Parameter Sensitivity Analysis and Optimization of the Laio Stochastic Model for Soil Moisture Dynamics in Ridge-Furrow Rainwater Harvesting Systems (Postprint)

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Abstract

Sensitivity analysis, optimization, and validation of hydrological model parameters are of great importance for improving the efficiency of model calibration and computational accuracy. To investigate the sensitivity of various parameters of the Laio soil moisture dynamic stochastic model (Laio model) in ridge-furrow rainwater harvesting systems, and simultaneously determine the optimal scheme for parameter optimization and model validation, this study combines multi-factor sensitivity analysis method with Improved Simplex Method (ISM), Particle Swarm Optimization (PSO), and Hybrid Particle Swarm Optimization (HPSO), utilizing measured data of rainfall, runoff, and soil moisture during the oat growing season of 2012-2013 from the ridge-furrow rainwater harvesting system at the Dingxi Arid Meteorology and Ecological Environment Experimental Base of China Meteorological Administration, to conduct sensitivity analysis, optimization, and validation of 13 parameters of the Laio model for ridge-furrow rainwater harvesting systems. The results indicate that mean precipitation and wilting coefficient sw are the most sensitive to the soil moisture probability density function, and the sensitivity to parameters is more pronounced at low soil water content, while the sensitivity to parameter sw is more pronounced at high soil water content; the optimized parameter values from the three algorithms (ISM, PSO, and HPSO) all yield satisfactory simulations of the soil moisture probability density function of ridge-furrow rainwater harvesting systems, with relative errors between measured and simulated values of peak value (CPV), peak position (PP), and 95% confidence interval (CI95%) all less than 10%, and CM indices all greater than 0.5; furthermore, the simulation performance and convergence speed of parameters optimized by the HPSO algorithm are significantly superior to those of the PSO and ISM algorithms, and can effectively overcome the deficiencies of the ISM and PSO algorithms. Therefore, the

HPSO algorithm can be considered as an alternative scheme for parameter optimization of soil moisture dynamic stochastic models in ridge-furrow rainwater harvesting systems. This research can provide scientific guidance for regional application of the Laio model and model parameter tuning.

Full Text

Sensitivity Analysis and Optimization of Laio Soil Moisture Dynamic Stochastic Model Parameters in Ridge-Furrow Rainwater Harvesting System

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Abstract: Sensitivity analysis, optimization, and validation of hydrological model parameters are crucial for improving model calibration efficiency and computational accuracy. To investigate the sensitivity of parameters in the Laio soil moisture dynamic stochastic model (Laio model) within ridge-furrow rainwater harvesting systems and to determine the optimal scheme for parameter optimization and model validation, this study combined multi-factor sensitivity analysis with the Improved Simplex Method (ISM), Particle Swarm Optimization (PSO), and Hybrid Particle Swarm Optimization (HPSO). Using measured rainfall, runoff, and soil moisture data from the Dingxi Arid Meteorology and Ecological Environment Experimental Station of the China Meteorological Administration during the 2012–2013 oat growing seasons, we conducted sensitivity analysis, optimization, and validation of 13 parameters in the Laio model for ridge-furrow rainwater harvesting systems. The results indicated that the mean precipitation amount per rainfall event ( ) and the wilting coefficient (s_w) were the most sensitive parameters affecting the soil moisture probability density function (p(s)). The sensitivity of p(s) to parameter was more pronounced under low soil moisture conditions, while its sensitivity to parameter s_w was more evident under high soil moisture conditions. All three algorithms (ISM, PSO, and HPSO) effectively simulated the soil moisture probability density function in ridge-furrow rainwater harvesting systems, with relative errors of the curve peak value (CPV), peak position (PP), and 95% confidence interval (CI95%) between measured and simulated values all less than 10%, and CM indices all greater than 0.5. Moreover, HPSO demonstrated significantly superior simulation performance and convergence speed compared to PSO and ISM, and could notably overcome the deficiencies inherent in both ISM and

PSO algorithms. Therefore, HPSO can be considered a promising alternative for parameter optimization of soil moisture dynamic stochastic models in ridge-furrow rainwater harvesting systems. This research provides scientific guidance for regional application and parameter calibration of the Laio model.

Keywords: ridge-furrow rainwater harvesting; soil moisture dynamics; Laio soil moisture dynamic stochastic model; sensitivity analysis; model parameter optimization

Water resource scarcity is a common challenge facing dryland agriculture worldwide. Effectively utilizing natural rainfall resources, ensuring food production security, and maintaining farmland ecosystem stability are critical for achieving coordinated and sustainable development of the "agriculture-ecology-economy" coupled system in dry farming regions. Meanwhile, developing and promoting efficient, low-cost, and environmentally friendly dryland cultivation techniques is of great significance for improving crop yield and water use efficiency in these areas. Ridge-furrow rainwater harvesting systems, which utilize field ridging, alternating furrow-ridge patterns, runoff generation on ridges, and efficient rainwater collection in furrows, have become an important water-saving irrigation measure in semi-arid farmland ecosystems due to their physiological and ecological effects of warming, evaporation suppression, and soil conservation. These systems play a crucial role in alleviating the contradictions between rapid population growth, increasing food shortages, and deteriorating agricultural ecology in dry farming regions.

Soil moisture is the primary water source for plants in semi-arid farmland ecosystems, serving as a carrier for nutrient cycling and flow, and playing a central and linking role in material and energy transformation within the soil-vegetation-atmosphere system. Understanding the relationship and interaction mechanisms between semi-arid farmland ecosystems and soil moisture is essential for revealing ecosystem stability and the dynamic processes of key soil and water elements. Since various factors affecting soil moisture dynamics (precipitation, evapotranspiration, soil heterogeneity, topography, etc.) are stochastic, particularly the randomness of rainfall events and rainfall amount distribution, soil moisture dynamic models only have practical significance when described in probabilistic form. Since Eagleson et al. first incorporated stochastic concepts into soil water balance equations, subsequent researchers including Milly, Rodriguez-Iturbe et al., Laio et al., Porporato et al., and Pan et al. have successively established stochastic mathematical models for soil moisture dynamics at different spatial and temporal scales, which have been widely applied. The Laio soil moisture dynamic stochastic model (Laio model) introduces two critical soil moisture thresholds (wilting coefficient and hygroscopic coefficient) in the evapotranspiration term, enabling more realistic description of soil moisture dynamics in arid and semi-arid farmland ecosystems and providing theoretical guidance for effective soil moisture utilization and management in dry farming regions.

Given that soil moisture in ridge-furrow rainwater harvesting systems in semi-arid regions is closely related to various physical, chemical, and biological processes, as well as rainfall, runoff, evapotranspiration, soil properties, microtopography, and mulching materials, and remains in a complex dynamic state, applying the Laio model to simulate and study soil moisture dynamics in this system is necessary. The Laio model involves 13 parameters related to soil, vegetation, and climate, some of which are difficult to obtain through direct observation and exhibit considerable uncertainty. Therefore, before model application, "localization"and "regionalization"issues must be considered, requiring sensitivity analysis and optimization of model parameters. Previous studies have addressed parameter sensitivity and acquisition methods for the Laio model. For instance, Yao et al. analyzed parameter sensitivity in the Horqin Sandy Land and classified parameters into three categories based on sensitivity levels. Miller et al. found that maximum evapotranspiration (E_max) and water stress point (s*) were the most difficult to obtain and showed the highest sensitivity. Ren et al. optimized Laio model parameters in a typical well-irrigated agricultural area in the Taihang Mountain Piedmont Plain using the PSO algorithm and found that calibrated parameters better simulated the stochastic variation characteristics of soil moisture during crop growth periods. However, systematic research on parameter sensitivity of the Laio model in ridge-furrow rainwater harvesting systems under different soil moisture conditions, as well as the fitness and effectiveness of various optimization algorithms for model parameter optimization, is still lacking. Therefore, establishing a convenient and feasible methodological system for parameter optimization, sensitivity analysis, and validation is crucial for improving parameter calibration efficiency, controlling model computational errors, and expanding model application domains.

This study utilized measured rainfall, runoff, and soil moisture data from the Dingxi Arid Meteorology and Ecological Environment Experimental Station during the 2012–2013 oat growing seasons in ridge-furrow rainwater harvesting systems. Multi-factor sensitivity analysis was employed to analyze and classify the sensitivity of Laio model parameters in semi-arid ridge-furrow rainwater harvesting systems. Based on ISM, PSO, and HPSO algorithms, 13 parameters of the Laio model were optimized and selected, and the effectiveness of parameters optimized by the three algorithms was validated and evaluated using measured data. The objective was to establish an effective methodological system for sensitivity analysis, optimization, and validation of Laio model parameters in ridge-furrow rainwater harvesting systems, providing scientific theoretical basis for model parameter correction and regional application.

1.1 Field Experiment and Data Measurement

The experiment was conducted during 2012–2013 at the Dingxi Arid Meteorology and Ecological Environment Experimental Station of the Lanzhou Institute of Arid Meteorology, China Meteorological Administration (35°33 N, 104°35 E, elevation 1,896.7 m). The station is located in the western loess hilly region of the Loess Plateau, characterized by a semi-arid climate and typical temperate continental monsoon climate. The average rainfall from 1971–2014 was 388 mm, with monthly averages of 20–80 mm in winter and 150–270 mm in summer. Rainfall distribution was extremely uneven throughout the year, with 86.9% of annual precipitation occurring from July to October. Evaporation was intense, with average annual potential evaporation reaching 1,500 mm. The experimental site had flat terrain with heavy loam soil. The average soil bulk density in the 0–100 cm layer was 1.38 $\text{g}\cdot\text{cm}^{3}$, field capacity was 25.6%, saturated water content was 43.87%, permanent wilting coefficient was 6.7%, groundwater depth was 10.4 m, and hydraulic connection between soil water and groundwater was weak. The local cropping system was single-crop-per-year, with main crops including spring wheat (*Triticum aestivum*), oats (*Avena sativa*), and potato (*Solanum tuberosum*), and main forages including alfalfa (*Medicago sativa*) and sainfoin (*Onobrychis viciaefolia*).

The experiment employed ridge-furrow rainwater harvesting technology with oats as the indicator crop. Ridges covered with mulch served as rainwater harvesting zones, while uncovered furrows served as planting zones. The experiment used a randomized complete block design with nine treatments (three furrow-ridge ratios × three mulching materials) and three replications. The three mulching materials were biodegradable film, common plastic film, and soil crust, while the three furrow-ridge ratios were 60 cm:30 cm, 60 cm:45 cm, and 60 cm:60 cm (furrow width:ridge width). Both biodegradable and common plastic films had a thickness of 0.08 mm. Soil ridges were manually compacted with original soil, forming natural soil crusts through wind and rain exposure. The representative symbols for soil ridges, biodegradable film ridges, and common plastic film ridges were SR, BMR, and CMR, respectively. Based on local planting experience, ridge slope was 40°, height was 25 cm, length was 10 m, and each plot contained four ridges and three furrows. The experimental design schematic is shown in [Figure 1: see original paper]. Planting management methods followed those described in previous studies. Since the three furrow-ridge ratios had no significant effect on soil moisture spatiotemporal dynamics in this ridge-furrow rainwater harvesting system, all results and discussion in this paper used average values across the three ratios for each mulching material.

Rainfall data during the experimental period were measured by an automatic rain gauge at the experimental station. Soil water content was measured before oat sowing (around April 10), after harvest (around August 20), and after rainfall events (>5 mm). Soil water content was determined using the oven-drying method (105 °C, 10 h) at depths of 0–140 cm with 20 cm intervals, totaling seven soil layers. Three sampling points were randomly selected in the furrow of each plot, and soil water content at each layer was averaged across the three points. Runoff from rainwater harvesting ridges was determined by back-calculating from rainfall data using the SCS-CN model developed by the United States Soil Conservation Service. Soil bulk density was measured using the core cutter method at 0–140 cm depth with 20 cm intervals and three replicates per layer, with mean values calculated. Root zone depth was determined through field investigation of oat root biomass distribution. Concurrent meteorological data were obtained from the nearby experimental station meteorological observatory.

1.2 Laio Soil Moisture Dynamic Stochastic Model

The theoretical foundation of soil moisture stochastic models is the mass balance principle: the change in soil water content per unit time equals the difference between soil water input and loss terms. Based on previous research on soil moisture stochastic models, Laio et al. established a daily time-scale soil water balance equation at a spatial point by introducing two critical soil water content thresholds (soil hygroscopic coefficient and soil wilting coefficient). The specific model (Laio model) can be expressed as:

$$nZ_r \frac{ds(t)}{dt} = \varphi[s(t), t] - \chi[s(t), t]$$

where: n is soil porosity; Z_r is root zone depth (cm); $s(t)$ is soil relative moisture content at time t (i.e., $s(t) = \theta(t)/n$, where $\theta(t)$ is soil volumetric water content (%) at time t); $\varphi[s(t), t]$ is rainfall infiltration rate ($\mathrm{cm} \cdot \mathrm{d}^{1}$), representing the portion of rainfall that actually reaches the soil, i.e., $\varphi[s(t), t] = R(t) - I(t) - Q[s(t), t]$, where $R(t)$ is rainfall intensity ($\mathrm{cm} \cdot \mathrm{d}^{1}$), $I(t)$ is plant interception rate ($\mathrm{cm} \cdot \mathrm{d}^{1}$), and $Q[s(t), t]$ is surface runoff rate ($\mathrm{cm} \cdot \mathrm{d}^{1}$); evapotranspiration and deep percolation constitute the loss term, i.e., $\chi[s(t), t] = E[s(t)] + L(s)$, where $E[s(t)]$ is evapotranspiration intensity ($\mathrm{cm} \cdot \mathrm{d}^{1}$) and $L(s)$ is deep percolation rate ($\mathrm{cm} \cdot \mathrm{d}^{1}$).

Combining the stochastic rainfall process with soil water loss terms of evapotranspiration and deep percolation forms the basis of soil moisture stochastic model establishment. Since rainfall is a stochastic process, a Soil Moisture Probability Density Function (p(s)) must be established to solve the soil water balance process (Equation 1). By transforming various soil water loss processes in the Laio model into the Chapman-Kolmogorov Forward Equation, the soil moisture probability density function can be analytically derived. Its specific expression is:

$$p(s) = \begin{cases} C \frac{s^{-\beta-1}}{(s-s_w)^{\beta-1}} \exp\left[-\frac{\gamma s}{s-s_w}\right] \exp\left[\frac{\eta s}{s-s_w}\right] & s_w < s \le s^* \\ C \frac{s^{-\beta-1}}{(s-s_w)^{\beta-1}} \exp\left[-\frac{\gamma s}{s-s_w}\right] \exp\left[\frac{\eta s^*}{s-s_w}\right] & s^* < s \le s_{fc} \\ C \frac{s^{-\beta-1}}{(s-s_w)^{\beta-1}} \exp\left[-\frac{\gamma s}{s-s_w}\right] \exp\left[\frac{\eta s^*}{s-s_w}\right] \exp\left[-\frac{\lambda(s-s_{fc})}{nZ_r K_s}\right] & s_{fc} < s \le 1 \end{cases}$$

where the constant C is solved by $\int_0^1 p(s)ds = 1$. The meanings and value ranges of other model parameters are shown in , and the initial values of parameters for each treatment were determined by the Monte Carlo trial-and-error method. Model assumptions and detailed derivation processes are described in the literature.

1.3 Principle of Model Parameter Sensitivity Analysis

Sensitivity analysis of model parameters investigates model responses caused by parameter variations and constitutes an important component of model parameter uncertainty analysis, as well as an indispensable step in model development and evaluation. Parameter sensitivity analysis also helps deepen understanding of model characteristics and improve model structural stability. For generality, the soil moisture probability density function is expressed as:

$$p = f(x_1, x_2, \cdots, x_i, \cdots, x_n)$$

where p is soil moisture probability density, x_i is the ith influencing factor, and n is the number of factors influencing p. When all factors change from $x_1, x_2, \cdots, x_i, \cdots, x_n$ to $x'_1, x'_2, \cdots, x'_i, \cdots, x'_n$, with changes of $\Delta x_1, \Delta x_2, \cdots, \Delta x_i, \cdots, \Delta x_n$, the soil moisture probability density changes correspondingly from p to p'. The total change $\Delta p = p' - p$ represents the combined effect of all factor changes. Using the Taylor expansion of multivariate functions:

$$\Delta p \approx \frac{\partial p}{\partial x_1}\Delta x_1 + \frac{\partial p}{\partial x_2}\Delta x_2 + \cdots + \frac{\partial p}{\partial x_i}\Delta x_i + \cdots + \frac{\partial p}{\partial x_n}\Delta x_n$$

where $\frac{\partial p}{\partial x_i}$ is the partial derivative of p with respect to x_i and Δx_i is the change in x_i. If only factor x_i changes while other factors remain constant (i.e., $\Delta x_i \neq 0$, $\Delta x_l = 0$ for $l \neq i$), the change in soil moisture probability density is denoted as Δp_i, representing the effect of x_i on p, expressed as $\Delta p_i \approx \frac{\partial p}{\partial x_i}\Delta x_i$. The total change is $\Delta p = \sum_{i=1}^{n} \Delta p_i$. Defining the ratio of the change in p to the change in factor x_i as the sensitivity A_i of p to x_i, then $A_i = \frac{\Delta p/p}{\Delta x_i/x_i}$, indicating that a 1% change in the ith factor x_i causes a A_i% change in soil moisture probability density p. A positive A_i indicates that p and x_i change in the same direction. Larger $|A_i|$ values indicate greater influence of factor x_i on soil moisture probability density p, meaning p is more sensitive to x_i, and x_i can be considered a sensitive factor.

The sensitivity A_i is calculated as:

$$A_i = \frac{\Delta p_i/p}{\Delta x_i/x_i} = \frac{\partial p}{\partial x_i} \cdot \frac{x_i}{p} \cdot \frac{\Delta x_i}{\Delta x_i} = \frac{\partial p}{\partial x_i} \cdot \frac{x_i}{p}$$

When obtaining the sensitivity of each parameter in the Laio model for ridge-furrow rainwater harvesting systems, we first used Mathematica 8.0.4 software to derive the general formula for parameter sensitivity, then selected the soil moisture content range ($0 \leq s \leq 1$) and calculated parameter sensitivities for each treatment using the initial parameter values in Table 1. This allowed us to identify sensitive parameters for different treatments. Additionally, the SOM neural network clustering method was used to classify parameter sensitivities, and we explored the characteristics of parameters within the same sensitivity category and the main reasons for differences between categories.

1.4 Principle of Model Parameter Optimization Algorithms

1.4.1 Principle of Improved Simplex Method The Simplex Method (SM), also known as the variable polyhedron search method, is a local search algorithm proposed by Nelder and Mead in 1965. Its basic principle is: in an N-dimensional Euclidean space, N+1 points form an initial simplex. By comparing the objective function values at these N+1 points, the worst point (with maximum function value) is reflected across the centroid of the remaining points to obtain a reflected point. This reflected point replaces the worst point to form a new simplex. Through repeated iterations, the function values at the vertices gradually decrease, approaching the objective function minimum using expansion, contraction, compression, and rejection operations until convergence criteria are met and the optimal point is found.

The Improved Simplex Method (ISM) used in this study modifies the step length based on the basic simplex method. Let the coordinate vectors of the best point (minimum function value F_b), second-best point (second-minimum function value F_n), and worst point (maximum function value F_w) be X_b, X_n, and X_w, respectively. The reflected point X_r has function value F_r. Based on the reflected value, different operations are performed: (1) If $F_r < F_b$, the reflection direction is correct, and the expansion point coordinate vector $X_e = X_c + \gamma_e(X_c - X_w)$ is calculated, where $X_c = \frac{1}{N}(\sum_{i=1}^{N+1} X_i - X_w)$. If expansion succeeds, X_e replaces X_w; otherwise, X_r is retained. (2) If $F_b \le F_r < F_n$, neither expansion nor contraction is performed, and X_r is retained. (3) If $F_n \le F_r < F_w$, the reflection direction is incorrect and contraction is needed, with the contraction point coordinate vector $X_t = X_c - \beta_t(X_c - X_w)$. (4) If $F_r \ge F_w$, contraction point coordinate vector $X_u = X_c + \beta_u(X_c - X_w)$ is calculated, and X_u replaces X_w.

Based on the above calculations, a new simplex is constructed, convergence is tested, and iterations continue until termination conditions are satisfied. ISM iteration termination is determined by predefined conditions, typically using the ratio of system response standard deviation S to mean system response $\bar{F}$ (coefficient of variation). If $COV = S/\bar{F} < 0.5\%$, optimization can be terminated.

1.4.2 Principle of Particle Swarm Optimization Algorithm Particle Swarm Optimization (PSO) is an evolutionary computation technique based on swarm intelligence, proposed by Kennedy and Eberhart in 1995. In PSO, each potential solution to the optimization problem is considered a particle in the search space. Each particle has a fitness value determined by the objective function and a velocity that determines its moving direction and distance. The algorithm iteratively searches for the optimal solution by tracking the individual extremum (personal best) of each particle and the global extremum (global best) of the entire swarm.

The basic mathematical description of PSO is: in a D-dimensional target search space, a swarm consists of N particles, where each particle' s position represents a potential solution. The ith particle is represented as a D-dimensional vector $X_i = (x_{i1}, x_{i2}, \cdots, x_{iD})$, $i = 1, 2, \cdots, N$. Substituting into the objective function yields its fitness value, which measures solution quality. The ith particle' s velocity is also a D-dimensional vector $V_i = (v_{i1}, v_{i2}, \cdots, v_{iD})$. The best position found so far by the ith particle is $P_i = (p_{i1}, p_{i2}, \cdots, p_{iD})$, and the best position found by the entire swarm is $P_g = (p_{g1}, p_{g2}, \cdots, p_{gD})$. Particle velocity and position are updated using:

$$v_{ij}(t+1) = wv_{ij}(t) + c_1 r_1 [p_{ij}(t) - x_{ij}(t)] + c_2 r_2 [p_{gj}(t) - x_{ij}(t)]$$

$$x_{ij}(t+1) = x_{ij}(t) + v_{ij}(t+1)$$

where r_1 and r_2 are random numbers following $U(0,1)$ distribution; c_1 and c_2 are acceleration factors, typically set to $c_1 = c_2 = 2$; w is the inertia factor. In each dimension, particles have a maximum velocity limit V_{max}. If velocity in any dimension exceeds V_{max}, it is limited to V_{max}.

1.4.3 Principle of Hybrid Particle Swarm Optimization Algorithm To improve global and local search capabilities and convergence efficiency, the Hybrid Particle Swarm Optimization (HPSO) algorithm was developed by incorporating the improved simplex search method into the PSO algorithm framework. In each iteration, PSO first performs global optimization on the swarm, then ISM conducts local searches around some elite particles in their better solution neighborhoods to find even better solutions.

The specific HPSO algorithm flow is: after each PSO optimization iteration, all P swarm particles (where P is population size) are sorted by fitness values. The top S particles with best fitness are selected to construct a "simplex" with S vertices. From these S vertices, the centroid X_c of the $S-1$ best-responding vertices is calculated. The remaining vertex X_s is reflected through centroid X_c with "stretching" mapping to generate new vertices, constructing a new "simplex." This process is repeated multiple times to produce S new particles. The fitness values of these updated particles are calculated, and the best-responding particle replaces the previous best individual in the original swarm, forming the next generation population with remaining individuals. The best personal best (pbest) position is then determined by comparing new individual fitness values with their own and the swarm' s pbest fitness values. If the swarm evolution reaches the maximum allowed generation T or the evaluation value is less than the given precision ε, the optimal result gbest is output and iteration terminates. Generally, the individual number S should not be too large, with values between 10% and 20% being appropriate.

1.5 Laio Model Transformation and Parameter Optimization

Nonlinear model parameter optimization involves determining the parameter set θ in the model structure (expressed as $y = f(x, \theta) + e$, where x is system input, y is system output, θ is the model parameter set to be optimized, and e is white noise with mean 0 and variance δ^2) based on known input-output data (x_i, y_i), $i = 1, 2, \cdots, n$, by minimizing the sum of squared deviations. For generality, when optimizing parameters for the Laio model in ridge-furrow rainwater harvesting systems, we solve the objective function:

$$F(\theta) = \sum_{i=1}^{n} [y_i - f(x_i, \theta)]^2$$

based on known data (x_i, y_i), $i = 1, 2, \cdots, n$. The resulting parameter set θ is the optimized model parameter value. Smaller $F(\theta)$ indicates more accurate optimized parameters, while larger values indicate less reliability. Objective function $F(\theta)$ optimization must consider model parameter constraints, making traditional algorithms difficult to apply. Since many modern non-traditional algorithms can solve complex optimization problems with nonlinear, non-differentiable, and multi-peak functions, this study selected the local search algorithm ISM, the global search algorithm PSO, and the hybrid search algorithm HPSO to optimize the sum of squared deviations of the Laio model, and analyzed and validated the convergence efficiency and effectiveness of optimized parameters for ISM, PSO, and HPSO algorithms.

1.6 Model Validation and Evaluation Methods

The effectiveness of parameter optimization by ISM, PSO, and HPSO algorithms was evaluated by comparing the characteristics of measured soil moisture probability density function curves with simulated curves using optimized parameters for different treatments in the study area. The validation metric selected was the Consistency Measure (CM) index, which indicates the degree of consistency between two target curves and is calculated as:

$$CM = \frac{2A_{12}}{A_1 + A_2}$$

where A_1 represents the area under target curve 1, A_2 represents the area under target curve 2, and A_{12} represents the common area under both target curves (see Figure 2: see original paper). Clearly, better consistency between target curve 1 and target curve 2 results in larger A_{12} and larger CM index; conversely, poorer consistency results in smaller A_{12} and smaller CM index. The CM index ranges from [0, 1], where CM = 1 indicates perfect consistency between target curves, and CM = 0 indicates no consistency (see Figure 2: see original paper).

2.1 Sensitivity Analysis of Laio Model Parameters

Using the initial parameter values for different treatments in Table 1 and MATLAB R2010a software, we calculated the sensitivity of each Laio model parameter under different soil moisture conditions. As shown in [Figure 3: see original paper], model parameter sensitivity varied under different soil moisture conditions and treatments. Overall, parameters n, β, s_h, and K_s had minimal impact on model output; s_{fc} and E_{max} had relatively small impact; parameters α and s_w had significant impact and were the most important factors affecting stochastic simulation of soil moisture dynamics in ridge-furrow rainwater harvesting systems. When soil moisture content was $s_h < s < s_w$, parameters s^*, Δ, E_w, and λ had relatively significant impact on model output, while Z_r had minimal impact. Under other soil moisture conditions, these parameters had relatively small impact. The variation trends of model parameter sensitivity with soil moisture content were generally consistent across treatments. Under the same soil moisture content, the sensitivity of sensitive parameters for treatment SR was significantly greater than for treatments CMR and BMR, with no significant difference between CMR and BMR. Under the same treatment, the sensitivity of sensitive parameters was significantly greater under low soil moisture conditions than under high moisture conditions. Additionally, except for parameters α, λ, and s_w which had positive sensitivity values, all other sensitive parameters had negative sensitivity values and were soil characteristic parameters.

The SOM neural network clustering method was used to classify the sensitivity of Laio model parameters in ridge-furrow rainwater harvesting systems ([Figure 4: see original paper]). The results showed that when 13 model parameters were divided into three categories, classification results under different soil moisture conditions shared similarities: n, β, s_h, and K_s were consistently weakly sensitive parameters; s_{fc} and E_{max} were moderately sensitive parameters; and α and s_w were strongly sensitive parameters. However, classification results for s^*, Δ, E_w, Z_r, and λ differed under various soil moisture conditions. By comprehensively considering classification results across different soil moisture conditions, model parameters in ridge-furrow rainwater harvesting systems can be divided into three categories: Category 1 (weakly sensitive) includes n, β, s_h, and K_s; Category 2 (moderately sensitive) includes s_{fc}, E_{max}, s^*, Δ, E_w, Z_r, and λ; Category 3 (strongly sensitive) includes α and s_w. These results are generally consistent with Yao et al., where α and s_w were strongly sensitive and n was weakly sensitive. However, they differ from Ren et al., where λ was classified as weakly sensitive but is strongly sensitive in this study. This discrepancy may be due to their failure to comprehensively consider parameter sensitivity under different soil moisture conditions. Additionally, differences in soil texture, vegetation, and climate characteristics among study areas may cause inconsistent parameter sensitivities, necessitating appropriate sensitivity analysis based on specific environmental conditions of the study area.

Since strongly sensitive parameters α and s_w have significantly greater impact on model simulation results than other parameters, their individual effects require further investigation. Using treatment CMR as an example, we examined the independent effects of α and s_w on the soil moisture probability density function $p(s)$ while keeping other parameters constant ([Figure 5: see original paper]). When parameter α varied from 0.026 to 0.135 cm · d ^{1}, increasing α caused nonlinear decreases in $p(s)$. When α varied from 0.135 to 1 cm · d ^{1}, increasing α first caused nonlinear increases then decreases in $p(s)$, with $p(s)$ reaching its maximum at $\alpha = 0.192$ cm · d ^{1}. When soil moisture content ranged from 0.026 to 0.25 cm^{3} · cm ^{3}, changes in parameter α caused significant fluctuations in $p(s)$, indicating that the sensitivity of $p(s)$ to α was more pronounced under low moisture conditions. This demonstrates that in arid and semi-arid regions, ridge-furrow plastic mulching systems can convert ineffective rainfall (<5 mm) into effective water stored in soil, significantly increasing soil moisture content. When parameter s_w varied from 0.026 to 0.56, $p(s)$ showed a nonlinear increasing trend across the soil moisture range, with larger changes in s_w causing greater changes in $p(s)$ at higher moisture contents. This indicates that the sensitivity of $p(s)$ to s_w was more evident under high moisture conditions, showing that ridge-furrow mulching systems can help crops escape drought stress and significantly improve soil water use efficiency through efficient rainwater collection.

2.2 Model Parameter Optimization Based on ISM, PSO, and HPSO Algorithms

Based on parameter sensitivity analysis results and optimization principles, we optimized 11 parameters (n, β, s_h, K_s, s_w, s^*, s_{fc}, E_w, Δ, E_{max}, α, λ) in the Laio model using ISM, PSO, and HPSO algorithms. Initial values and ranges for these parameters are listed in Table 1.

During parameter optimization, algorithm settings were: population size = 80; maximum iterations = 1000; fitness function used sum of squared deviations (see Section 1.5); ISM expansion coefficient $\gamma_e = 2$, contraction coefficient $\beta_t = 0.5$; PSO and HPSO inertia weight w decreased linearly from $w_{max} = 0.9$ to $w_{min} = 0.4$; learning factors $c_1 = c_2 = 2$; contraction coefficient $\beta_u = 0.5$; HPSO used $S = 6$ elite individuals for simplex local search; expansion coefficient $\gamma_e = 2$; the simplex method performed 100 search steps on selected elite individuals; parameters n, Z_r, and α were determined based on measured values. To ensure optimization accuracy and effectiveness, computation stopped when any of three termination criteria were met: (1) distance between parameters corresponding to optimal objective function values in two iterations was less than precision $\varepsilon_1 \leq 10^{-5}$; (2) difference between objective function values in two iterations was less than precision $\varepsilon_2 \leq 10^{-5}$; (3) maximum cycle count was reached. To minimize randomness effects, each optimization algorithm was run independently 10 times with random seeds, and average optimal solutions and computational efficiency were recorded. Algorithm performance and optimized parameter validation are shown in Table 2 and [Figure 6: see original paper].

2.3 Validation and Evaluation of Optimized Parameters

Using measured rainfall, runoff, and soil moisture data from 2012–2013 for each treatment (SR, CMR, and BMR) in the experimental ridge-furrow rainwater harvesting system, we compared the matching degree between measured and simulated soil moisture probability density functions using parameters optimized by the three algorithms (ISM, PSO, and HPSO) in terms of curve shape (peak value, peak position, 90% confidence interval) and CM index to evaluate optimization effectiveness and select superior parameter values and algorithm types. [Figure 6: see original paper] and Table 3 present the validation and evaluation results.

As shown in [Figure 6: see original paper] and Table 3, optimized parameters from all three algorithms accurately depicted curve shapes, captured peak positions, and described main characteristics of soil moisture probability density functions. Relative errors of CPV, PP, and CI95% between measured and simulated values were all less than 10%, and CM indices were all greater than 0.5, indicating good simulation performance. However, HPSO optimization (mean CM index = 0.901) outperformed PSO (mean CM index = 0.848), which in turn outperformed ISM (mean CM index = 0.678). Moreover, HPSO convergence speed (mean generations = 285) was faster than PSO (mean generations = 503). This demonstrates that HPSO-optimized parameters can significantly improve simulation accuracy and efficiency for soil moisture probability density in ridge-furrow rainwater harvesting systems, making HPSO a promising alternative for parameter optimization. However, previous studies have shown that hybrid algorithms combining global and local search methods, while improving local convergence speed and performance, may increase the risk of falling into local minima. Therefore, developing hybrid optimization algorithms that significantly enhance local search capability while maintaining high probability of finding global optima requires further research.

This study employed multi-factor sensitivity analysis to analyze and classify the sensitivity of Laio model parameters in semi-arid ridge-furrow rainwater harvesting systems. Based on ISM, PSO, and HPSO algorithms, 13 parameters of the Laio model were optimized and selected, with validation and evaluation using measured data. The main conclusions are:

1) By comprehensively considering parameter sensitivity under different soil moisture conditions, among the 13 parameters in the ridge-furrow rainwater harvesting system Laio model, mean annual precipitation (α) and wilting coefficient (s_w) had the most significant impact on model output and were strongly sensitive parameters.

2) When strongly sensitive parameters α and s_w varied within certain mois-

ture ranges, the sensitivity of soil moisture probability density function $p(s)$ to parameter α was more pronounced under low moisture conditions, while sensitivity to parameter s_w was more evident under high moisture conditions.

3) Optimized parameters from all three algorithms effectively simulated soil moisture probability density functions in ridge-furrow rainwater harvesting systems, with relative errors of CPV, PP, and CI95% between measured and simulated values all less than 10% and CM indices all greater than 0.5.

4) HPSO demonstrated significantly superior simulation performance and convergence speed compared to PSO and ISM, and could notably overcome deficiencies in both ISM and PSO algorithms. Therefore, HPSO can serve as a promising alternative for parameter optimization of soil moisture dynamic stochastic models in ridge-furrow rainwater harvesting systems.

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Note: Figure translations are in progress. See original paper for figures.