Fractal Transport Properties of Reservoir Pores Based on the Multiscale Dimensional Unification Principle (Postprint)

Authors: Wang Zhiguo^{1,2}, Zhang Conghui^{1}, Jia Yongying^{1,2}, Yin Chailing^{1}, Liu Xiaoyan^{1,2}

Date: 2018-01-05T00:00:00+00:00

Abstract

Oil reservoir pore distributions exhibit fractal characteristics. Current research methods on applying fractal description approaches to investigate fluid transport properties in reservoir porous media have certain limitations, primarily due to the lack of consideration for dimensional consistency principles and scale effects. This paper, based on the principle of dimensional consistency, employs fractal description methods to derive permeability and seepage flow rate for transport processes in reservoir porous media, obtaining unit scale conversion coefficients for transport characteristic parameters. This enables analysis of macroscopic transport processes based on microscopic fractal characteristic descriptions. The fractal transport description equation established herein facilitates conversion between different scale hierarchies, yielding macroscopic characteristic parameter results that are more consistent with measured values. Using the model developed in this study, data from an actual oil reservoir were analyzed and validated, demonstrating that the variation patterns of transport flow rate and permeability align with the distribution patterns of real oil reservoirs.

Full Text

Study on Fractal Transport Characteristics of Reservoir Pores Based on the Principle of Multi-Scale Dimensional Unity

WANG Zhiguo[1,2], ZHANG Conghui[1], JIA Yongying[1,2], YIN Chailing[1], LIU Xiaoyan[1,2]

[1]College of Civil and Construction Engineering, Northeast Petroleum University, Daqing 163318, China

[2]Heilongjiang Key Laboratory of Disaster Prevention, Mitigation and Protection Engineering, Northeast Petroleum University, Daqing 163318, China

Abstract

Reservoir pore distributions exhibit fractal characteristics. However, current research methods for applying fractal description techniques to fluid transport properties in reservoir porous media suffer from certain limitations, primarily the failure to consider dimensional unity principles and scale effects. Based on the principle of dimensional unity, this paper employs fractal description methods to derive permeability and seepage flow in porous media, obtaining unit scale conversion coefficients for transport characteristic parameters. This enables macroscopic transport process analysis based on microscopic fractal feature descriptions. The proposed fractal transport equations facilitate conversion between different scale levels, yielding macroscopic characteristic parameter results that more closely match measured values. Using actual reservoir data, the model was validated, demonstrating that the variation patterns of transport flow and permeability align with the distribution laws of real reservoirs.

Keywords: transport characteristics; fractal; dimensional unity; scale conversion; reservoir pores

Introduction

Transport in porous media refers to the transfer of fluids, gases, electricity, heat, sound, waves, etc., within pore spaces. Research on fluid transport characteristics in porous media is typically associated with reservoir development, as the properties of reservoir storage media can directly or indirectly reflect subsurface reservoir conditions, enabling understanding and prediction of reservoir dynamics to enhance oil and gas recovery.

Studies of pore structures can be traced back to the 1920s when Nutting first investigated reservoir pore characteristics using capillary curve theory combined with thin section methods. Since then, researchers have gradually recognized that actual reservoir pore structures are extremely complex and cannot be adequately described using traditional mathematical methods. In the 1980s, B.B. Mandelbrot introduced the concept of fractals after studying irregular patterns in nature, providing a tool for handling extremely irregular complex geometries. Subsequent studies demonstrated that reservoir rocks exhibit excellent fractal characteristics at both macroscopic and microscopic scales. Avnir et al. proved the fractal nature of reservoir rock pore structures using molecular adsorption methods. Katz and Thompson observed rock fractures using scanning electron microscopy, finding that various sandstones, shales, and carbonate rocks are self-similar across three to four orders of magnitude. They described pore structure characteristics using fractal dimensions ranging from 2.55 to 2.87, with statistical self-similarity scales between 0.2 and 50. They also predicted rock porosity values using statistical fractal methods, showing good agreement with measured values. Hansen derived the volume fractal dimension, surface fractal dimension, and their relationship with rock permeability for sandstones using the box-counting method, demonstrating that rock permeability depends on fractal dimension magnitude. Kim Tae H et al. developed fractal discrete fracture network code based on fractal theory, establishing FDFN models to obtain reservoir porosity values. Xie S Y et al. used electron microscopy to present two-dimensional images and calculated fractal dimensions using box-counting and multifractal parameters to establish relationships with reservoir porosity and permeability for evaluating reserves in different formations.

With scientific and technological advances and deepening oil and gas exploration, domestic scholars have also made significant contributions. Tong Dengke et al. used fractal dimensions and fractal exponents to describe seepage characteristics in fractal reservoirs, analyzing the influence of fractal parameters on reservoir pressure with predictions closely matching actual reservoir data. Yu Boming et al. established fractal seepage models for reservoir porous media, deriving analytical solutions for various parameters. Liu Junliang et al. derived porosity, permeability, and other characteristic parameters for fractal porous media using Sierpinski carpets, discussing relationships between permeability and fractal dimension under different models. Kong Xiangyan et al. derived formulas for seepage velocity, permeability, and porosity in dual-fractal-dimension porous media, plotting type curves for reservoir formation analysis. Wang Ziming et al. combined fractal characteristics of carbonate reservoir fractures with permeability data to establish permeability variogram functions, proving consistency with actual conditions. Liu Bo et al. obtained fractal dimensions of low-permeability reservoir rocks from mercury injection capillary pressure data, comparing them with other rock heterogeneity parameters and concluding that higher average fractal dimensions correspond to stronger heterogeneity, lower oilfield production, and higher water cut. Yu Benfu et al. established a fractal seepage model for reservoir pressure distribution, deriving dynamic reservoir pressure prediction formulas and demonstrating the influence of fractal dimensions on reservoir pressure.

In summary, researchers studying fluid transport characteristics in reservoir porous media using fractal theory have primarily established fractal reservoir seepage models to approximately describe geological pore structures, achieving certain scientific results. However, actual pore structures exhibit non-uniform variations in internal properties and spatial distribution, resulting in structural anisotropy across different spatial directions. For instance, some large pores exist at millimeter or centimeter scales, while tiny pores are only nanometer or micrometer scale, causing pore sizes to span several scale levels from molecular to microscopic or microscopic to macroscopic scales. Although reservoir pores exhibit fractal characteristics across different scale ranges, their impact on transport properties varies significantly. Large scales reflect overall reservoir pore properties and patterns, while small scales represent local characteristics. This creates fractal scale effect issues when simply combining both scales for reservoir analysis—patterns summarized at one scale level may not apply at another, causing substantial deviations from actual values. Therefore, this paper employs fractal description methods based on dimensional unity principles to derive transport process model equations for reservoir porous media and conducts practical analysis.

---

1. General Fractal Description Method for Seepage

Transport characteristics in porous media are described by a series of characteristic parameters, with porosity and permeability being the two most important physical parameters. Research on these quantities has been a major focus. Studies have shown that naturally formed porous media and some artificially manufactured materials exhibit fractal characteristics in their microstructures. Using fractal geometry theory, various theoretical models have been proposed to describe porous media microstructures and establish relationships between transport problems and fractal dimensions.

Permeability has a functional relationship with porosity, fractal dimension, and other parameters, expressed as:

$$\langle MATH_0 \rangle$$

Based on fluid flow states, permeability expressions can be derived using capillary bundle models:

$$\langle MATH_1 \rangle$$

where N represents the total number of tortuous capillaries or pores in a unit cell, τ is the average tortuosity defined as the ratio of actual capillary length to macroscopic length, and δ is the ratio of minimum to maximum pore radii. However, this derivation lacks precision as coefficients in the calculation of total number N lack clear concepts, and the definition of average tortuosity τ is ambiguous.

A second model is the porous media seepage mechanics model, which derives permeability from the logarithmic linear relationship between Fanning friction coefficient f and Reynolds number Re under certain conditions:

$$\langle MATH_2 \rangle$$

where C is a constant obtainable from f and Re. However, this formula requires restrictive conditions that limit seepage velocity in porous media, and the proposed correlations have not been validated against experimental results.

Yu Boming et al. established a fractal analytical solution model combining the Hagen-Poiseuille equation to derive the total flow relationship:

$$\langle MATH_3 \rangle$$

where ΔP is the pressure difference between inlet and outlet, μ is fluid viscosity, L is the apparent length of tortuous capillaries or pores, D_f is the porous medium fractal dimension, and D_T is the tortuosity fractal dimension. Combining the flow formula with Darcy' s law yields the permeability function:

$$\langle MATH_4 \rangle$$

where A is the cross-sectional area of a unit cell. This formula contains no empirical constants, and each parameter has clear physical meaning. However, it fails to address dimensional unity issues. Without understanding pore variation patterns and scale dependencies, parameters are arbitrarily substituted without theoretical basis, creating dimensional inconsistencies that inevitably deviate from actual results. Since permeability and flow are macroscopic properties while fractal dimensions characterize microscopic pore structures, a medium is needed to unify dimensions when calculating properties across scales. This paper focuses on deriving unit scale conversion coefficients to modify existing formulas, achieving dimensional unity and reducing errors for more realistic values.

---

2. Fractal Seepage Description Based on Dimensional Unity Principle

Previous description methods have significant defects. First, the single-capillary flow expression involves two different scale-level parameters: capillary length L (macroscopic scale) and capillary diameter λ (microscopic scale), which affect pore characteristics differently. Second, the derivation process fails to define the dimensional scale for each characteristic parameter, resulting in inconsistent dimensions and units on the right side of the equation when reflecting flow distribution patterns in pores. Therefore, this basic seepage formula is incomplete and imperfect. When combining these terms, dimensional unity must be achieved through a scale conversion coefficient.

Starting from equations containing both length L and diameter λ, we unify them to macroscopic scale to obtain:

$$\langle MATH_5 \rangle$$

where α represents the order of magnitude needed for consistency between different scale levels, determined by the relationship between artificially defined scales and internationally unified dimensions.

Substituting the converted capillary length equation into the flow equation yields:

$$\langle MATH_6 \rangle$$

The flow through a unit cell cross-section is:

$$\langle MATH_7 \rangle$$

Using Darcy' s law, the permeability formula becomes:

$$\langle MATH_8 \rangle$$

where $\beta = \alpha^{1-D_T}$ is called the flow scale conversion coefficient.

---

3. Validation and Application of the Modified Fractal Seepage Formula

Based on the established flow and permeability model equations for reservoir porous media, this section numerically calculates macroscopic transport characteristic parameters (flow and permeability) for four rock cores to analyze the influence of microscopic pore features on macroscopic transport parameters.

3.1 Experimental Data and Basic Parameters Four different oilfield core samples were used for numerical calculations, including three sandstones and one carbonate rock. The original three-dimensional data were obtained through micro-CT scanning imaging systems, enabling intuitive description of core characteristics. [Figure 1: see original paper] shows the three-dimensional image of sandstone B1 digital core, where transparent parts represent pores and colored parts represent solid skeletons, with X, Y, Z representing different directions. [Figure 2: see original paper] displays a two-dimensional cross-section of B1 in the Z-direction, where white indicates pores and black indicates solids.

Basic physical parameters were calculated to more accurately describe and analyze the reservoir. Experimental data and basic parameters are shown in .

3.2 Fractal Dimension Using core B1 as an example to validate the modified formula, scanning equipment provided the following data: minimum and maximum pore diameters, permeability $K = 1286$ mD, dynamic viscosity $\mu = 1$ mPa · s, porosity $\phi = 0.196$, and pressure difference $\Delta P = 1$ MPa. Based on microscopic parameters, the pore distribution fractal dimension D_f was calculated as 2.484, and the tortuosity fractal dimension D_T as 1.198. The other three core samples were processed similarly.

According to Darcy' s law, all related physical quantities in the formula are macroscopic, thus having no scale level issues and can be directly substituted into the flow equation to obtain the new permeability formula:

$$\langle MATH_9 \rangle$$

This yields the fractal permeability formula conversion coefficient $\beta = 0.831$. The significance of this modified formula lies in prioritizing calculations under specific dimensional scales when describing macroscopic physical properties, then combining them with scale conversion coefficients to achieve dimensional unity on both sides of the equation.

3.3 Comparative Calculation Analysis Comparative calculations of seepage flow and permeability were performed using both original and modified formulas, as detailed in and . The numerical calculations yield the following conclusions:

1. Comparing flow and permeability values at micrometer and millimeter scales reveals that different scale levels produce varying degrees of error, demonstrating the scale effect problem. This occurs because reservoir porous media heterogeneity creates pore structures of different scales at various spatial positions.

2. For core sample , millimeter-scale values are closer to measured values than micrometer-scale values, but both deviate significantly from actual measurements. Analysis suggests that either the extracted core sample contains predominantly large pores in this region, making it more suitable for larger-scale calculations, or the fractal characteristics are not prominent within this scale range, resulting in large errors. This indicates that while microscopic pore structures are being analyzed, smaller scales do not necessarily yield more accurate results, and single scales cannot reflect overall reservoir characteristics.

3. Comparing pre- and post-modification permeability and flow values shows that modified results have smaller errors and are closer to actual values, validating the modified formula. The correction achieves dimensional unity by combining microscopic-scale values with their corresponding dimensional conversion coefficients as a bridge to obtain final macroscopic results.

3.4 Parameter Variation Characteristic Analysis Parameter variation analysis was conducted using core sample . By varying core sample size, minimum and maximum pore diameters, and porosity, different permeability and flow values were obtained to analyze the influence of microscopic pore structure parameters on macroscopic transport characteristics.

Effect of Porosity: As shown in [Figure 3: see original paper], under constant pressure, internal flow and permeability increase with porosity. When porosity ϕ increases, the microscopic pore fractal dimension D_f increases, enhancing pore structure heterogeneity and creating uneven pore distributions that affect seepage performance in different directions. Simultaneously, increased porosity reduces average tortuosity τ and tortuosity fractal dimension D_T, causing pore channels to gradually straighten and facilitating fluid flow, thus increasing permeability and flow.

Effect of Minimum Pore Diameter: [Figure 4: see original paper] demonstrates that under constant porosity, permeability and flow decrease as minimum pore diameter increases. As minimum pore diameter λ_{min} gradually increases, the microscopic pore fractal dimension D_f decreases, making pore size distributions more uniform. However, tortuosity fractal dimension D_T increases, indicating more tortuous fluid flow paths that impede smooth passage through pores, thereby reducing flow and permeability.

Effect of Maximum Pore Diameter: [Figure 5: see original paper] shows that increasing maximum pore diameter gradually increases core permeability and flow. With constant porosity ϕ and unchanged minimum pore diameter λ_{min}, increasing maximum pore diameter λ_{max} enlarges the pore fractal dimension D_f, intensifying heterogeneity and increasing the magnitude difference between maximum and minimum pores. Tiny pores become negligible and have minimal impact on overall flow and permeability. Additionally, increasing λ_{max} reduces D_T, straightening pore channels and facilitating macroscopic transport.

Effect of Core Size: [Figure 6: see original paper] reveals that flow and permeability gradually decrease with increasing core sample size. As the apparent length dimension L increases, the fractal dimension D_f remains unchanged because core length does not affect internal structure, making D_f unsuitable for judging its influence on macroscopic transport parameters. Although increasing core size reduces tortuosity fractal dimension D_T, the pressure difference between core ends remains constant, so flow and permeability still decrease with increasing distance.

---

4. Conclusions

Porous media structures are extremely complex and difficult to describe precisely. Researchers have attempted to introduce fractal theory for description and analysis. However, due to scale differences in pore spatial distribution, scale effects significantly impact macroscopic transport characteristics and should not be ignored. This paper considers dimensional unity principles and scale effects to modify and improve related fractal transport models for porous media. The calculations and analysis yield the following conclusions:

1. Addressing the unclear unit and scale issues in existing porous media frac-

tal seepage models, this paper re-derives relevant fractal transport analysis models and validates them through application to reservoir porous media.

2. Combining microscopic pore scales with macroscopic calculations yields relevant transport characteristic parameters for analyzing scale effects between different levels. Practical analysis demonstrates that smaller scales do not necessarily produce more accurate results for reservoirs.
3. Based on the proposed fractal transport model, oilfield reservoir data were analyzed and calculated to examine the influence of microscopic pore characteristic parameters on macroscopic transport parameters. Results show that flow and permeability increase with porosity and maximum pore diameter but decrease with minimum pore diameter and sample size. The simulated results align with actual reservoir transport characteristic patterns.

References

[4] Mandelbrot B B. How Long Is the Coast of Britain: Statistical Self-Similarity and Fractional Dimension[J]. Science, 1967, 156(3775):636-638

[5] Adler P M, et al. Real Porous Media: Local Geometry and Macroscopic Properties[R]. Applied Mechanics Reviews, 1998, 51(9): 537-585

[6] Thovert J F, Wary F, et al. Thermal Conductivity of Random Media and Regular Fractals[J]. Journal of Applied Physics, 1990,68(8): 3872-3883

[7] Pfeifer P, Avnir D. Chemistry in Non-Integral Dimensions Between Two and Three[J]. The Journal of Chemical Physics, 1983, 7(7) :3369-3558

[8] Katz, A.J and Thompson, A.H. Fractal Sandstone Pores: Implications for Conductivity and Pore Formation[J]. Physical Review Letters, 1985, 54:1325-1328

[9] Hansen, J.P. and Skjeltorp, A.T.. Fractal Pore Space and Rock Permeability Implications[J]. Physical Review B, 1988, 38(4):2635-2638

[10] Kim, Tae H, Schechter, David S. Estimation of Fracture Porosity of Naturally Fractured Reservoirs with No Matrix Porosity Using Fractal Discrete Fracture Networks[J]. Reservoir Evaluation and Engineering, 2009, 12(2):232-242

[11] XIE Shuyun, CHENG Qiuming, et al. Fractal and multifractal analysis of carbonate pore-scale digital images of petroleum reservoirs[J]. Marine and Petroleum Geology,2010, 27(2):476-485

[12] TONG Dengke, GE Jiali. An Exact Solution For Unsteady Seepage Flow Through Fractal Reservoir[J]. Acta Mechanica Sinica,1998,30(5):621-627

[13] LIU Junliang, TIAN Changan, et al. Fractal Dimensionality Dependence of Microstructural Parameters and Permeability in Fractal Porous Media[J]. Advances in Water Science,2006, 17(6):812-817

[14] SUN Xiangyan, LI Daolun, et al. Basic Formulas of Fractal Seepage and Type-Curves of Fractal Reservoirs[J]. Advances in Water Science（Natural Science）,2007, 22(2): 1-5

[15] WANG Ziming, SONG Wenjie, et al. Application of Fractal Simulation Technique to Modeling of Fractured Carbonate Reservoir[J]. Natural Gas Industry,2007, 27(11):73-75

[16] LIU Bo, FAN Xiaodong, et al. Fractal Characterization of Heterogeneity in Low Permeability Rock and Its Effect on Production Performance[J]. Journal of Liaoning Technical University（Natural Science）, 2009, 28(1):150-152

[17] YU Benfu, YAN Xiangzhen et al. Dynamic Reservoir Pressure Prediction of Depleted Reservoir Gas Storage Considering the Fractal Feature of Porous Medium[J]. Petroleum Science, 2013, 34(5):1017-1022

Note: Figure translations are in progress. See original paper for figures.