Hopf bifurcation. The system has one linear stability boundary and one nonlinear stability boundary, with a bistable region between them.

(3) Bifurcation diagrams are obtained for system parameters including non-dimensional heating power K, heater position xf, damping coefficient c1, and time delay . The bifurcation diagrams for non-dimensional heating power K, damping coefficient c1, and time delay exhibit only one Hopf bifurcation point, while the bifurcation diagram for heater position xf exhibits two Hopf bifurcation points.

References

[References section preserved as in original]

Note: Figure translations are in progress. See original paper for figures.

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