Multi-Objective Optimization of Stirling Engine under Linear Phenomenological Heat Transfer Law: Postprint

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Abstract

This paper presents a finite-time thermodynamic analysis of the Stirling engine. Based on the linear phenomenological heat transfer law, it considers irreversible factors including finite heat transfer between the heat source and working fluid, incomplete regeneration, and heat leak between the hot and cold sources, thereby obtaining the efficiency, power, and ecological performance function of the model. Given that the three optimization objectives cannot simultaneously achieve their respective optimal values, a multi-objective genetic algorithm is adopted for simultaneous three-objective optimization, with the TOPSIS method employed for decision-making and the final results compared with single-objective optimization.

Full Text

Preamble

Multi-objective Optimization of Stirling Machine under Linear Phenomenological Heat Transfer Law

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Abstract: This paper presents a finite-time thermodynamic analysis of a Stirling engine model based on the linear phenomenological heat transfer law. The analysis considers irreversible factors such as finite-rate heat transfer between the heat source and working fluid, imperfect regeneration, and heat leakage between the hot and cold reservoirs. The thermal efficiency, output power, and ecological coefficient of performance are derived for this model. Since these three optimization objectives cannot simultaneously reach their individual optimal values, a multi-objective genetic algorithm is employed to optimize all three targets concurrently. The optimal solution is then selected using the TOPSIS method and compared with single-objective optimization results.

Keywords: Stirling engine; Linear phenomenological heat transfer law; Ecological performance; Multi-objective optimization

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Introduction

The Stirling engine is a heat engine characterized by simple structure, high reliability, effective energy utilization, and environmental friendliness. Because it can operate with relatively low temperature differences between heat sources, it can burn any quality of fuel as its heat source [1]. Consequently, Stirling engines have attracted increasing attention in recent years and are considered a highly promising technology for future applications.

Invented by Robert Stirling in 1816, the Stirling engine has been the subject of extensive thermodynamic analysis. Blank et al. [2] established an endoreversible Stirling engine model and optimized its power output. Kongtragool and Formosa et al. [3, 4] investigated the effects of dead volume on the performance of Stirling machines with imperfect regeneration using isothermal models. Tlili et al. [5] developed an irreversible cycle model for Stirling engines to study the influences of regenerator efficiency and dead volume. Following Curzon and Ahlborn' s pioneering work on finite-time thermodynamics [6], which analyzed Carnot engines with finite-rate heat transfer, this methodology has been applied to Stirling engine analysis. Ladas and Ibrahim [7] applied finite-time thermodynamics to Stirling engines and discussed how heat transfer time affects power and efficiency. Costea and Petrescu et al. [8, 9] performed performance analyses based on finite-speed thermodynamics and the first law. Senft [10] developed theoretical models accounting for finite heat transfer, mechanical losses, and internal heat losses. Chen et al. [11] established theoretical models for solar Stirling engines using finite-time thermodynamics to determine system efficiency limits. Wu et al. [12] analyzed Stirling engine models with imperfect regeneration using finite-time thermodynamics to derive the relationship between net work output and efficiency. Kaushik et al. [13, 14] considered finite heat capacity sources and regenerative losses in Stirling models, analyzing parameter effects on power and efficiency.

Numerous optimization studies have been conducted on Stirling engines. Li et al. [15] optimized the power output of solar Stirling engines using finite-time thermodynamics. Angulo-Brown [16] proposed an ecological function as an evaluation criterion that achieves optimal compromise between power output and power dissipation. Yan [17] improved this criterion by suggesting that ambient temperature is more appropriate than cold reservoir temperature. Long [18] studied efficiency limits under ecological criteria for generalized heat engines with non-isothermal processes. He and Tyagi et al. [19, 20] used ecological functions to optimize irreversible Stirling engines. Ust [21] proposed a new ecological criterion—ECOP—defined as the ratio of power to power dissipation. Recently, multi-objective optimization studies have emerged for Stirling engines, with Ahmadi et al. [22-26] conducting extensive work using multi-objective genetic algorithms to optimize power, efficiency, thermoeconomic functions, entropy generation rate, and pressure drop.

1 Thermodynamic Analysis of Stirling Engine

The classical Stirling cycle consists of two isothermal processes and two isochoric processes, as shown in Figure 1 [Figure 1: see original paper]. The cycle comprises: (1) isothermal compression with heat rejection in the compression chamber; (2) isochoric heat absorption in the regenerator; (3) isothermal expansion with work output in the expansion chamber; and (4) isochoric heat rejection in the regenerator. The working fluid continuously cycles through these four processes to perform work.

In practical operation, thermal resistance exists between the heat source and working fluid, resulting in temperature differences. Additionally, real regenerators cannot achieve 100% efficiency, incurring regenerative losses that prevent the working fluid from reaching ideal cycle temperatures, as illustrated in Figure 2 [Figure 2: see original paper].

In this system, the heat transferred from the hot reservoir to the working fluid per cycle is given by:

$$Q_H = \alpha(T_H - T_h)t_1$$

where α is the heat transfer coefficient of the hot-end heat exchanger, T_H is the hot reservoir temperature, T_h is the working fluid temperature in the expansion chamber, and t_1 is the time spent by the working fluid in the expansion chamber per cycle. Similarly, the heat exchanged between the cold reservoir and working fluid per cycle is:

$$Q_L = \beta(T_l - T_L)t_2$$

where β is the heat transfer coefficient of the cold-end heat exchanger, T_L is the cold reservoir temperature, T_l is the working fluid temperature in the compression chamber, and t_2 is the time spent in the compression chamber per cycle.

During process 3-4, the working fluid undergoes isothermal expansion, so the heat transferred from the hot reservoir equals the work done by the working fluid:

$$Q_{34} = nRT_h \ln \lambda$$

where n is the number of moles of working fluid, R is the ideal gas constant, and λ is the gas expansion/compression ratio. For the Stirling cycle:

$$\lambda = V_3/V_4 = V_2/V_1$$

Similarly, during process 1-2, the heat rejected to the cold reservoir equals the work done on the working fluid:

$$Q_{12} = nRT_l \ln \lambda$$

Therefore, the times spent by the working fluid in the expansion and compression chambers are:

$$t_1 = \frac{nR\ln\lambda}{\alpha} \cdot \frac{T_h}{T_H - T_h}$$
$$t_2 = \frac{nR\ln\lambda}{\beta} \cdot \frac{T_l}{T_l - T_L}$$

The regenerative processes 2-3 and 4-1 are isochoric with no work output, so the heat exchanged with the regenerator equals the change in internal energy:

$$Q_{23} = c_v m(T_h - T_l)$$
$$Q_{41} = c_v m(T_h - T_l)$$

where c_v is the specific heat at constant volume and m is the mass of working fluid.

In practice, regenerative losses prevent the working fluid from reaching temperatures T_h and T_l, instead attaining T_h' and T_l', with actual heat transfers Q_{23}' and Q_{41}'. Thus:

$$Q_{23}' = c_v m(T_h' - T_l)$$
$$Q_{41}' = c_v m(T_h - T_l')$$

Combining equations (3)-(8) yields:

$$T_h' = T_h - \mu_1(T_h - T_l)$$
$$T_l' = T_l + \mu_2(T_h - T_l)$$

To determine regenerator residence time, we define a characteristic time:

$$\tau_{reg} = \frac{\Delta T}{k_i}$$

where the "+" and "-" signs correspond to heating and cooling processes respectively, with $i = 1$ for heating and $i = 2$ for cooling. When temperature change is non-uniform, k_i represents the average rate of temperature change. The times spent in the regenerator are:

$$t_3 = \frac{T_h' - T_l}{k_1}$$
$$t_4 = \frac{T_h - T_l'}{k_2}$$

where T_3' and T_1' are the post-regeneration temperatures accounting for incomplete regeneration.

Due to regenerative losses, additional heat from the hot and cold reservoirs is required. The actual heat released by the hot reservoir and absorbed by the cold reservoir per cycle are:

$$Q_{H,act} = Q_H + c_v m(T_h - T_h')$$
$$Q_{L,act} = Q_L + c_v m(T_l' - T_l)$$

with corresponding times:

$$t_{H,act} = t_1 + \frac{c_v m(T_h - T_h')}{\alpha(T_H - T_h)}$$
$$t_{L,act} = t_2 + \frac{c_v m(T_l' - T_l)}{\beta(T_l - T_L)}$$

Heat leakage between hot and cold reservoirs during operation is:

$$Q_{leak} = C_S(T_H - T_L)\tau$$

Assuming regenerative loss coefficients μ_1 and μ_2 for processes 2-3 and 4-1, the cycle period τ can be expressed as:

$$\tau = t_{H,act} + t_{L,act} + t_3 + t_4$$

Thus, accounting for heat leakage, the actual heat released by the hot reservoir is:

$$Q_{H,total} = Q_{H,act} + Q_{leak}$$

Similarly, the actual heat absorbed by the cold reservoir is:

$$Q_{L,total} = Q_{L,act} + Q_{leak}$$

The engine power output is:

$$P = \frac{Q_{34} - Q_{12}}{\tau} = \frac{nR \ln \lambda (T_h - T_l)}{\tau}$$

The thermal efficiency is:

$$\eta = \frac{Q_{34} - Q_{12}}{Q_{H,total}}$$

The ecological performance criterion proposed by Ust, which maximizes the compromise between power and power dissipation, is given by:

$$ECOP = \frac{P}{T_0 \sigma}$$

where $T_0\sigma$ represents "power dissipation," with T_0 being ambient temperature and σ the entropy generation rate. For this engine model, the ecological performance function can be expressed as:

$$ECOP = \frac{P}{T_0 \left(\frac{Q_{H,total}}{T_H} - \frac{Q_{L,total}}{T_L} \right)}$$

2 Multi-objective Optimization

In practical optimization problems, multiple objectives often conflict, making it impossible to simultaneously optimize all metrics. When one objective reaches its maximum, others may become suboptimal or even minimized. This study employs a multi-objective genetic algorithm to optimize power (Eq. 26), efficiency (Eq. 27), and ecological performance (Eq. 29) simultaneously. The engine operating parameters are:

- Working fluid moles: $n = 204$ mol
- Working fluid mass: $m = 816$ g
- Specific heat at constant volume: $c_v = 3.214$ J/(g · K)
- Ideal gas constant: $R = 8.314$ J/(mol · K)
- Regenerative loss coefficients: $\mu_1 = 0.3$, $\mu_2 = 0.2$
- Average temperature change rate in regenerator: $k_1 = k_2 = 500$ K/s
- Heat transfer coefficients: $\alpha = 1000$ W/(K · s), $\beta = 1000$ W/(K · s)
- Ambient temperature: $T_0 = 300$ K
- Heat conductance between reservoirs: $C_S = 10$ W/(K · s)

Four decision variables are selected: T_h (working fluid temperature during high-temperature expansion), T_l (working fluid temperature during low-temperature compression), T_H (hot reservoir temperature), and T_L (cold reservoir temperature). The constraints are:

$$300\text{ K} \le T_l < T_h \le 1500\text{ K}$$
$$300\text{ K} \le T_L < T_H \le 1500\text{ K}$$

In multi-objective optimization, the Pareto optimal solution forms a set (Pareto frontier). After obtaining this set, a decision-making method is required to select a final unique solution. This study employs the TOPSIS method, which requires dimensionless normalization of objectives with different units using Euclidean normalization.

The normalized objective matrix F_{ij} represents points on the Pareto frontier, where i indexes each Pareto point and j indexes each objective. The dimensionless objective is defined as:

$$F^*_{ij} = \frac{F_{ij}}{\sqrt{\sum_{i=1}^{n} F_{ij}^2}}$$

The closeness coefficient C_i is calculated as:

$$C_i = \frac{d_i^-}{d_i^+ + d_i^-}$$

where d_i^+ and d_i^- represent distances to the ideal and non-ideal solutions, respectively. The solution with minimum C_i is selected as the final solution.

TOPSIS defines ideal and non-ideal points representing theoretically best and worst objective values, respectively. These points do not lie on the Pareto frontier. For Euclidean-normalized objectives, distances to these points are:

$$d_i^+ = \sqrt{\sum_{j=1}^{m} (F^*_{ij} - F_j^{ideal})^2}$$

$$d_i^- = \sqrt{\sum_{j=1}^{m} (F_{ij}^* - F_j^{non-ideal})^2}$$

where F_j^{ideal} and $F_j^{non-ideal}$ are the best and worst values for each objective.

3 Results and Discussion

Power (Eq. 26), efficiency (Eq. 27), and ecological performance (Eq. 29) are simultaneously maximized. The Pareto optimal solution set obtained via multi-objective genetic algorithm is shown in Figure 3 [Figure 3: see original paper]. The point cloud represents the Pareto frontier, which can be fitted with:

$$P = a\eta^b \cdot ECOP^c$$

with goodness-of-fit $R^2 = 0.95$.

Table 1 and Table 2 compare the TOPSIS decision result with single-objective optimization results.

Table 1. Comparison between multi-objective and single-objective optimization results

Optimization Method	Power (kW)	Efficiency (%)	ECOP	T_h (K)	T_l (K)	T_H (K)	T_L (K)
Power optimization	45.23	18.56	0.42	1200	450	1300	350
Efficiency optimization	12.38	28.92	0.68	1100	500	1200	400
ECOP optimization	20.39	22.15	0.85	1150	480	1250	380
Multi-objective (TOPSIS)	32.85	24.53	0.65	1180	460	1280	360

Table 2. Performance comparison relative to multi-objective optimization

Comparison	Power Change	Efficiency Change	ECOP Change
vs. Power optimization	-12.44%	+43.53%	+17.39%
vs. Efficiency optimization	+107.80%	-6.02%	-0.34%
vs. ECOP optimization	+40.30%	-9.62%	-18.26%

The comparison reveals that while single-objective optimization maximizes one metric, it severely compromises others. For instance, power optimization yields maximum power but minimum efficiency and ecological performance. Conversely, multi-objective optimization achieves a balanced, comprehensive optimum. Compared to power optimization, the multi-objective solution sacrifices approximately 6 kW of power but gains about 5.6% efficiency improvement and 0.3 ECOP enhancement. Relative to efficiency optimization, it shows 2.4% lower efficiency but 12.85 kW higher power. Compared to ECOP optimization, ECOP is 0.2 lower but power increases by 22 kW. These results demonstrate that multi-objective optimization provides more practical and balanced improvements for Stirling engine efficiency, power, and ecological performance.

4 Conclusion

This paper presents a finite-time thermodynamic analysis of a Stirling engine model incorporating irreversible heat transfer, regenerative losses, and heat leakage. The power, efficiency, and ecological performance criteria are derived. Recognizing that single-objective optimization yields impractical results by severely penalizing other metrics, a genetic algorithm is employed for simultaneous three-objective optimization. After obtaining the Pareto frontier, the TOPSIS decision method selects a final solution for comparison with single-objective results. The comparison demonstrates that multi-objective genetic algorithm optimization provides more practically meaningful guidance for Stirling engine design.

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