Research on the Thermal Radiation Transport Solution Method Library for Reentry Vehicle Flow Fields F Postprint

Authors: Wang Jingying (1); Hao Jia'ao (2); Du Guangsheng (1); Li Chunxuan (2)

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Abstract

Based on a comparative analysis of one-dimensional radiative equilibrium and temperature discontinuity case results from several thermal radiation transport solution methods, a thermal radiation transport solution method library constructed from the discrete ordinates method, P1 approximation, and optically thick limit approximation is proposed. This method library can be effectively coupled with the governing equations for thermochemical nonequilibrium flow fields of reentry vehicles. Using this method library, a spherical nose case at 60 km altitude and Mach 35 was simulated. Calculations demonstrate that thermal radiation has a significant "cooling" effect on high-temperature flow fields; radiative heat transfer is comparable in magnitude to convective heat transfer, and the radiative heat flux calculated from flow field simulations without coupled thermal radiation will be 27% higher than the coupled calculation value. Numerical simulation studies under such flight conditions should consider thermal radiation coupling effects.

Full Text

Preamble

Thermal Radiation Solving Method Library for Reentry Vehicle Flowfield Simulation

WANG Jing-Ying[1], **HAO Jia-Ao**[2], **DU Guang-Sheng**[1], **LEE Chun-Hian**[2] [1]School of Energy and Power Engineering, Shandong University, Jinan 250100, China
[2]School of Aeronautic Science and Engineering, Beihang University, Beijing 100191, China

Abstract: Based on comparative analysis of one-dimensional radiation equilibrium and temperature discontinuity model cases using various thermal radiation transport solving methods, this paper proposes a thermal radiation solving method library consisting of the discrete ordinates method, P1 approximation, and optically limiting approximation. This library can achieve effective coupled simulation with thermo-chemical nonequilibrium flowfield governing equations for reentry vehicles. The method library was applied to simulate a spherical nose case at 60 km altitude and Mach 35. The calculations demonstrate that thermal radiation exerts a remarkable "cooling" effect on the high-temperature flowfield, with radiative heating comparable to convective heating. The radiative heat flux from uncoupled radiation calculations exceeds the coupled value by 27%, indicating that radiation coupling effects must be considered in numerical simulations under such flight conditions.

Keywords: radiation; hypersonic; aerothermodynamics; reentry

Reentry vehicles are the primary platforms for deep space exploration missions such as manned lunar missions. When returning to Earth' s atmosphere at extremely high speeds (approximately 10 km/s), the surrounding high-temperature shock layer flowfield (reaching 10^4 K) subjects the vehicle to severe aerodynamic heating, posing a grave threat to vehicle safety and potentially causing mission failure [1]. Therefore, accurately predicting the aerothermal environment of reentry vehicles to provide a basis for thermal protection design has become a critical technical challenge for corresponding deep space exploration missions.

The high-temperature atmospheric flowfield characteristics of high-speed reentry vehicles are extremely complex, with thermochemical nonequilibrium processes such as molecular vibrational energy excitation and multi-component chemical reactions strongly coupled with thermal radiation. For aerodynamic heating prediction under these conditions, thermal radiation coupling effects must be considered in addition to conduction and convection. Candler and Park [2] employed a one-dimensional tangent slab formula to solve thermal radiation transport coupled with thermochemical nonequilibrium flowfield calculations, reporting a stagnation point radiative heat flux of approximately 6\$×10ˆ{4}\$ W/m^2 for the AFE model at 78 km altitude and 9 km/s velocity. Feldick [3] combined one-dimensional tangent slab approximation, Monte Carlo simulation, and a P1-Monte Carlo hybrid method with the DPLR (Data Parallel Line Relaxation) program to conduct nonequilibrium flowfield coupled radiation simulations for the MPCV (Multi-Purpose Crew Vehicle). The study found that the tangent slab approximation overestimated radiative heat flux by 30% compared to Monte Carlo simulation. Gao Tiesuo et al. [4] obtained radiative heat flux by integrating emission coefficients along spatial paths under the optically thin limit assumption, with the high-temperature flowfield obtained by solving chemically nonequilibrium N-S equations, concluding that radiative heat flux contributions in the severe heating region of reentry capsules could account for up to 30% of total heat flux.

Monte Carlo simulation results can be considered accurate solutions for thermal radiation transport when the sample size is sufficiently large [6]; therefore, the results from tangent slab approximation and Monte Carlo simulation serve as benchmarks for the one-dimensional cases in this study. The optically thin approximation generally requires a characteristic optical thickness τL 1, while the optically thick approximation requires τL 1. Detailed principles and expressions of each method can be found in references [6] and [7]. All methods were implemented through self-programmed FORTRAN codes.

[Figure 1: see original paper] Sketch of one-dimensional radiative transfer model

1 One-Dimensional Thermal Radiation Transport Model

A unified one-dimensional model case was adopted for comparative analysis of existing thermal radiation transport solving methods. The geometric description follows reference [5] and is illustrated in [Figure 1: see original paper]: the space between two infinite, isotropic, diffuse parallel plates is filled with participating gray medium, with thermodynamic and radiative properties independent of temperature. The parallel plates are separated by distance L, with normal coordinate y and optical thickness coordinate $\tau =$ y, where is the medium' s radiative absorption coefficient. The temperatures of walls 1 and 2 are Tw1 and Tw2, respectively, with emissivities 1 and 2. The radiation intensity I forms angle γ with the y-axis and azimuthal angle ϕ. This model possesses one-dimensional axisymmetric properties. For convenience, uniform spatial discretization is employed with total discrete points N+1, physical spacing Δy = L/N, and optical thickness spacing $\Delta\tau = \tau$L/N.

2 Comparative Analysis of Various Methods Using One-Dimensional Model Cases

Multiple commonly used thermal radiation transport solving methods were selected for comparative analysis using one-dimensional model cases. The methods examined include tangent slab approximation (TSA), Monte Carlo method (MC), discrete ordinates method (DOM), P1 approximation (P1), and two optically limiting approximation methods: optically thin approximation (THIN) and optically thick approximation (THICK). The tangent slab approximation is essentially a one-dimensional integral analytical solution of the thermal radiation transport equation. For the configuration shown in [Figure 1: see original paper], the radiative heat flux from tangent slab approximation is given by:

where T represents temperature, and E2 and E3 are second- and third-order exponential integral functions [6], with:

To determine the number of discrete points N, grid independence calculations were performed for the benchmark tangent slab approximation method, with results shown in [Figure 2: see original paper]. The figure demonstrates that when the number of discrete grid points N > 30, the radiative heat flux values from tangent slab approximation become essentially stable, and the dimensionless temperature distribution data no longer change with grid refinement. Therefore, all N values in this study' s one-dimensional cases exceed 30 to ensure calculation accuracy of the tangent slab approximation method. For the radiative equilibrium cases in this section, N = 50.

Currently, domestic and international research predominantly employs a single existing thermal radiation transport solving method directly coupled with flowfield numerical schemes. However, the high-fidelity methods' characterization of thermal radiation across infinite spectral and directional dimensions undoubtedly introduces massive computational and storage requirements for coupled simulations, while simplified radiation models suffer from insufficient prediction accuracy. Consequently, a single thermal radiation transport solving method cannot meet the current demands for efficient simulation of reentry vehicle aerothermal environments. Based on analysis of one-dimensional thermal radiation transport model cases and 充分考虑 (full consideration of) reentry flight condition flowfield optical thickness characteristics and numerical scheme features, this paper constructs a thermal radiation solving method library suitable for coupled simulation with reentry vehicle thermochemical nonequilibrium flowfield equations and successfully implements flowfield-radiation coupled simulation for a spherical nose case under Ma = 35 reentry flight conditions.

2.1 Radiative Equilibrium

Assuming radiative equilibrium of the medium between the two walls in [Figure 1: see original paper], the spatial gradient of radiative heat flux qr is zero, i.e., dqr/dy = 0 W/m^3. With wall temperatures Tw1 = 1000 K and Tw2 = 700 K, emissivities 1 = 2 = 1.0, calculations were performed for absorption coefficients = 0.7, 1.5, and 3.0 m^{-1}. L = 1.0 m. The dimensionless radiative heat flux is defined as:

[Figure 2: see original paper] Grid independent simulation for tangent slab approximation: (a) nondimensional radiative heat transfer, (b) distribution of nondimensional temperature

[Figure 3: see original paper] Temperature distributions of one-dimensional radiative equilibrium calculated by different methods

[Figure 3: see original paper] presents temperature distribution results from various methods for the radiative equilibrium cases. The discrete ordinates method (DOM) shows almost identical temperature values to the benchmark tangent slab approximation (TSA) and Monte Carlo simulation (MC), while P1 approximation (P1) values also closely match the benchmark solution. The optically thin approximation (THIN) yields constant temperature values, whereas the optically thick approximation (THICK) shows significant differences from tangent slab approximation data, though its prediction accuracy gradually improves as the total optical thickness τL increases.

Dimensionless radiative heat flux results from each method are listed in , where errors are relative errors calculated using tangent slab approximation heat flux values as the benchmark.

The differences between the two benchmark methods (tangent slab approximation and Monte Carlo simulation) are extremely small. The discrete ordinates method demonstrates high prediction accuracy for radiative heat flux, while P1 approximation radiative heat flux errors are within 5%. However, both optically limiting approximation methods exhibit large heat flux value errors.

Comparison of non-dimensional equilibrium radiative heat fluxes calculated by different methods

2.2 Temperature Discontinuity

A temperature discontinuity case was designed to mimic the high-temperature shock layer flowfield of reentry vehicles: the medium temperature between the two walls is divided into two uniform layers at position y/L = 0.3. The high-temperature layer near wall 1 has temperature identical to wall temperature Tw1 = 10000 K, while the low-temperature layer near wall 2 has temperature identical to wall temperature Tw2 = 300 K. Calculations were performed for absorption coefficients = 0.01, 0.1, 1.0, and 5.0 m^{-1}. L = 1.0 m, N = 1000. The dimensionless radiative energy source term is defined as:

Dimensionless radiative energy source term values from each method are shown in [Figure 4: see original paper]. Under all optical thickness conditions, the discrete ordinates method results are highly consistent with the benchmark tangent slab approximation, while P1 approximation shows slight deviations. At small optical thickness τL = 0.01, the optically thin approximation yields results similar to tangent slab approximation, but gradually loses validity as τL increases. The optically thick approximation radiative energy source term values remain zero at all times.

[Figure 4: see original paper] Non-dimensional radiative energy source terms of temperature discontinuity using different methods

Dimensionless radiative heat flux values from each method at τL = 0.1 are presented in , where errors are again relative errors calculated using tangent slab approximation heat flux values as the benchmark. The discrete ordinates method and tangent slab approximation yield essentially identical radiative heat flux values, and P1 approximation also provides correct values. Under such small optical thickness conditions, optically thin approximation results possess certain reliability, while optically thick approximation can only produce zero heat flux results. Similar characteristics are observed under other total optical thickness conditions.

Non-dimensional radiative heat transfers at two walls calculated by different methods with τL = 0.1

3 Thermal Radiation Solving Method Library

Coupled solution of reentry vehicle thermochemical nonequilibrium flowfields with thermal radiation inevitably consumes greater computational resources. As demonstrated in Section 2, various thermal radiation transport solving methods exhibit significant performance differences, making it difficult for a single method to meet coupled simulation requirements [3]. This paper proposes the following solution: integrate multiple methods to construct a thermal radiation solving method library, and select methods from the library according to their characteristics, on-demand, and in stages to execute thermal radiation coupled simulations. Methods in the solving library should satisfy: accurate radiative heat flux prediction, minimized computational cost, compatibility with flowfield numerical scheme 协同求解 (collaborative solution), and suitability for large-scale parallel computing. Based on the comparative analysis of model cases in Section 2 and considering the actual solution execution processes of each method, the performance characteristics are summarized in . The prediction accuracy in the table primarily concerns radiative energy source term and radiative heat flux calculations.

The tangent slab approximation is a one-dimensional integral analytical solution that is exact for one-dimensional problems. However, it can only integrate along one-dimensional paths, making it unreasonable for high-dimensional calculations with difficult-to-define accuracy. Monte Carlo simulation can be considered an exact solution when sample sizes are sufficient, but this brings computational cost issues. The discrete ordinates method shows high consistency with benchmark methods (tangent slab approximation and Monte Carlo simulation) in one-dimensional solutions. It is a direct discretization method based on the complete radiation transport equation and is easily extensible to high dimensions. P1 approximation solves a simplified radiation transport equation with moderate accuracy. Optically thin and thick approximations are only reasonable under extremely small (generally $\tau L < 0.01$) or extremely large optical thickness conditions.

Characteristics of different methods solving radiative transfer

Regarding computational resource consumption, tangent slab approximation performs numerical integration along one-dimensional paths, requiring modest storage but demanding upstream-downstream point-by-point integration at each grid point. Each calculation requires N^2 algebraic operations (where N is the number of discrete grid points), making the computational cost considerable. Monte Carlo simulation requires tracking the history of each energy bundle’ s absorption and emission, with enormous energy bundle sample sizes often consuming substantial storage resources and computational effort. The discrete ordinates method requires storing radiation intensities in each angular direction and solving M algebraic equations (where M is the number of discrete angular directions) at each grid point, occupying certain storage and computational costs. P1 approximation only solves a second-order differential equation for incident radiation, with modest storage and computational requirements. Optically limiting approximations explicitly calculate radiation quantities directly from the temperature field, resulting in very low solution cost. lists CPU time consumed by each method for solving the radiative equilibrium cases in Section 2, where the energy bundle sample size for Monte Carlo simulation was:

As can be seen from Section 4, the actual computational time consumed by each method is consistent with the qualitative analysis in . Tangent slab approximation and Monte Carlo simulation consume substantial computational time, while discrete ordinates method and P1 approximation demonstrate higher computational efficiency, and optically limiting methods consume almost no time.

Except for tangent slab approximation, which is difficult to extend to high dimensions, the remaining methods are easily extensible to high-dimensional cases. Meanwhile, tangent slab approximation also encounters cross-domain integration issues during parallel solution processes, requiring massive data transfer between parallel regions. Discrete ordinates method and P1 approximation both transfer thermal radiation-related quantities at interfaces between adjacent regions. Optically limiting approximations only require temperature values at interfaces between adjacent regions.

CPU time consumption of solving radiative equilibrium case by different methods (unit: s)

In summary, this paper abandons tangent slab approximation (unfavorable for high-dimensional solution and parallel processing) and Monte Carlo simulation (high computational cost). The final thermal radiation solving method library consists of three methods at different levels: discrete ordinates method, P1 approximation, and optically limiting approximation (including optically thin and thick approximations). The general execution principle for this method library when facing thermal radiation transport solution is: use discrete ordinates method directly for uncoupled precise solution with flowfield; for coupled solution with flowfield, if the flowfield' s optical thickness satisfies limiting conditions, first use optically limiting approximation for coupled simulation, then switch to discrete ordinates method after convergence to save computational cost; if limiting conditions are not satisfied, first use P1 approximation for coupled solution, then switch to discrete ordinates method after convergence. These usage principles are illustrated in [Figure 5: see original paper].

[Figure 5: see original paper] Operation procedures of the thermal radiation solving method library

4 Reentry Flight Case Coupled Simulation Demonstration

A spherical nose model with R = 1.0 m was selected for coupled simulation of high-temperature flowfield and thermal radiation under reentry flight conditions at Ma = 35, demonstrating the effectiveness of the constructed thermal radiation solving method library coupled with flowfield numerical schemes. Free stream density ∞ = 3.1$\times10^{-4}$ kg/m^3, temperature T∞ = 247 K, and atmospheric mass fraction composition YN_2 = 0.77, YO_2 = 0.23 correspond to Earth atmospheric parameters at 60 km altitude. High-temperature flowfield solution employs numerical schemes from references [8-10], using Park' s two-temperature thermodynamic nonequilibrium model [11] and Gupta' s 11-species 20-reaction chemical kinetics model [12]. Thermal radiation properties are calculated using Anderson' s two-step model [7]. Wall temperature Tw = 1000 K, wall emissivity = 1.0, with fully catalytic wall condition. Flowfield-radiation coupling algorithm is described in reference [8]. Both flowfield solution and thermal radiation solving method library are implemented through self-programmed FORTRAN codes. Computational grid size is 60$\times70, with the first grid spacing normal to the wall at 1.0\times10^{-6}$ m [13] as required for aerothermal prediction. The computational cost for this spherical nose model is relatively small, allowing direct use of the high-precision discrete ordinates method from the solving method library for flowfield-radiation coupled solution.

[Figure 6: see original paper] presents pressure and temperature distributions from flowfield coupled with radiation calculations. The shock layer is very thin, with shock standoff distance less than 1/15 of the spherical nose radius. Post-shock stagnation region translational-rotational temperature and vibrational-electronic temperature both exceed 10000 K. [Figure 7: see original paper] shows thermal radiation variable distributions in the flowfield, where radiation energy source term and total absorption coefficient distributions correspond completely to the flowfield temperature distribution. Although maximum incident radiation occurs within the shock layer, high incident radiation values also extend to the upstream preshock flow, exhibiting greater diffusivity.

Under Ma = 35 flight conditions, the radiation energy source term reaches considerable magnitude (approximately 10^8 W/m^3), with thermal radiation exerting significant effects on the flowfield. However, the shock layer optical thickness remains very small, less than 0.005. Therefore, the optically thin approximation from the solving method library was also used for coupled calculation with the flowfield. [Figure 8: see original paper] compares stagnation line temperature results from uncoupled radiation (uncoupled), coupled optically thin approximation (coupled_{thin}), and coupled discrete ordinates method (coupled_{dom}). Compared with uncoupled results, coupled calculations show lower shock layer temperatures and thinner shock layers, highlighting the radiation cooling effect [7]. The optically thin approximation shows more pronounced radiation cooling than discrete ordinates method coupled calculation because it neglects radiation absorption and only considers radiation emission, resulting in larger radiation energy source term values and more significant flowfield effects.

[Figure 9: see original paper] presents stagnation line N^+ and e^- mass fraction distributions from uncoupled radiation and coupled optically thin approximation and discrete ordinates method calculations. Coupled calculations yield significantly lower N^+ and e^- mass fractions than uncoupled values, likely because radiation cooling effects inhibit ionization reactions. Therefore, significant coupling effects exist between high-temperature atmospheric chemical reactions and thermal radiation under high-speed reentry flight conditions. Additionally, the optically thin approximation coupled calculation shows more significant radiation cooling, with N^+ and e^- mass fractions lower than discrete ordinates method coupled calculation values.

[Figure 10: see original paper] shows stagnation line thermal radiation quantities from uncoupled radiation and coupled optically thin approximation and discrete ordinates method calculations. Coupled calculations yield smaller absolute thermal radiation values than uncoupled values, with optically limiting approximation coupled calculations showing more pronounced reduction in absolute thermal radiation values.

[Figure 6: see original paper] Flowfield simulation results coupled with radiation: (a) pressure (unit: Pa), (b) translational-rotational temperature (unit: K), (c) vibrational-electronic temperature (unit: K)

presents stagnation point convective heat flux (qc) and radiative heat flux (qr) values from uncoupled radiation and coupled optically thin approximation and discrete ordinates method calculations. Uncoupled radiative heat flux values are obtained by solving with discrete ordinates method based on converged flowfield results; optically thin approximation cannot provide radiative heat flux values, so its radiative heat flux is similarly obtained by solving with discrete ordinates method after coupled calculation convergence.

The results show that uncoupled and coupled radiation calculations yield similar convective heat flux values, but uncoupled radiative heat flux is significantly higher than coupled values—48% higher than optically thin approximation coupled results and 27% higher than discrete ordinates method coupled values. Radiative heat transfer under these conditions is comparable to convective heat transfer levels. Therefore, flowfield-radiation coupling effects must be considered when conducting aerothermal research under such reentry flight conditions.

[Figure 7: see original paper] Contours of radiation variables of coupled radiation simulation: (a) radiation source (unit: W/m^3), (b) total absorption coefficient (unit: m^{-1}), (c) incident radiation (unit: W/m^3)

[Figure 8: see original paper] Temperature distributions along the stagnation line of uncoupled and coupled radiation simulations

[Figure 9: see original paper] Mass fraction distributions along the stagnation line of uncoupled and coupled radiation simulations: (a) N^+, (b) e^-

[Figure 10: see original paper] Radiation results along the stagnation line of uncoupled and coupled radiation simulations: (a) radiation source (unit: W/m^3), (b) total absorption coefficient (unit: m^{-1}), (c) incident radiation (unit: W/m^3)

Radiative heat transfers at the stagnation point of uncoupled and coupled radiation simulations with $Ma = 35$ (unit: MW/m^2)

Conclusion

This study designed two one-dimensional thermal radiation transport model cases—radiative equilibrium and temperature discontinuity—to comparatively analyze the performance of various thermal radiation transport solving methods. Considering storage requirements, computational cost, and parallelization characteristics during actual solution processes, the one-dimensional exact solution (tangent slab approximation) and computationally expensive Monte Carlo simulation were abandoned. A thermal radiation solving method library was proposed, constructed from three methods at different levels: discrete ordinates method, P1 approximation, and optically limiting approximation. The usage principles for this method library were also presented. The solving method library can accurately solve thermal radiation transport, efficiently couple with flowfield numerical schemes, and is suitable for large-scale parallel computing.

Using the constructed thermal radiation solving method library coupled with reentry vehicle high-temperature thermochemical nonequilibrium flowfield numerical solution schemes, a demonstration simulation was successfully implemented for a spherical nose case at Ma = 35. Under corresponding reentry flight conditions, significant flowfield-radiation coupling effects exist, with thermal radiation exerting a "cooling" effect on the flowfield. Uncoupled and coupled radiation calculations yield similar convective heat transfer, but uncoupled radiative heat flux is much larger than coupled values. Coupled simulations predict radiative heat transfer comparable to convective heat transfer levels. Flowfield-radiation coupling effects must be considered when conducting simulations under related reentry flight conditions.

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Note: Figure translations are in progress. See original paper for figures.