Effects of Higher-Order Spatial Harmonics in Linear Accelerators

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Abstract

This study investigates the effects of higher-order spatial harmonics on beam dynamics in linear accelerators.

Full Text

Preamble

The Effect of High Spatial Harmonics on Beam Dynamics in RF Linac

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Abstract

This paper extends our previous work [1] analyzing the effect of high spatial harmonics on beam dynamics in RF linacs. We derive an approximate solution for radial motion and present numerical results obtained using the mapping technique.

1 Introduction

An RF linac accelerating structure is a type of periodic slow-wave guide in which TM modes are excited and propagate to accelerate electrons. Typically, the TM01 mode serves as the main accelerating mode (synchronous mode). In addition to the fundamental mode, spatial harmonic modes can also be excited according to Floquet' s theorem, though with amplitudes smaller than that of the main mode. In previous studies [1, 2, 3, 4], we analyzed the effects of high harmonic modes in linacs and identified the bunch lengthening phenomenon in photocathode RF guns. This paper continues that work by deriving an approximate solution for radial motion that accounts for high harmonic modes and presenting additional numerical results.

2 EM Field in Accelerating Structure

The general RF TM electromagnetic field in an RF linac accelerating structure can be expressed as [1, 2]:

$$Ez = E0\sum b_n \cos(\omega t - k_n z), \quad Er = -$$

where $k_n = k_0 + \frac{2\pi}{D}n$, D is the periodic length of the accelerating structure, k_0 is the fundamental mode wavenumber, $k_n (n \in \mathbb{Z})$ represents the high harmonics mode wavenumbers, and z is the longitudinal position along the beam axis.

3 Longitudinal Motion for Single Particle

The longitudinal motion equations are expressed as [1]:

$$\sum b_n \cos(\Delta\phi - = k_0((\gamma^2 - 1) - 1).$$

As derived in [1], assuming $\gamma_n = \gamma(nD)$ and $\Delta\phi_n = \Delta\phi(nD)$, we obtain the standard mapping under the case of $b_0 = b_n = 1$ [1]:

$$\gamma_{n+1} = \gamma_n + \frac{mc^2}{\cos(\Delta\phi_n)}, \quad n - 1)$$

$$\Delta\phi_{n+1} = \Delta\phi_n + k_0 D(-1).$$

We concluded in [1] that the high harmonic mode increases both the energy spread and the bunch length.

4 Transverse Motion for Single Particle

The radial electromagnetic force on a relativistic charged particle ($v = \beta c \simeq c$) due to the transverse electromagnetic field is given by the radial motion equation:

$$F_r = -(\gamma \dot{r}) = -$$

For the fundamental synchronous mode, we have $\omega t = k_0 z + \Delta\phi$. Therefore, in position coordinates, the radial motion equation is expressed as:

$$r'' = -\frac{2\gamma m \beta^2 c^2}{\sum(b_n \cos(\Delta\phi - nz))}.$$

We divide the field into two parts: the fundamental mode field and all the high spatial harmonic fields, which constitute a small perturbation compared to the main fundamental synchronous field. To solve for r, we expand r as $r_0 + r_1$ (omitting higher-order terms) to obtain:

$$r_0'' = -\frac{qE_0 r_0}{2\gamma m\beta^2 c^2}(b_0 \cos(\Delta\phi)),$$

$$r_1'' = -\frac{qE_0 r_0}{2\gamma m\beta^2 c^2}\left(\sum(b_n \cos(\Delta\phi - nz)) + \sum(b_n \cos(\Delta\phi - nz))\right).$$

Assuming $\Delta\phi$ is unchanged, we can obtain $r_0 =$ constant from the first equation. From the equation for r_1, we get approximately:

$$r_1' = -\frac{qE_0 r_0}{2\gamma m\beta^2 c^2}\left(\sum(b_n \cos(\Delta\phi - nz)) + \sum(b_n \cos(\Delta\phi - nz))\right).$$

We can finally obtain the approximate first-order solution for r:

$$r = r_0 + \frac{qE_0 r_0}{\gamma m\beta^2 c^2}\left(\sum b_n \sin(\Delta\phi - nz) + \sum b_n \sin(\Delta\phi - nz)\right).$$

It is evident that high harmonic mode components exist in the transverse motion in addition to the fundamental mode part. According to Fourier spectrum analysis principles, the amplitude of each high harmonic mode is small, and the effect of individual high harmonics is weak. However, the cumulative effect of all harmonics can be significant if the high harmonic modes are not effectively damped.

Alternatively, we can use the mapping technique to derive the following 2D mapping under the special case of $b_0 = b_n = 1$:

$$r_{n+1}' - r_n' = -\frac{qE_0 r_n}{2\gamma_n mc^2}\cos(\Delta\phi_n)D, \quad r_{n+1} - r_n = Dr_n'$$

where $r_n = r(nD)$.

5 Numerical Results

For convenience, we adopt parameters from an S-band electron linac: working frequency of 2856 MHz, $\lambda = 10.5$ cm, $k_0 = \frac{2\pi}{\lambda}$, and $D = 3.5$ cm. The typical accelerating electric field is 20 MV/m (SLAC S-band accelerating structure). We use numerical methods to solve the mappings for longitudinal and transverse motion, with typical phase space shown in Figure 1 [Figure 1: see original paper].

From the figure, we can observe that the radial motion includes high harmonic components. However, the condition $b_0 = b_n = 1$ is quite strong, as the amplitude of high harmonic modes is weaker than that of the fundamental mode. Therefore, the numerical results presented above overestimate the effect of high harmonic modes to some extent.

6 Discussion

In this paper, we have established a series of equations and mappings to analyze the effect of high harmonic modes on beam dynamics in RF linacs. Numerical results are also provided using standard SLAC S-band electron linac parameters.

References

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Note: Figure translations are in progress. See original paper for figures.

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