Postprint: Performance Study of Quantum Stirling Refrigeration Cycle

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Abstract

An irreversible quantum Stirling refrigeration cycle model is established with particles in infinitely many infinite potential wells as the working substance. The distribution of particles in the potential wells over energy levels is determined by the Gibbs distribution function. This refrigeration cycle consists of two isoenergetic processes and two processes of constant potential well width, and features heat leak between high- and low-temperature reservoirs as well as incomplete regeneration factors. Analytical expressions for the coefficient of performance, refrigeration rate, entropy production rate, and ecological objective function are derived; the relationship curve between refrigeration rate and ecological function is a twisted leaf shape returning to the origin. The influence of the heat leak coefficient and incomplete regeneration factor on the coefficient of performance, refrigeration rate, and ecological function is analyzed.

Full Text

Study on the Performance of Quantum Stirling Refrigeration Cycle

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Abstract

An irreversible quantum Stirling refrigeration model is established in which the working substance consists of countless replicas of a single particle contained in a one-dimensional infinite potential well. The occupation probabilities of the particle in each eigenstate are governed by the Gibbs distribution. The cycle comprises two iso-energy processes and two isochoric processes (constant potential well width), incorporating heat leakage between high- and low-temperature reservoirs and imperfect regeneration. Analytical expressions for the coefficient of performance (COP), cooling rate, entropy generation rate, and ecological objective function are derived. The relationship curve between cooling rate and ecological function exhibits a loop-shaped characteristic that returns to the origin. The effects of heat leakage coefficient and imperfect regeneration factor on COP, cooling rate, and ecological function are analyzed.

Keywords: finite time thermodynamics; one-dimensional infinite potential well; quantum refrigeration cycle; ecological performance optimization

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Introduction

At micro-nanoscales, low temperatures, or high density conditions, the quantum characteristics of working substances in thermodynamic cycles must be considered. Applying finite-time thermodynamics theory and methods enables the establishment of various quantum cycle models to investigate their thermodynamic properties and optimize their operating regimes. In practical heat engines, irreversibilities such as thermal resistance, heat leakage, friction, vortex effects, and non-equilibrium phenomena render cycles irreversible. Several scholars have studied the influence of irreversible factors on quantum thermodynamic cycle performance. Jin et al. introduced bypass heat leakage considering weak coupling between high- and low-temperature heat sources. Feldmann and Kosloff incorporated an internal friction coefficient to describe quantum non-adiabatic phenomena. Wu et al. examined the effects of heat leakage, thermal resistance, and finite-rate heat transfer on irreversible quantum Stirling refrigerators. Jin also investigated optimal performance from an exergoeconomic perspective.

Building upon previous work [3,10,14,16,23,28,30], this paper establishes an irreversible Stirling refrigeration cycle model that incorporates both heat leakage between high- and low-temperature reservoirs and imperfect regeneration. The working substance consists of numerous particles confined in one-dimensional infinite potential wells. We derive analytical expressions for the coefficient of performance, cooling rate, entropy generation rate, and ecological function, and analyze the relationships between these key performance parameters and both the heat leakage rate and imperfect regeneration factor.

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1. Quantum Mechanical Description of the System

This study investigates a particle confined in a one-dimensional infinite potential well of width L. Its stationary wavefunction satisfies the Schrödinger equation. Applying the boundary conditions $\psi(0) = 0$ and $\psi(L) = 0$, the wavefunction solutions are:

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$$

where $\varphi_n(x)$ represents the normalized eigenfunction. The coefficients a_n satisfy the normalization condition $\sum_n |a_n|^2 = 1$. The system' s energy eigenvalues are:

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}$$

where m is the particle mass and n is the quantum number. The Hamiltonian expectation value is:

$$\langle H \rangle = \sum_n p_n E_n$$

with $p_n = |a_n|^2$ representing the probability density. In classical cycles, a piston undergoes reciprocating motion within a cylinder. Assuming the potential well walls behave like classical pistons, both the wavefunction and energy levels vary with the well width L.

The generalized force is defined as:

$$Y_n = -\frac{\partial E_n}{\partial L}$$

Thus, the force acting on the potential well wall can be expressed as:

$$F = \sum_n p_n Y_n = \sum_n p_n \left(-\frac{\partial E_n}{\partial L}\right)$$

2. Quantum Stirling Refrigeration Cycle

We consider particles confined in a one-dimensional infinite potential well. For simplicity, only two energy levels are considered. The working substance comprises countless such particles, each confined in its own infinite potential well, with energy level occupation probabilities determined by the Gibbs distribution. The cycle consists of two iso-energy processes and two isochoric regeneration processes, corresponding to classical isothermal and isochoric processes, respectively.

Accounting for imperfect regeneration and heat leakage between high- and low-temperature reservoirs, the system absorbs heat from a low-temperature source at T_L and rejects heat to a high-temperature source at T_H, as illustrated in [Figure 1: see original paper]. This constitutes an irreversible quantum Stirling refrigeration cycle.

Process 1-2 (Iso-energy): During this process, the total system energy remains constant. The system absorbs heat from the low-temperature source, which is entirely converted into work:

$$Q_{in} = W_{12} = \int_{L_1}^{L_2} F\,dL$$

Process 3-4 (Iso-energy): Similarly, the heat rejected to the high-temperature source equals the work done on the system:

$$Q_{out} = W_{34} = \int_{L_2}^{L_1} F\,dL$$

The probability density of particles occupying a particular energy level follows the Gibbs distribution:

$$p_n = \frac{e^{-E_n/kT}}{Z}$$

where Z is the partition function.

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3. Cycle Period

Processes 2-3 and 4-1 are regeneration processes. In ideal regeneration, the heat exchanged in the regenerator satisfies $Q_{23} = Q_{41}$. Assuming the potential well walls move with velocity $v(t)$ and average speed $\bar{v}$, the time required for iso-energy processes 1-2 and 3-4 is:

$$\tau_{12} = \tau_{34} = \frac{L_2 - L_1}{\bar{v}}$$

Processes 2-3 and 4-1 occur at constant well width, during which the internal energy changes. Assuming the internal energy changes at a constant rate, and since the internal energy is a single-valued function of temperature when L is constant, we have:

$$\frac{dU}{dt} = \pm M$$

where M is a constant independent of system energy, determined solely by regenerator material properties. The positive and negative signs correspond to processes 2-3 and 4-1, respectively. For simplicity, we assume $M_1 = M_2$.

Thus, the time required for regeneration processes 2-3 and 4-1 can be expressed as:

$$\tau_{23} = \frac{\Delta U_{23}}{M}, \quad \tau_{41} = \frac{\Delta U_{41}}{M}$$

Assuming heat leakage exists between the high- and low-temperature reservoirs, the heat leakage rate is:

$$\dot{Q}_{leak} = \alpha(T_H - T_L)$$

where α is a constant. The heat leakage per cycle is:

$$Q_{leak} = \alpha(T_H - T_L)\tau$$

where τ is the cycle period.

In the regeneration process 4-1, ideal regeneration would terminate at state 1. However, due to internal irreversibilities, actual Stirling refrigerators experience regeneration losses, ending the regeneration process at state $1'$. We introduce an imperfect regeneration factor μ ($\mu > 0$) to quantify this loss:

$$Q_{r,loss} = \mu Q_{41}$$

The total cycle period is:

$$\tau = \tau_{12} + \tau_{23} + \tau_{34} + \tau_{41}$$

Consequently, the net heat absorbed from the low-temperature source and the net heat rejected to the high-temperature source are:

$$Q_L = Q_{in} - \frac{1}{2}Q_{leak} - Q_{r,loss}$$

$$Q_H = Q_{out} + \frac{1}{2}Q_{leak} + Q_{r,loss}$$

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4. Key Performance Parameters of the Irreversible Stirling Refrigeration Cycle

From equations (12) and (13), the input work is:

$$W = Q_H - Q_L$$

Combining equations (19) and (30) yields the cooling rate:

$$R = \frac{Q_L}{\tau} = \frac{(2/\pi)^2 \frac{\hbar^2}{2m}(1/x^2 - 1) - \alpha(T_H - T_L) - \mu M}{\tau}$$

where $x = L_2/L_1$ represents the potential well width ratio. The coefficient of performance (COP) of this irreversible quantum Stirling refrigerator is:

$$\text{COP} = \frac{Q_L}{W} = \frac{Q_L}{Q_H - Q_L}$$

The entropy generation rate is:

$$\dot{S}_{gen} = \frac{Q_H}{T_H} - \frac{Q_L}{T_L}$$

The exergy output rate is:

$$\dot{E} = Q_L\left(\frac{T_0}{T_L} - 1\right) - W$$

The ecological objective function is defined as:

$$E = R - T_0\dot{S}_{gen}$$

where T_0 is the environmental temperature. The dimensionless ecological objective function is:

$$E^* = \frac{E}{E_{max}}$$

From equations (27), (28), and (31), for given T_H, T_L, T_0, L_1, M, α, and μ, the cooling rate, COP, and ecological objective function all depend on the potential well width ratio x.

[Figure 2: see original paper] shows the relationship between COP and x for various values of the imperfect regeneration factor μ, with $T_L = 200\,\text{K}$, $T_0 =$

300 K, $M = 15$, and $\alpha = 0.1\,\mathrm{kW/K}$. The COP decreases with increasing x and diminishes as μ increases.

[Figure 3: see original paper] illustrates the effect of heat leakage coefficient α on COP versus x for $\mu = 0.01$, with other parameters identical to [Figure 2: see original paper]. The COP decreases as α increases.

[Figure 4: see original paper] depicts the cooling rate as a function of x for different μ values, with $\alpha = 0.1$. The cooling rate exhibits a parabolic relationship with x. The extremum condition yields the optimal relationship between cooling rate and potential well width ratio.

[Figure 5: see original paper] shows the effect of α on cooling rate versus x for $\mu = 0.01$. The cooling rate decreases with increasing heat leakage.

[Figure 6: see original paper] presents the optimization relationship between dimensionless ecological objective function and cooling rate (E^*-R). The E^*-R curve is loop-shaped, featuring a maximum ecological objective function point and a maximum cooling rate point. Each ecological objective function value (except at the maximum) corresponds to two cooling rate values, with the larger cooling rate being the preferred operating point. While heat leakage does not alter the curve type, it reduces both the ecological objective function value and cooling rate. The impact of heat leakage is more pronounced at higher ecological objective function values.

[Figure 7: see original paper] demonstrates the influence of μ on the E^*-R relationship. Comparing [Figure 6: see original paper] and [Figure 7: see original paper] reveals that heat leakage has a more significant effect on the E^*-R relationship than the imperfect regeneration factor, indicating that heat leakage is the more critical irreversibility.

Conclusion

This paper establishes an irreversible quantum Stirling refrigeration cycle model using countless particles in infinite potential wells as the working substance. Based on solutions to the Schrödinger equation, we analyze the performance of this irreversible quantum Stirling refrigerator, deriving expressions for COP, cooling rate, entropy generation rate, and ecological objective function. The effects of heat leakage coefficient and imperfect regeneration factor on these performance metrics are investigated. The relationship between cooling rate and ecological function exhibits a loop-shaped characteristic returning to the origin, with distinct maxima for both ecological objective function and cooling rate. The region between these two maxima represents the optimal operating range for the refrigerator. These results enhance understanding of irreversible quantum Stirling refrigeration cycle performance using particles in infinite potential wells as working substances.

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Note: Figure translations are in progress. See original paper for figures.