Postprint: Optimal Design of Single Expansion Ramp Nozzle Based on Discrete Adjoint Method

Authors: Song Hongchao Li Xin Ji Lucheng

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Abstract

Currently, the primary design methods for hypersonic nozzles are the method of characteristics and optimization approaches based on CFD analysis. When employing the method of characteristics, the flow must be simplified under inviscid and irrotational assumptions, which cannot guarantee optimal design under realistic flow conditions. When using optimization algorithms such as genetic algorithms or conventional gradient-based methods, an increase in the number of design variables poses tremendous computational challenges. To overcome these limitations, this paper develops a design methodology for single expansion ramp nozzles based on the discrete adjoint method, leveraging the characteristic that computational cost is nearly independent of the number of design variables. Using the control of area distribution along the nozzle flow path as the parameterization approach, refined adjoint optimization is achieved upon a prototype nozzle with excellent preliminary design performance, resulting in a thrust coefficient improvement of 0.8 percentage points over the prototype.

Full Text

Preamble

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Research on Optimization Design of Unilateral Expansion Nozzle Based on Discrete Adjoint Method

HongChao Song, Li Xin, Ji Lucheng
(Beijing Institute of Technology, Beijing, 100081, China)

Abstract

Currently, the main design methods for hypersonic nozzles are the characteristic line method and optimization methods based on CFD analysis. The characteristic line method requires simplifying the flow based on inviscid and irrotational assumptions, which cannot ensure optimal design under real flow conditions. When using optimization algorithms such as genetic algorithms or conventional gradient-based methods, the increase in the number of design variables poses a significant computational challenge. To overcome the limitations of these methods, this paper develops a design method for unilateral expansion nozzles based on the discrete adjoint method, leveraging its characteristic that computational cost is almost independent of the number of design variables. Using control of the nozzle’ s area distribution along the flow direction as the parameterization method, fine-grained adjoint optimization design is achieved on the basis of a prototype nozzle with excellent performance from preliminary design. The optimized thrust coefficient is improved by 0.8 percentage points compared to the prototype nozzle.

Keywords: Discrete Adjoint Method; Unilateral Expansion Nozzle; Parameterization Method

Unilateral expansion nozzles are widely used in scramjet engines for hypersonic vehicles. Currently, design methods for such hypersonic nozzles mainly include the characteristic line method and optimization design methods.

The characteristic line method has been extensively applied to supersonic flow profile design, capable of producing nozzle internal flow fields that are smooth and shock-free. A typical characteristic line method is the maximum-thrust nozzle proposed by Rao[1]. This method calculates the flow field using the characteristic line approach under constraints of given nozzle length and mass flow rate to obtain a maximum-thrust contour. Zudov et al.[2] constructed and solved a variational problem for maximizing thrust in supersonic two-dimensional asymmetric nozzles, using the characteristic line method in their solution process. In China, Cao Deyi et al.[3] also employed the characteristic line method to directly design afterbody unilateral expansion nozzles. Early applications of the characteristic line method achieved certain success in hypersonic internal flow component profile design. However, the characteristic line method requires assumptions of inviscid and irrotational flow, which deviate significantly from real flow conditions. Additionally, nozzles designed using this method typically have long ducts that require truncation, resulting in substantial performance losses.

With the rapid development of computational fluid dynamics and computer technology, optimization design methods based on CFD analysis have made significant progress in recent decades. Chen Bing[4] established a two-dimensional nozzle model based on NS equation optimization design and solved it using the complex method. Subsequently, combined with genetic algorithms and efficient, high-precision space-marching methods[5], aerodynamic optimization design studies were conducted on two-dimensional supersonic scramjet nozzles. Gan Wenbiao et al.[6] constructed an optimization methodology combining experimental design methods, surrogate model techniques, and genetic algorithms, applying it to integrated afterbody/nozzle design for hypersonic vehicles.

The genetic algorithms and conventional gradient-based methods employed in the aforementioned hypersonic nozzle designs, while effective, suffer from computational costs that grow geometrically or exponentially with the number of design variables, making it difficult to meet the demands of reduced design cycles, increased design automation, and refined design in full three-dimensional engineering applications.

Compared with conventional optimization methods, the adjoint method offers the distinct advantage that its computational cost is almost independent of the number of design variables, overcoming the computational limitations of conventional methods. Through the efforts of Jameson and Giles et al.[7-11] over more than two decades, aerodynamic adjoint optimization design has matured for applications ranging from airfoils, wings, wing-body configurations to complete aircraft. This paper employs the discrete adjoint optimization method to design hypersonic unilateral expansion nozzles, truly addressing the design problem in a refined design mode and providing technical reference for subsequent integrated inlet and propulsion system design.

1.1 Discrete Adjoint Optimization Theory

For steady flow optimization problems, the residual R of the flow equations can be considered as a function of the design variables D, grid X, and flow variables Q, i.e., = . This residual equation essentially represents a constraint for the steady flow optimization problem. In the initial objective function (objective function L is:

The coefficient being 0 can simplify the calculation accordingly. Equation (3) is the discrete adjoint equation for the flow problem. Based on satisfying the adjoint equation, the simplified objective function is:

1.2.1 Flow Control Equations

The three-dimensional dimensionless Reynolds-averaged Navier-Stokes equations can be expressed as follows:

where n is the outward normal unit vector at the control volume boundary, Q is the cell-averaged conserved variable, F_i is the inviscid flux, and F_v is the viscous flux. Laminar viscosity calculations employ the perfect gas assumption and follow Sutherland' s formula.

1.2.2 Turbulence Model

Numerous experiments have shown that the SA turbulence model selected in this paper exhibits strong robustness and is highly suitable for general internal flow aeronautical engineering applications. The dimensionless SA turbulence model control equation is as follows:

1.3 Optimization Method

The optimization program developed in this work comprises seven modules: objective function establishment, grid generation, flow field solution, adjoint field solution, sensitivity calculation, optimization algorithm, and grid deformation. Detailed optimization procedures (such as flow solution methods, adjoint solution methods, optimization algorithms, and grid deformation) are described in Reference 12.

1.3.1 Objective Function Gradient-based algorithms are a type of local optimization method whose ultimate goal is to identify an extremum rather than the global optimum of the objective function within the optimization domain. To maximize the thrust coefficient of the hypersonic nozzle, the objective function selected in this paper is:

where m represents the number of design variables, p__i represents the exponent of the i-th design variable, and n__n represents the unit normal vector at point n.

1.3.2 Initial and Boundary Conditions The nozzle has a supersonic inlet. According to characteristic theory, all aerodynamic parameters must be specified at the inlet. The inlet conditions are: Mach number 1.4, static pressure 145600 Pa, static temperature 1500 K, and specific heat ratio 1.4. The outlet employs a pressure boundary condition with the assumption of adiabatic and isentropic conditions. If the outlet flow is supersonic, all aerodynamic parameters are calculated by extrapolation. The wall condition uses a no-slip wall assumption. The current program initializes the entire flow field using freestream parameters.

1.3.3 Geometric Parameterization Method In this paper, 500 nodes on the nozzle surface are selected as design variables. When perturbed, each design variable moves within the plane perpendicular to the streamline direction, thereby changing the nozzle' s area distribution along the flow path. Therefore, the parameterization object is the perturbation of the aerodynamic shape rather than the shape itself. By controlling the nozzle's area distribution, the key factor in nozzle design is captured while reducing the number of optimization design variables. The nozzle parameterization method is:

where v is the vector containing design variables, r__n(v) is the nozzle surface coordinate at point n, r__n0 is the surface coordinate of the initial model, and Δr__n is the perturbation coordinate. The specific expression for the perturbation coordinate is:

2 Prototype Nozzle Design

To address the issue that gradient-based algorithms cannot guarantee global optimality, a feasible approach is to pre-determine the local optimization domain where the optimal solution lies, narrowing the optimization interval for the adjoint method to ensure result validity. Therefore, the design of the prototype nozzle is crucial. The prototype nozzle serves as the starting point for adjoint optimization, acting as a "genetic inheritor," and its performance largely determines the effectiveness of subsequent adjoint optimization.

The unilateral expansion nozzle designed in this study operates at Mach 4.5 and an altitude of 18.5 km, where the standard atmospheric static pressure is 6994.8 Pa. To enhance performance, the prototype nozzle design should employ an optimization algorithm with global search capability. For this purpose, the parameters controlling the initial nozzle geometry must be minimized to reduce computational cost in prototype design.

In the prototype nozzle design process, the nozzle inlet cross-section is specified as circular and the outlet as rectangular, with the inlet/outlet shapes and nozzle length held constant throughout the optimization. Based on this, each spanwise cross-section along the flow direction is defined as a super-ellipse, with the area and n-exponent distributions along the flow path serving as optimization parameters. By adjusting nine parameters including the area distribution law, super-ellipse exponent n distribution law, and lower wall angle, the nozzle geometry can be uniquely determined, as shown in Figure 1 [Figure 1: see original paper].

Figure 1. Diagram of Nozzle Modeling

This modeling concept enhances three-dimensional surface control capability in nozzle design, with particular emphasis on the area distribution along the flow direction and the application of dihedral angle principles.

When designing the prototype nozzle using a genetic algorithm, the mesh contains approximately 600,000 cells. To ensure computational accuracy, the Y-plus value is kept below 1, as shown in Figure 2 [Figure 2: see original paper].

Figure 2. Nozzle Grids

Based on these nine control parameters, this paper employs a genetic algorithm for evolutionary optimization to bring the initial nozzle configuration close to the global optimum. The genetic algorithm uses a population size of 10. As shown in Figure 3 [Figure 3: see original paper], when the population evolves to the eighth generation, the thrust coefficient rapidly increases from less than 91% to approximately 93%, and the optimization process stabilizes. The result from the eighth generation of the genetic algorithm optimization is taken as the final prototype nozzle design, providing the initial geometry for subsequent adjoint optimization. Figure 4 [Figure 4: see original paper] shows the Mach number contour distribution in the flow symmetry plane and along the flow path for the prototype nozzle. It can be observed that the Mach number distribution at the exit is relatively uniform.

Figure 3. Convergence History of the Genetic Algorithm

Figure 4. Mach Number Distribution of the Original Nozzle

3 Adjoint Optimization Results Analysis

Starting from the prototype nozzle, fine-grained adjoint optimization design is performed on the nozzle using 500 design variables on the nozzle surface—a level of refinement difficult to achieve with genetic algorithms and other conventional gradient-based algorithms. Figure 5 [Figure 5: see original paper] shows the objective function iteration curve during the optimization process. It can be observed that in the first three optimization iterations, the improvement in the objective function is relatively slow, which is related to the optimization algorithm, objective function formulation, and parameterization method. Meanwhile, the greatest improvement occurs during the fourth optimization process, with thrust increasing by approximately 0.8 percentage points. Subsequently, the objective function essentially converges, indicating that the optimization has reached an extremum and the process is complete. This adjoint optimization required only 7 optimization cycles, with computational cost equivalent to just 14 flow field calculations.

Figure 5. Convergence History of the Objective Function Optimized by the Adjoint Method

The adjoint optimization process employs conventional geometric constraint methods, keeping the inlet and outlet geometry unchanged. Figure 6 [Figure 6: see original paper] compares the geometry before and after optimization, revealing substantial changes, with the circled area marking the location of maximum change. Figure 6(b) shows that the dihedral angle region exhibits particularly noticeable changes.

Figure 6. Comparison of Geometry before and after Optimization

Figure 7 [Figure 7: see original paper] shows Mach number and static pressure contours in the flow symmetry plane before and after optimization. The flow field changes are evident: the prototype nozzle has smooth geometric variation from inlet to outlet, and the flow expands gradually. In contrast, the optimized nozzle exhibits lower expansion in the front section and more dramatic expansion in the rear section, particularly on the lower wall. Figure 8 [Figure 8: see original paper] presents the Mach number distribution along the flow path, also reflecting that the optimized nozzle has smaller expansion in the front section but greater expansion at the exit compared to the prototype nozzle. Especially near the corner regions, the four corners at the optimized nozzle exit experience more complete expansion, with corner flow significantly improved.

Figure 7. Comparison of Flow Field before and after Optimization

Figure 8. Comparison of Mach Number Distribution along the Streamline before and after Optimization

This paper first constructs the nozzle with fewer parameters and employs a genetic algorithm for global optimization design to obtain a well-performing prototype nozzle. Subsequently, the discrete adjoint method is applied for fine-grained optimization design of the prototype nozzle, achieving a final design result with thrust coefficient improved by an additional 0.8 percentage points.

The design system developed in this paper for unilateral expansion nozzles using the adjoint method requires no simplified assumptions about the flow and is not limited by the number of design variables, overcoming some limitations of existing design methods and meeting the development trend of fine-grained design. Meanwhile, the parameterization model proposed in this paper provides a key technical approach for integrated optimization design of inlets and propulsion systems.

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Note: Figure translations are in progress. See original paper for figures.