# Coupling of Error Factors in the Concentrating Process of Parabolic Trough Collectors (Postprint)

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## Abstract

During actual operation, the concentrating process of parabolic trough solar collectors may be influenced by various practical factors, including solar incidence angle, installation errors, mirror surface errors, and tracking errors. This paper employs a spatial coordinate transformation method combined with the Monte Carlo ray tracing method to calculate the heat flux distribution of parabolic trough collectors when various practical factors exist individually and when multiple practical factors are coupled. The results demonstrate that installation errors and tracking errors increase the non-uniformity of the focused heat flux, whereas mirror surface errors decrease the non-uniformity of the focused heat flux; the calculated heat flux distribution under coupled practical factors can provide a basis for analyzing the optical and thermal performance of parabolic trough collectors.

## Full Text

## Investigation on Coupled Optical Factors of Concentration Process in Parabolic Trough Collectors

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### Abstract

The concentration process of a parabolic trough collector can be affected by incidence angle, installation error, reflector slope error, and tracking error in operation. The coordinate transformation method and the Monte Carlo Ray Trace method are integrated to compute heat flux distributions under individual and coupled optical errors respectively in this study. It is shown that installation error and tracking error increase the non-uniformity of the heat flux distribution while reflector slope error tends to decrease it. Besides, the heat flux distribution under coupled optical errors can provide foundations for thermal and optical analysis of parabolic trough collectors.

**Key words:** parabolic trough collector; concentration process; flux distribution; optical factor

## Introduction

Parabolic trough collectors (PTCs) represent an important technology for solar thermal power generation. In PTC research, calculating the concentrated heat flux distribution on the absorber tube surface is often the first step. He et al. [1] used the Monte Carlo Ray Tracing (MCRT) method to calculate the concentrated heat flux for an ideal PTC, demonstrating the practicality of the MCRT approach, which has since been widely applied [2-3]. Compared with ideal conditions, the concentration process in PTCs is subject to various practical factors including solar incidence angle, installation error, tracking error, and reflector slope error. Zhao et al. [4] employed spatial coordinate transformation to study the effects of absorber tube installation error, tracking error, and solar incidence angle on PTC heat flux distribution, while Cheng et al. [5] conducted detailed sensitivity analyses of reflector slope error and tracking error. Both studies found that these practical factors significantly alter the concentrated heat flux distribution, necessitating investigation of heat flux distribution on the absorber tube exterior under such conditions.

Current literature often calculates PTC concentrated heat flux for only a single practical factor at a time. However, in practical applications, the concentration process may be simultaneously affected by multiple factors. This paper first calculates and analyzes the concentrated heat flux for individual practical factors, then uses spatial coordinate transformation based on the MCRT method to compute the heat flux distribution under coupled multiple practical factors.

## 1. Heat Flux Distribution Simulation Method

### 1.1 Methodology

[Figure 1: see original paper] shows a schematic diagram of a parabolic trough collector. For simplicity, the glass envelope is not illustrated. The reflector aperture width, focal length, and collector length are denoted by $w$, $f$, and $L$, respectively, with the coordinate origin established at the center of the absorber tube inlet cross-section.

The details of the MCRT method can be found in reference [1]. In this method, if the incident ray direction vector is $\mathbf{d}_i$ and the reflective surface normal vector is $\mathbf{n}$, then the reflected ray direction vector $\mathbf{d}_r$ is calculated as:

$$\mathbf{d}_r = \mathbf{d}_i - 2(\mathbf{d}_i \cdot \mathbf{n})\mathbf{n}$$

Various practical factors essentially affect the absorber tube position, incident ray direction, or mirror normal direction. In the MCRT algorithm, these three aspects operate independently. To investigate each factor’ s influence, one only needs to modify the absorber tube position or calculate new $\mathbf{d}_i$ or $\mathbf{n}$ using spatial coordinate transformation methods.

### 1.2 Installation Error

When installation errors cause translation of the absorber tube, assuming the new coordinates of the tube centerline in the $y$ and $z$ directions are $y_1$ and $z_1$, respectively, the absorber tube outer wall equation becomes:

$$(y - y_1)^2 + (z - z_1)^2 = r^2$$

where $r$ is the outer radius of the absorber tube. Installation errors or support structure deformation may also cause the absorber tube position to become non-parallel with the focal line. For PTCs with dual-axis tracking systems, the $x$-coordinate where a photon is received by the absorber tube equals the $x$-coordinate of the incident point on the mirror surface (denoted as $x_0$). Therefore, once the centerline coordinate equation of the absorber tube is known, the photon arrival position can still be determined.

For simplicity, assuming the inlet cross-section center coordinate is $(0, 0, 0)$ and the outlet cross-section center coordinate is $(L, y_1, z_1)$, the absorber tube outer wall coordinate equation in the plane perpendicular to the focal line containing the reflected ray becomes:

$$(y - \frac{y_1}{L}x)^2 + (z - \frac{z_1}{L}x)^2 = r^2$$

Substituting the new absorber tube coordinate equation (2) or (3) into the MCRT algorithm while keeping other components unchanged yields the non-uniform heat flux distribution on the tube exterior under installation errors.

### 1.3 Solar Incidence Angle

PTC sun tracking employs either single-axis or dual-axis systems. Only dual-axis tracking can maintain direct (0°) incidence, but due to structural complexity, single-axis tracking is generally adopted in engineering practice, where the solar incidence angle is often non-zero, as shown in [Figure 2: see original paper].

Sunlight ray AB strikes point B on the reflector edge at incidence angle $\theta$, reflects to point C on the absorber tube, with distance CD representing the length that cannot receive reflected sunlight from ray passing through B. Reference [6] derived the maximum and minimum values of CD as:

$$L_{\max} = \frac{w}{2}\tan\theta$$

$$L_{\min} = \frac{w}{2}\tan\theta - \frac{r}{\cos\theta}$$

where $L_{\max}$ is the maximum distance from the end affected by end loss, beyond which the total concentrated heat flux remains constant, while tube sections with distance less than $L_{\min}$ from the end cannot receive reflected sunlight.

Figure 3: see original paper shows ray 1 as the direct incident ray calculated by MCRT, and ray 2 as the oblique incident ray with incidence angle $\theta$. The coordinate system O-$x'y'z'$ in Figure 3: see original paper is obtained by rotating the O-$xyz$ system about the $y$-axis by angle $\theta$. The coordinates of ray 1 in this new system represent the coordinates of ray 2 in the original O-$xyz$ system, expressed as $\mathbf{d}_i' = \mathbf{M}_A \cdot \mathbf{d}_i$, where $\mathbf{M}_A$ is the spatial coordinate transformation matrix from O-$xyz$ to O-$x'y'z'$:

$$\mathbf{M}_A = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

Substituting the resulting $\mathbf{d}_i'$ into equation (1) while keeping other components unchanged yields the heat flux distribution at non-zero solar incidence angles. For single-axis tracking systems, determining the intersection between reflected rays and the absorber tube exterior becomes more complex due to incidence angle effects and is not investigated in this study.

### 1.4 Tracking Error

Tracking error refers to deviation of the collector position due to inaccurate tracking, as shown in Figure 4: see original paper. Due to tracking error, collector 1 is rotated counterclockwise by angle $\beta$ relative to its correct position (collector 2). As shown in Figure 4: see original paper, the spatial coordinate transformation matrix is:

$$\mathbf{M}_B = \begin{bmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The modified incident ray coordinates become $\mathbf{d}_i'' = \mathbf{M}_B \cdot \mathbf{d}_i'$. Substituting this into equation (1) while keeping other components unchanged yields the heat flux distribution under tracking errors.

### 1.5 Reflector Slope Error

Reflector slope error accounts for reflection errors caused by microscopic defects or macroscopic deformation of the reflective surface. After superimposing all types of reflector errors, the deviation between actual and ideal mirror normal directions can be used as a substitute [7]. This deviation angle is assumed to have two components in the $x$ and $y$ directions that follow identical Gaussian distributions [8]. The actual mirror normal vector $\mathbf{n}'$ can be obtained from the ideal mirror normal direction $\mathbf{n}$ through two coordinate transformations:

$$\mathbf{n}' = \mathbf{M}_B \cdot \mathbf{M}_A \cdot \mathbf{n}$$

where $\mathbf{M}_A$ and $\mathbf{M}_B$ are calculated using equations (7) and (8), with angles $\theta$ and $\beta$ having the same probability density function:

$$f(\theta) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{\theta^2}{2\sigma^2}\right)$$

where $\sigma$ is the standard deviation representing the reflector slope error angle. Substituting the resulting $\mathbf{n}'$ into equation (1) while keeping other components unchanged yields the heat flux distribution with reflector slope error.

### 1.6 Coupling of Practical Factors

For installation errors, the absorber tube coordinates change relative to the ideal model, which can be accounted for using equation (2) or (3). For tracking error, reflector slope error, solar incidence angle, and other practical factors, the reflected ray direction can be calculated as:

$$\mathbf{d}_r = \mathbf{d}_i' - 2(\mathbf{d}_i' \cdot \mathbf{n}')\mathbf{n}'$$

where $\mathbf{d}_i'$ and $\mathbf{n}'$ are the actual incident ray and mirror normal vectors considering tracking error, reflector slope error, and solar incidence angle.

It is important to note that since matrix multiplication is not commutative, the order of coordinate transformations is not arbitrary. For example, when simultaneously considering incidence angle and tracking error, the actual incident ray coordinates $\mathbf{d}_i'' = \mathbf{M}_B \cdot \mathbf{M}_A \cdot \mathbf{d}_i$ differ from $\mathbf{d}_i'' = \mathbf{M}_A \cdot \mathbf{M}_B \cdot \mathbf{d}_i$. Matrix $\mathbf{M}_A$ transforms direct rays into oblique incident rays, while matrix $\mathbf{M}_B$ accounts for tracking error. Considering the actual physical process, the transformation order should be determined based on how errors are introduced in the system.

## 2. Results and Discussion

Using spatial coordinate transformation matrices, this section simulates and analyzes individual practical factors including installation error and solar incidence angle, followed by calculations under coupled multiple factors. Since the relative magnitude of heat flux distribution on the tube exterior is independent of direct normal irradiance, the local concentration ratio (LCR) is used in the analysis. The LS-3 parabolic trough collector is adopted as the physical model for simulation, with geometric parameters listed in . Conclusions for other collector types can be obtained through similar analysis.

Parameters of the LS-3 PTC

| Parameter | Value |
|---|---|
| Absorber tube radius (m) | 0.035 |
| Focal length (m) | 1.71 |
| Reflector aperture width (m) | 5.76 |
| Glass envelope transmittance | 0.965 |
| Absorber tube absorptance | 0.965 |
| Reflector reflectance | 0.93 |

### 2.1 Effects of Installation Error

Installation errors in the $y$ and $z$ directions are defined as [4]: $Y = y_1/w$ and $Z = z_1/f$. [Figure 5: see original paper] shows the heat flux distribution under translational installation errors. When installation errors reach the millimeter scale, the heat flux pattern changes significantly. [Figure 6: see original paper] and [Figure 7: see original paper] compare the effects of $y$- and $z$-direction errors when both are present. [Figure 6: see original paper] maintains constant $Z$ while varying $Y$, showing substantial changes in heat flux distribution shape. [Figure 7: see original paper] maintains constant $Y$ while varying $Z$, showing relatively minor changes. Therefore, $y$-direction installation error has a greater impact than $z$-direction error, requiring particular attention to minimize $y$-direction errors during absorber tube installation.

[Figure 8: see original paper] illustrates the effect of non-parallel installation on heat flux distribution. With inlet cross-section center at $(0, 0, 0)$ and outlet cross-section center at $(L, y_1, z_1)$ corresponding to $Y = 0.1\%$ and $Z = 0.1\%$, the axial heat flux distribution is non-uniform but can be considered as a continuous arrangement of numerous translational installation error distributions.

### 2.2 Effects of Incidence Angle

Figure 9: see original paper compares axial heat flux distributions for dual-axis and single-axis tracking systems under equal direct normal irradiance, with the vertical axis representing total energy absorbed per cross-section. For the single-axis system, $L_{\max}$ and $L_{\min}$ positions are marked, showing excellent agreement between MCRT simulation and formula calculations, validating the accuracy of MCRT for oblique incidence simulation. In regions unaffected by end loss, each cross-section receives total concentrated heat flux equal to that under direct

incidence with equivalent irradiance, and can be considered axially uniform. Figure 9: see original paper shows that at the absorber tube end, the received concentrated energy increases continuously from zero to a circumferentially non-uniform, axially uniform distribution.

### 2.3 Effects of Reflector Slope Error and Tracking Error

[Figure 10: see original paper] shows the effect of reflector slope error on the concentration process. The circumferential heat flux distribution remains symmetric, but reflector slope error tends to make the concentrated heat flux more uniform, which helps reduce tube wall temperature differences, with larger errors producing more pronounced effects. Figure 10: see original paper shows the intercept factor variation in regions unaffected by end loss. When reflector slope error exceeds 0.12°, the intercept factor falls below 1, which should be avoided.

[Figure 11: see original paper] illustrates the effect of tracking error on the concentration process. Tracking error causes sunlight to deviate from the direction normal to the reflector aperture plane, increasing heat flux non-uniformity. Figure 11: see original paper shows the intercept factor variation with tracking error angle. When tracking error exceeds 0.6°, the intercept factor drops below 1.

### 2.4 Coupling of Practical Factors

[Figure 12: see original paper] presents the heat flux distribution under coupled practical factors: 30° solar incidence angle, $Y = 0.1\%$, $Z = 0.1\%$, 0.2° tracking error, and 0.1° reflector slope error. Figure 12: see original paper shows the circumferential heat flux in regions unaffected by end loss. Due to varying magnitudes of different errors, the concentrated heat flux pattern exhibits diverse shapes. Calculating the coupled heat flux distribution provides a basis for three-dimensional temperature field analysis inside the absorber tube.

## Conclusions

This paper analyzed the effects of practical factors including absorber tube installation error, solar incidence angle, reflector slope error, and tracking error on PTC heat flux distribution, and calculated the coupled heat flux distribution under multiple factors. The main conclusions are:

1. When absorber tube installation errors reach the millimeter scale, the concentrated heat flux pattern changes significantly. $Y$-direction installation error has greater impact than $Z$-direction error, requiring special attention to minimize $Y$-direction errors during installation.
2. In the end loss affected region, the concentrated energy received by the absorber tube increases continuously from zero along the axial direction. Outside this region, each cross-section receives total concentrated heat

flux equivalent to that under direct incidence with equal irradiance, and can be considered axially uniform.

3. Reflector slope error makes concentrated heat flux more uniform, helping reduce tube wall temperature differences, with larger errors producing more pronounced effects. Tracking error increases heat flux nonuniformity, affecting collector safety. When reflector slope error exceeds 0.12° or tracking error exceeds 0.6°, the intercept factor falls below 1.
4. Due to varying magnitudes of different practical errors, the concentrated heat flux pattern exhibits diverse shapes. Calculating the coupled heat flux distribution provides a basis for optical and thermal performance analysis.

## References

[1] He Y L, Xiao J, Cheng Z D, et al. A MCRT and FVM coupled simulation method for energy conversion process in parabolic trough solar collector[J]. Renewable Energy, 2011, 36(3):976-985.

[2] Wang F, Shuai Y, Yuan Y, et al. Thermal stress analysis of solar receiver using eccentric concentrated radiation[J]. Solar Energy, 2010, 84(10):1809-1815.

[3] Wu Z, Li S, Yuan G, et al. Three-dimensional numerical study of heat transfer characteristics of parabolic trough receiver[J]. Applied Energy, 2014, 113(113):902-911.

[4] Zhao Dongming, Xu Ershu, Wang Zhifeng, et al. Influences of installation and tracking errors on the optical performance of a solar parabolic trough collector[J]. Renewable Energy, 2016, 94:197-212.

[5] Cheng Z D, He Y L, Cui F Q, et al. Comparative and sensitive analysis for parabolic trough solar collectors with a detailed Monte Carlo ray-tracing optical model[J]. Applied Energy, 2014, 115(4):559-572.

[6] Xu C, Chen Z, Li M, et al. Research on the end loss effect compensation for parabolic trough solar collectors[J]. Applied Energy, 2014, 115(4):128-139.

[7] Cooper T, Steinfeld A. Derivation of the Angular Dispersion Error Distribution of Mirror Surfaces for Monte Carlo Ray-Tracing Applications[J]. Journal of Solar Energy Engineering, 2011, 133(4):125-134.

[8] Grena Roberto. Optical simulation of a parabolic solar trough collector[J]. International Journal of Sustainable Energy, 2010, 29(1):19-36.

*Note: Figure translations are in progress. See original paper for figures.*