Modified Multi-Relaxation-Time Lattice Boltzmann Method for Herschel-Bulkley Fluids: Postprint

Authors: Wu Weiwei, Huang Xiaodiao, Yuan Hong

Date: 2018-01-02T00:00:00+00:00

Abstract

As an important class of non-Newtonian fluids, Herschel-Bulkley fluids have been widely applied in engineering fields. To improve the stability and accuracy of LBM simulations of Herschel-Bulkley fluid flows, an improved method is proposed based on the multi-relaxation-time LBM (MRT LBM). Using Poiseuille flow as an example, the improved algorithm is introduced in detail, with comparative analyses performed for shear-thinning and shear-thickening fluids separately. The variation trend of relative error with increasing lattice number under different exponent conditions is compared, demonstrating the effectiveness of this improved method. Finally, through application in a cement 3D printing nozzle and comparison with simulation results from the multi-relaxation-time LBM (MRT LBM), the feasibility of this improved method is further validated.

Full Text

Preamble

ChinaXiv Collaborative Journal No.: 2016-0209

A Modified Multiple-Relaxation-Time Lattice Boltzmann Method for Herschel-Bulkley Fluids

WU Wei-Wei, HUANG Xiao-Diao, YUAN Hong

(School of Mechanical and Power Engineering, Nanjing Tech University, Nanjing 211800, China)

Abstract

Herschel-Bulkley fluids represent an important class of non-Newtonian fluids with wide engineering applications. To enhance the stability and accuracy of lattice Boltzmann method (LBM) simulations for Herschel-Bulkley fluid flows, we propose a modified approach based on the multiple-relaxation-time LBM (MRT-LBM). Using Poiseuille flow as a benchmark case, we present a detailed formulation of the improved algorithm and compare its performance for both shear-thinning and shear-thickening fluids. The variation trends of relative errors with increasing lattice resolution under different power-law indices demonstrate the effectiveness of the proposed method. Finally, through application to cement paste flow in a 3D printing nozzle and comparison with standard MRT-LBM results, we further validate the feasibility of this modified approach.

Keywords: Herschel-Bulkley fluids; modified MRT LBM; Poiseuille flow; cement 3D printing; MRT LBM

0 Introduction

The lattice Boltzmann method (LBM) provides an effective alternative to finite element methods for fluid flow simulation, offering clear physical interpretation and computational simplicity. It has been widely applied to numerous complex fluid flows [?], including extensive research on non-Newtonian fluid simulations. In analyzing non-Newtonian fluid flows, the relaxation time varies dynamically with local fluid viscosity. It is well established that maintaining the relaxation time within the range of 1/2 to 1 ensures simulation stability and accuracy; beyond this range, numerical instability and poor precision readily occur [?]. To address this issue, researchers have proposed various solutions. Gabbanelli et al. introduced a truncation model for power-law fluids and validated its effectiveness using Poiseuille flow analysis [?]. Malaspinas et al. employed a local method to investigate both power-law and Carreau fluids, demonstrating good stability within certain exponent ranges [?]. Boyd et al. applied a local method to pipe flow of power-law fluids, showing superior computational accuracy and efficiency compared to Gabbanelli' s truncation model [?]. In 2001, D' Humières et al. proposed the multiple-relaxation-time LBM (MRT-LBM) for three-dimensional cavity flows, proving its enhanced stability over conventional LBM [?]. Subsequently, MRT-LBM has been adapted for non-Newtonian fluid simulations. Chai et al. analyzed power-law and Bingham fluids using MRT-LBM, reporting excellent stability and accuracy [?]. Fallah et al. simulated power-law fluid flow around a rotating cylinder with MRT-LBM, examining rotational speed and power-law index effects [?]. Chen et al. modeled three-dimensional Bingham plastic flow between parallel plates using MRT-LBM, resolving instability issues inherent in traditional LBM [?]. Li et al. investigated power-law fluid flow in a two-dimensional square cavity, concluding that MRT-LBM is well-suited for non-Newtonian fluid simulation, though Reynolds number and power-law index significantly influence results [?].

Despite these advances, research has primarily focused on common non-Newtonian fluids such as power-law and Bingham fluids, while less common types like Herschel-Bulkley fluids remain understudied. This paper addresses Herschel-Bulkley fluids by proposing a modified MRT-LBM approach that more effectively improves simulation stability. Using Poiseuille flow as a benchmark, we detail the methodology, compare results with analytical solutions, analyze relative errors, and finally apply the method to cement paste flow in a 3D printing nozzle to further demonstrate its feasibility.

1.1 Herschel-Bulkley Fluid Constitutive Equation

The constitutive model of Herschel-Bulkley fluids is expressed as a piecewise function. Due to the existence of yield stress, the shear rate equals zero when the stress is below this threshold. The specific formulation is:

$$\tau = \tau_0 + K\dot{\gamma}^n \quad \text{when} \quad \tau > \tau_0$$
$$\dot{\gamma} = 0 \quad \text{when} \quad \tau \leq \tau_0$$

To avoid the inconvenience caused by the piecewise function in analysis, a modified constitutive equation is adopted:

$$\tau = \tau_0[1 - \exp(-m\dot{\gamma})] + K\dot{\gamma}^n$$

Here, parameter m primarily controls the stress growth to avoid the inherent discontinuity of the constitutive equation. As m approaches zero, the equation reduces to the power-law fluid constitutive model; as m approaches infinity, it becomes the ideal Herschel-Bulkley fluid. When $\dot{\gamma} = 0$, the modified equation matches the original piecewise formulation. In practice, m is typically assigned a relatively large value, though care must be taken not to make it arbitrarily large to avoid convergence issues. From this formulation, the apparent viscosity equation can be derived [**?**]:

$$\mu = \frac{\tau_0[1 - \exp(-m\dot{\gamma})]}{\dot{\gamma}} + K\dot{\gamma}^{n-1}$$

1.2 MRT LBM Fluid Analysis

The MRT lattice Boltzmann equation is:

$$f(r + e_i\delta t, t + \delta t) - f(r,t) = -M^{-1}S[m(r,t) - m^{eq}(r,t)]$$

where the distribution functions $f(r,t)$ and $f^{eq}(r,t)$ are transformed to moment space as $m(r,t)$ and $m^{eq}(r,t)$:

$$m(r,t) = Mf(r,t)$$
$$m^{eq}(r,t) = Mf^{eq}(r,t)$$

Matrix M defines the transformation, and S is the diagonal relaxation matrix:

$$S = \mathrm{diag}(s_0, s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8)$$

In the model, density and momentum are conserved quantities, so the corresponding relaxation parameters s_0, s_3, and s_5 are set to zero. Parameters s_7 and s_8 are set to $1/\tau$, where τ is the relaxation time in the BGK model. The remaining parameters are assigned values slightly greater than 1, specifically $s_1 = s_2 = 1.4$ and \$s_4 = s_6 = 1.2.

The evolution of the MRT model consists of two processes: collision and streaming. The collision step differs from the BGK model and follows:

$$f^+(r,t) = f(r,t) - M^{-1}S[m(r,t) - m^{eq}(r,t)]$$

where $f^+(r,t)$ represents the post-collision distribution function. The streaming step is identical to the BGK model:

$$f(r + e_i\delta t, t + \delta t) = f^+(r,t)$$

Compared with the BGK model, the MRT model primarily involves transformations between moment space and velocity space. Through Chapman-Enskog expansion, the kinematic viscosity can be obtained as:

$$\nu = c_s^2\left(\frac{1}{s_7} - \frac{1}{2}\right)\delta t = c_s^2\left(\frac{1}{s_8} - \frac{1}{2}\right)\delta t$$

The macroscopic definitions of density and velocity can also be derived, consistent with the BGK model.

1.3 Modified MRT LBM

Based on the above analysis, MRT-LBM applied to non-Newtonian fluids is primarily affected by the local relaxation time, which can be viewed as being influenced by viscosity, which in turn is related to the strain rate through the stress tensor. For Herschel-Bulkley fluids, $n < 1$ corresponds to shear-thinning fluids, $n = 1$ represents Bingham fluids, and $n > 1$ corresponds to shear-thickening fluids. Direct application of MRT-LBM to non-Newtonian fluids with exponential terms encounters stability and accuracy issues. When the shear rate approaches zero, for shear-thinning fluids ($n < 1$), the viscosity tends toward infinity, causing divergence; for shear-thickening fluids ($n > 1$), the viscosity tends toward zero. Previous studies have not addressed solutions specific to Herschel-Bulkley fluid analysis.

In reality, most non-Newtonian fluids exhibit non-Newtonian characteristics only within a certain shear rate range, beyond which they typically behave like Newtonian fluids with constant viscosity [?]. Based on this phenomenon, we propose a modified MRT-LBM method for Herschel-Bulkley fluids, presented in piecewise function form. Due to the yield stress in Herschel-Bulkley fluids, to ensure constant viscosity outside the effective shear rate range, the Bingham fluid constitutive model is used as a substitute.

MRT-LBM can perform accurate and stable simulations when the dynamic viscosity falls within a certain range. However, instability and divergence occur when the relaxation time approaches 1/2, corresponding to viscosity approaching zero ($\nu \leq 0.001$). Conversely, when the relaxation time exceeds 1, the accuracy of MRT-LBM decreases, corresponding to relatively large viscosity ($\nu \geq 1/6$). Therefore, upper and lower viscosity limits are set for Eq. (11): $\nu_{\min} = 0.001$ and $\nu_{\max} = 0.16$. Evidently, for shear-thinning fluids, the maximum viscosity corresponds to shear rates approaching zero, while for shear-thickening fluids, the minimum viscosity corresponds to shear rates approaching zero.

2 Flow Between Parallel Plates

Poiseuille flow is commonly used to verify stability and accuracy due to the existence of analytical solutions. Here, we use Poiseuille flow to illustrate the detailed implementation of this modified model. Two parallel plates are separated by distance H in the y-direction, with a constant pressure gradient ∇P applied in the x-direction, yielding the velocity component u_y distribution along y. Exploiting symmetry, the y-domain is divided into $[-H/2, 0]$ and $[0, H/2]$; analysis of the $[0, H/2]$ range suffices to determine the complete velocity profile. Within $[0, H/2]$, four distinct regions A, B, C, and D can be identified. Region A corresponds to the high shear rate range near the wall, where the shear rate exceeds $\dot{\gamma}_\infty$, with y_h as the boundary value; in this region, the fluid is treated as Bingham fluid. Region B represents the effective Herschel-Bulkley fluid region, with y_l as the critical boundary. Region C corresponds to the low shear rate range, where the fluid is again treated as Bingham fluid. Region D corresponds to the stage where stress is below the yield stress, resulting in zero shear rate, with y_τ as the boundary value. Combining the velocity derivation for Poiseuille flow, the theoretical velocity distribution function for this modified method can be obtained.

$$u(y) = \begin{cases} \alpha_1(y_h - y) + \alpha_2(y_h^{1+1/n} - y^{1+1/n}) + \alpha_3(y - y_\tau) & \text{for } y_\tau \leq y \leq y_l \\ \alpha_1(y_h - y) + \alpha_2(y_h^{1+1/n} - y^{1+1/n}) & \text{for } y_l \leq y \leq y_h \\ \alpha_1(y_h - y) & \text{for } y_h \leq y \leq H/2 \\ 0 & \text{for } 0 \leq y \leq y_\tau \end{cases}$$

The constant terms α_1, α_2, and α_3 in the above equation are expressed as:

$$\alpha_1 = \frac{\tau_0}{\mu_B}, \quad \alpha_2 = \frac{K}{\mu_B}\left(\frac{\nabla P}{K}\right)^{1/n}, \quad \alpha_3 = \frac{\nabla P}{2\mu_B}$$

Differentiating the velocity distribution function and applying stress continuity conditions yields the corresponding boundary values:

$$y_\tau = \frac{\tau_0}{\nabla P}, \quad y_l = \left(\frac{\tau_0}{K}\right)^{n/(n+1)}\left(\frac{1}{\nabla P}\right)^{1/(n+1)}, \quad y_h = \frac{H}{2} - \frac{\mu_B \dot{\gamma}_\infty}{\nabla P}$$

where $\partial P/\partial x = \nabla P$. Different magnitudes of ∇P produce different region partitions.

Based on Eq. (14), we primarily consider four factors affecting the final simulation results: initial yield stress τ_0, pressure gradient ∇P, power-law index n, and lattice number N. Examining Eq. (14) reveals that to ensure the existence of y_l and/or y_h, the following condition must be satisfied:

$$\nabla P > \frac{2\tau_0}{H}$$

To investigate the effectiveness of this truncation model for different indices n, we consider the range $0.2 \leq n \leq 5$. Here, we select $n = 0.5$ and $n = 1.5$, representing shear-thinning and shear-thickening fluids respectively, to clearly demonstrate the influence of initial yield stress on simulation results.

For $n = 0.5$ (shear-thinning), the parameters are: pressure gradient $\nabla P = -1.5 \times 10^{-6}$, fluid consistency index $K = 0.001$, plate height $H = 10$, lattice number $N = 200$, fluid density $\rho = 1$, viscosity limits $\nu_{\min} = 0.001$ and $\nu_{\max} = 0.16$. Since $n < 1$, we have $\nu_0 = 0.16$ and $\nu_\infty = 0.001$, with initial yield stress $\tau_0 = 3.0 \times 10^{-6}$. The comparison between numerical and simulation results is shown in Figure 1 [Figure 1: see original paper]. The solid line represents the analytical solution from Eq. (12), circles denote LBM simulation results, dashed lines represent extensions of Bingham and Herschel-Bulkley fluids, and vertical dashed lines indicate region boundaries.

For $n = 1.5$ (shear-thickening), the parameters are: $\nabla P = -2 \times 10^{-6}$, $K = 0.01$, $H = 10$, $N = 200$, $\rho = 1$, $\nu_{\min} = 0.001$, $\nu_{\max} = 0.16$. Since $n > 1$, $\nu_0 = 0.001$ and $\nu_\infty = 0.16$, with $\tau_0 = 4.0 \times 10^{-6}$. The results are shown in Figure 2 [Figure 2: see original paper], with the same legend conventions as Figure 1.

Figures 1 and 2 demonstrate good agreement between numerical and analytical solutions regardless of initial yield stress variations. The initial yield stress τ_0 and pressure gradient ∇P determine the extent of the yielded region (Region D).

To evaluate the effectiveness of this truncation model across different power-law indices, we conducted a series of simulations for four fluids with different indices, comparing relative errors between MRT-LBM simulations and analytical solutions. The relative error is calculated as:

$$\text{Error} = \frac{\sum |u_{\text{LBM}} - u_{\text{analytical}}|}{\sum |u_{\text{analytical}}|}$$

Since LBM simulation accuracy depends on viscosity, to isolate the relationship between relative error and lattice number, we maintain constant viscosity during unit conversion using $\nu^* = \nu(\delta x^2/\delta t)$ and set $\delta x^2/\delta t = 1$. When increasing the lattice number N, we scale δx and δt proportionally to preserve viscosity. Given that $1/\delta x \propto N$, we have $\delta t \propto 1/N^2$. With N varying from 10 to 200, the relative errors under different indices and pressure gradients are shown in Figure 3 [Figure 3: see original paper].

The solid line in the figure represents a line with slope -1. In the legend, case (a) denotes situations where the low shear rate region (C) is small, while case (b) represents larger low shear rate regions. The comparison reveals that both cases produce essentially identical relative errors, which continuously decrease with increasing lattice number. Further grid refinement can reduce errors below 0.01%. The results also show that different power-law index n values produce noticeably different relative error characteristics.

3 Application in Cement 3D Printing Nozzle

During cement 3D printing, screw extrusion is employed to achieve cement paste shaping, making understanding of paste flow within the nozzle crucial. Under specific water-cement ratios, cement paste exhibits pronounced Herschel-Bulkley fluid characteristics. At room temperature (20°C), water-cement ratio of 0.5, and hydration time of 5 minutes, the paste follows a typical Herschel-Bulkley rheological model described by [**?**]:

$$\tau = 3.899 + 1.103\dot{\gamma}^n$$

Converting this to the form of Eq. (11), where parameter m controls stress growth, we take $m = 500$:

$$\tau = 3.899[1 - \exp(-500\dot{\gamma})] + 1.103\dot{\gamma}^n$$

The cement paste flows in the screw channel region shown in Figure 4 [Figure 4: see original paper], where the screw rotates at constant speed N. Assuming a stationary screw, the barrel can be considered to rotate in the opposite direction at speed N. Unfolding the entire structure yields a rectangular cross-section cavity [**?**], ignoring the slight inclination of the screw flights [**?**]. The expansion schematic is shown in Figure 5 [Figure 5: see original paper]. Thus, analyzing cement paste flow in the screw (Figure 4) transforms into analyzing flow in the rectangular cavity (Figure 5).

Based on the nozzle expansion schematic, we select the central cross-section $(Y\text{-}Z)_{\text{Center}}$ and employ the D2Q9 model. Only the top surface is assigned a velocity along the Z-direction at $y = h$, while the other three walls have zero velocity. Wall slip effects are neglected. The actual cross-section dimensions are $W = 16$ mm and $h = 6$ mm. Considering the nozzle scanning speed requirements and cement paste setting time, the extrusion speed is relatively low; the screw rotation speed is set to 30 r/min with a lead angle $\theta = 20°$. Using similarity criteria with Reynolds number Re as the key dimensionless parameter, we obtain the simulation model parameters. Viscosity correction via Eq. (13) ensures stability and accuracy, yielding the streamline pattern shown in Figure 6 [Figure 6: see original paper]. The flow exhibits a recirculation pattern centered near $(0.5W, 0.7h)$, with no significant fluid motion in the lower left and right corners.

Using the modified MRT-LBM with different grid resolutions and comparing with standard MRT-LBM, we obtain the velocity field distributions shown in Figure 7 [Figure 7: see original paper]. The modified MRT-LBM uses grid sizes of 288×108 and 160×60, while standard MRT-LBM uses 288×108 for comparison. Figure 7(a) shows the relationship between velocity component u and channel depth at width 8 mm. Figure 7(b) shows u versus channel width at depth 5.4 mm. Figure 7(c) shows velocity component v versus channel depth at width 14.4 mm. Figure 7(d) shows v versus channel width at depth 3 mm. The results demonstrate that this modified method can effectively simulate velocity field distributions. Although some discrepancies exist between different grid resolutions, the results are essentially consistent.

4 Conclusions

For generalized Newtonian fluids, specifically Herschel-Bulkley fluids, we propose a modified MRT-LBM that effectively improves simulation stability and prevents divergence.

(1) We detailed the methodology using Poiseuille flow, deriving the analytical solution formula. Relative error calculations show that errors decrease continuously with increasing lattice number, reaching approximately 0.05%. Both shear-thinning and shear-thickening fluids exhibit similar convergence trends. Furthermore, we demonstrate that the size of the low shear rate region has minimal impact on relative error, while the power-law index n significantly influences error characteristics.

(2) We applied this method to analyze cement paste flow in a 3D printing nozzle and compared results with standard MRT-LBM, finding consistent velocity field distributions that further validate the effectiveness of the improved method.

(3) Flow analysis in the cement 3D printing nozzle reveals that due to paste viscosity and nozzle geometry, the cross-sectional flow exhibits recirculation characteristics with the vortex center located approximately at $(0.5W, 0.7h)$. The streamline pattern can be used to identify flow regions. The analysis shows minimal fluid motion in the lower left and right corners of the cavity. To improve flow velocity, one can either increase screw rotation speed or enlarge the channel width.

References

[?] Sheikholeslami M, Gorji-Bandpy M, Ganji D D. Lattice Boltzmann Method for MHD Natural Convection Heat Transfer Using Nanofluid[J]. Powder Technology, 2014, 254: 82-93.

[?] Ashorynejad H R, Mohamad A A, Sheikholeslami M. Magnetic Field Effects on Natural Convection Flow of a Nanofluid in a Horizontal Cylindrical Annulus Using Lattice Boltzmann Method[J]. International Journal of Thermal Sciences, 2013, 64: 240-250.

[?] SHEN Ching, TIAN Dongbo, XIE Chong, et al. Examination of the LBM in Simulation of Microchannel Flow in Transitional Regime[C]//ASME 2003 1st International Conference on Microchannels and Minichannels. America: American Society of Mechanical Engineers, 2003: 405-410.

[?] WANG Wentan, LIU Zhe, JIN Yong, et al. LBM Simulation of Droplet Formation in Micro-channels[J]. Chemical Engineering Journal, 2011, 173(3): 828-836.

[?] GUO Zhaoli, Zhao T S, SHI Yong. A Lattice Boltzmann Algorithm for Electro-osmotic Flows in Microfluidic Devices[J]. The Journal of Chemical Physics, 2005, 122(14): 144907.

[?] YUAN Yudong, Rahman S. Extended Application of Lattice Boltzmann Method to Rarefied Gas Flow in Micro-channels[J]. Physica A: Statistical Mechanics and its Applications, 2016, 463: 25-36.

[?] Niu X D, Shu C, Chew Y T, et al. Investigation of Stability and Hydrodynamics of Different Lattice Boltzmann Models[J]. Journal of Statistical Physics, 2004, 117(3-4): 665-680.

[?] Gabbanelli S, Drazer G, Koplik J. Lattice Boltzmann Method for Non-Newtonian (power-law) Fluids[J]. Physical Review E, 2005, 72(4): 046312.

[?] Malaspinas O, Courbebaisse G, Deville M. Simulation of Generalized Newtonian Fluids with the Lattice Boltzmann Method[J]. International Journal of Modern Physics C, 2007, 18(12): 1939-1949.

[?] Boyd J, Buick J, Green S. A second-order accurate lattice Boltzmann non-Newtonian flow model[J]. Journal of Physics A: Mathematical and General, 2006, 39(46): 14241-14247.

[?] D' Humières D. Multiple-relaxation-time lattice Boltzmann models in three dimensions[J]. Philosophical Transactions of the Royal Society of London A: Mathematical, Physical and Engineering Sciences, 2002, 360(1792): 437-451.

[?] CHAI Zhenhua, SHI Baochang, GUO Zhaoli, et al. Multiple-relaxation-time Lattice Boltzmann Model for Generalized Newtonian Fluid Flows[J]. Journal of Non-Newtonian Fluid Mechanics, 2011, 166(5): 332-342.

[?] Fallah K, Khayat M, Borghei M H, et al. Multiple-relaxation-time Lattice Boltzmann Simulation of Non-Newtonian Flows Past a Rotating Circular Cylinder[J]. Journal of Non-Newtonian Fluid Mechanics, 2012, 177: 1-13.

[?] CHEN Songgui, ZHANG Chuanhu, FENG Yuntian, et al. Three-dimensional Simulations of Bingham Plastic Flows Using the Multiple-relaxation-time Lattice Boltzmann Model[J]. Engineering Applications of Computational Fluid Mechanics, 2016, 10(1): 347-360.

[?] LI Qiuxiang, HONG Ning, SHI Baochang, et al. Simulation of Power-law fluid Flows in Two-dimensional Square Cavity Using Multi-relaxation-time Lattice Boltzmann Method[J]. Communications in Computational Physics, 2014, 15(01): 265-284.

[?] Mitsoulis E. Flows of Viscoplastic Materials: Models and Computations[J]. Rheology Reviews, 2007, 2007: 135-178.

[?] Papanastasiou T C, Boudouvis A G. Flows of Viscoplastic Materials: Models and Computations[J]. Computers & Structures, 1997, 64(1): 677-694.

[?] Alexandrou A N, McGilvreay T M, Burgos G. Steady Herschel-Bulkley Fluid Flow in Three-dimensional Expansions[J]. Journal of Non-Newtonian Fluid Mechanics, 2001, 100(1): 77-96.

[?] Buick J M, Cosgrove J A. Numerical Simulation of the Flow Field in the Mixing Section of a Screw Extruder by the Lattice Boltzmann Model[J]. Chemical Engineering Science, 2006, 61(10): 3323-3326.

[?] Mohamad A A. Lattice Boltzmann Method: Fundamentals and Engineering Applications with Computer Codes[M]. Springer Science & Business Media, 2011.

[?] Cruz D O A, Pinho F T. Analysis of Isothermal Flow of a Phan-Thien-Tanner Fluid in a Simplified Model of a Single-screw Extruder[J]. Journal of Non-Newtonian Fluid Mechanics, 2012, 167: 95-105.

[?] Kim S J, Kwon T H. Development of Numerical Simulation Methods and Analysis of Extrusion Processes of Particle-filled Plastic Materials Subject to Slip at the Wall[J]. Powder Technology, 1995, 85(3): 227-239.

[?] Sastrohartono T, Jaluria Y, Esseghir M, et al. A Numerical and Experimental Study of Three-dimensional Transport in the Channel of an Extruder for Polymeric Materials[J]. International Journal of Heat and Mass Transfer, 1995, 38(11): 1957-1973.