Review of Methods for Determining Indicator Weights in Multi-Criteria Evaluation: Postprint

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Abstract

[Purpose/Significance] To systematically review the common methods for determining indicator weights in multi-factor evaluation, providing a reference for relevant researchers to rationally select weight determination methods when evaluating specific problems.

[Method/Process] Through in-depth investigation of the basic principles, underlying ideas, and specific application cases of several typical subjective weighting methods, objective weighting methods, and integrated subjective-objective weighting methods, this paper analyzes and summarizes their respective advantages, disadvantages, and applicable scopes.

[Result/Conclusion] To date, no completely universal and generally applicable method for determining indicator weights exists. Different methods are based on different principles and guiding philosophies, resulting in varying applicable scopes. When conducting practical evaluations, weighting methods should be selected rationally according to the characteristics of the evaluation object to enhance the accuracy and effectiveness of comprehensive evaluation.

Full Text

Review on the Weighting Methods of Indexes in the Multi-Factor Evaluation

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Abstract

[**Purpose/significance**] This paper systematically reviews commonly used weighting methods for indexes in multi-factor evaluation, providing a reference for relevant personnel to select reasonable weighting methods for specific evaluation problems. [**Method/process**] Through in-depth analysis of the fundamental principles, underlying concepts, and specific application cases of several common subjective weighting methods and integrated weighting methods, this paper examines their advantages, disadvantages, and applicable scopes. [**Result/conclusion**] To date, there is no fully universal and general-purpose method for determining index weights. Different weighting methods are based on different principles and concepts, resulting in varying applicable scopes. When conducting practical evaluations, methods should be selected based on the characteristics of the evaluation object to improve the accuracy and effectiveness of comprehensive evaluation.

Keywords: multi-factor evaluation; index weight; weighting methods

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The determination of index weights is a critical component in multi-factor comprehensive evaluation, and whether the weights are determined reasonably directly affects the reliability and validity of evaluation results. To date, scholars both domestically and internationally have conducted extensive research on weighting methods for indexes in multi-factor evaluation and achieved fruitful results. A comprehensive review of these studies reveals that while there are diverse methods for determining index weights, they can generally be categorized into three major types: subjective weighting methods, objective weighting methods, and integrated subjective-objective weighting methods. Subjective weighting methods primarily rely on the knowledge, experience, or preferences of decision-makers and experts to subjectively judge the importance of each index. Objective weighting methods mainly calculate weights based on sample data analysis. Integrated subjective-objective weighting methods combine the weights obtained from both approaches to leverage their respective strengths and compensate for their weaknesses. Different weighting methods are founded on different principles and concepts, leading to variations in their theoretical models, original data requirements, data processing approaches, and ultimately resulting in significant differences in weight allocation. Therefore, different index systems should select appropriate weighting methods based on their own characteristics to ensure more reasonable weight distribution.

2 Common Subjective Weighting Methods

Generally, subjective weighting methods determine weights primarily based on the knowledge, experience, or preferences of decision-makers and experts, comparing indexes by importance, assigning values, or calculating weights. These methods treat weights as quantitative representations of the relative importance of evaluation indexes to evaluation objectives. While such methods exhibit relatively strong subjectivity and arbitrariness, the ranking of index weights generally aligns with the actual situation of evaluation objects. Currently, commonly used subjective weighting methods can be classified into four categories: expert estimation method, analytic hierarchy process, binomial coefficient method, and chain ratio scoring method.

2.1 Expert Estimation Method

The expert estimation method involves domain experts subjectively judging the importance of each index based on their experience and knowledge. The final weight allocation can be derived directly from the average of weights independently provided by K experts [1], or determined using frequency statistics, where the K weight values for each index are grouped by certain intervals, and the midpoint of the group with the highest frequency serves as the final weight for that index [2].

As early as 1986, F. Shands et al. [3] applied the expert estimation method to determine index weights for a teacher performance evaluation system, submitting evaluation indexes to several domain experts through questionnaires for two rounds of scoring to allocate weights. The resulting performance evaluation model achieved good results in practical school applications. In recent years, Chinese scholars Liu Lu et al. [4] also employed this method to determine index weights for heating system energy efficiency evaluation, calculating weights from the average of coefficients provided by multiple domain experts. When compared with comprehensive evaluation results from analytic hierarchy process and variation coefficient method, the expert estimation method showed the best alignment with actual system operation, primarily because heating source systems involve many practical influencing factors that experts can more comprehensively consider when allocating weights.

The expert estimation method offers three main advantages: First, it fully utilizes expert experience and knowledge, enabling comprehensive consideration of various external influencing factors based on expert judgment, thus ensuring high reliability. Second, weight calculation relies on traditional descriptive statistics such as mean and frequency, making it simple and straightforward. Third, it is not constrained by the availability of sample data and can provide probabilistic estimates for numerous non-technical, non-quantifiable indexes. However, the method also has certain limitations: First, weight allocation is entirely dependent on expert experience and knowledge, and different expert compositions may yield different evaluation results, exhibiting considerable subjectivity and arbitrariness. Second, when there are many indexes, maintaining consistency in the judgment process becomes difficult, making it challenging to ensure complete objectivity and reasonableness.

Overall, this method has broad applicability and is suitable for various evaluation systems with a moderate number of indexes, particularly for practical problems lacking sample data or mathematical models, where it proves effective.

2.2 Analytic Hierarchy Process

The basic concept of the analytic hierarchy process is to decompose the indexes of a complex problem into several ordered hierarchical structures based on their interrelationships. Within each level, domain experts conduct pairwise comparisons of indexes using a specific ratio scale, quantifying subjective judgments to form judgment matrices. Mathematical methods are then employed to calculate the weight of each index in the judgment matrix relative to the upper level, followed by hierarchical total ranking to compute the weight coefficients of all indexes relative to the overall objective [5]. Currently, there are over 20 methods for calculating index weight coefficients in judgment matrices, including eigenvector method, least squares method, root method, and linear programming method, with different methods producing certain variations in weight ranking.

The analytic hierarchy process has been widely applied to determine index weights in various evaluation systems. For instance, as early as the last century, R. Shen et al. [6] used this method to evaluate labor intensity in industry, synthesizing quantitative judgments on index relative importance from dozens of domain experts to obtain judgment matrices, then calculating index weights using the eigenvector method and passing consistency tests. The constructed evaluation model achieved good results in practical applications. Chinese scholars Chu Cunkun et al. [7] applied it to a three-level evaluation index system for university library subject service models dominated by qualitative indexes, also achieving favorable evaluation outcomes.

The analytic hierarchy process offers three main advantages: First, it quantifies qualitative judgments based on subjective experience and knowledge, organically combining qualitative and quantitative analysis to leverage both strengths. This approach incorporates the decision-maker’ s logical reasoning and theoretical analysis while employing objective deduction and precise calculation, making the decision-making process highly scientific and credible. Second, it hierarchically decomposes complex evaluation problems into clear, structured levels, making evaluation more explicit and organized. Third, it is not constrained by sample data availability and can solve practical problems that traditional optimization techniques cannot handle. However, the method also has limitations: First, index weight determination primarily depends on expert experience and knowledge, and different expert selections may lead to variations in weight allocation results, exhibiting subjectivity and uncertainty. Second, judgment matrices in the analytic hierarchy process are prone to serious inconsistency issues. When there are many indexes at the same level and the nine-level ratio scale method is difficult to master accurately, decision-makers can easily make contradictory and confusing relative importance judgments. To address this issue, Ma Nongle et al. [8] proposed using a three-level scale method instead of the nine-level scale to construct judgment matrices, making it easier to measure index importance and eliminating the need for consistency tests. However, this approach results in more concentrated weight distribution, making it difficult to distinguish weights among multiple indexes.

Overall, the analytic hierarchy process has broad applicability, particularly suitable for evaluation systems with complex target structures, lacking sample data, and where domain experts have clear understanding of index relative importance, provided the number of indexes is moderate.

2.3 Binomial Coefficient Method

The binomial coefficient method [9] involves K experts independently conducting pairwise comparisons of the importance of n indexes. Through multiple rounds of comparison and statistical processing, priority values representing the order of importance are obtained for each index. These indexes are then arranged from the middle outward based on their values to form an index priority sequence. After renumbering the indexes in the sequence from left to right as 1, 2, ···, the weight allocation value for the i-th index is $w_i = C_{n-1}^{i-1} / 2^{n-1}$ according to the binomial coefficient principle.

The binomial coefficient method for index weight determination was first proposed by Chinese scholar Cheng Mingxi [9] in 1983 and subsequently gained relatively wide application in China. For example, early on, Zhao Shuli [10] applied this method to multi-index evaluation of laboratory equipment investment, determining index priorities through the average scores of several experts, then calculating index weights using binomial expansion coefficients. The resulting optimal evaluation provided reliable basis for university decision-makers in equipment investment. In recent years, Liu Fuqiang et al. [11] used it to determine index weights for factors affecting the excavation schedule of pumped-storage projects. Given the numerous influencing factors that are difficult to quantify subjectively, domain experts directly judged index priorities, and weights were calculated using binomial coefficients. The final evaluation results aligned with those obtained using the entropy weight method.

The binomial coefficient method has four main advantages: First, it organically combines qualitative analysis with quantitative calculation, quantifying subjective experience to enhance scientific rigor and systematic organization. Second, it does not require specific quantification of importance magnitude, only relative comparisons between indexes, making expert judgment relatively easy and avoiding contradictory or confused assessments. Third, it employs simple binomial expansion for weight calculation, making the method straightforward to implement. Fourth, it is not constrained by sample data availability and can solve practical problems that traditional optimization techniques cannot handle. However, the method also has certain defects: First, weight determination primarily depends on subjective expert judgment, introducing randomness and uncertainty. Second, when calculating weights for indexes with different priorities using the binomial coefficient formula, identical weights may occur—symmetrically positioned indexes in the priority sequence receive the same weight values, creating certain deviations from actual conditions. Third, the method focuses only on the ordinal ranking of index importance without considering the degree of difference in relative importance, leading to potential biases in weight allocation.

Overall, this method has no restrictions on sample data availability and broad applicability, particularly suitable for multi-factor evaluation problems with moderate numbers of indexes that lack precedents and quantitative weighting experience.

2.4 Chain Ratio Scoring Method

The chain ratio scoring method [12] relies on expert knowledge and experience to compare each index with its adjacent successor in terms of importance. The importance ratios between adjacent indexes are determined by synthesizing judgments from multiple experts. Using the last index as a benchmark, the comparative weights of all indexes are calculated in reverse order and further normalized to obtain the final index weights.

The chain ratio scoring method was first proposed by Chinese scholar Lu Mingsheng [12] in 1986 and has since gained relatively wide application both domestically and internationally. For example, Chen Zhigang et al. [13] applied this method to evaluate Shanghai' s innovative city development stage, relying on experts to determine chain ratio values between evaluation indexes, then performing correction and normalization to obtain index weights. The final evaluation results aligned with Shanghai' s actual development at that time. J. Xie et al. [14] also employed this method in evaluating highway emergency plans, having experts conduct top-down pairwise comparisons of adjacent indexes to determine importance, followed by benchmarking and normalization to obtain weights. The evaluation results matched real-world choices, demonstrating the method' s effectiveness.

The chain ratio scoring method offers four main advantages: First, it organically combines qualitative judgment with quantitative calculation, making the evaluation process more systematic and scientific. Second, experts need to determine relatively few importance values, simplifying the assignment process. Third, by sequentially determining relative importance in one direction, it avoids judgment contradictions and eliminates the need for consistency tests required in analytic hierarchy process, effectively solving complex decision problems. Fourth, it is not constrained by sample data availability and can solve practical problems that traditional optimization techniques cannot handle. However, the method also has certain limitations: First, it demands high-level expert knowledge, requiring clear understanding of index importance and accurate quantitative comparisons between each adjacent index pair; otherwise, the entire index system' s weight allocation may deviate significantly. Second, weight determination primarily depends on subjective experience, introducing considerable uncertainty and arbitrariness.

Overall, this method has no restrictions on sample data availability and broad applicability, particularly suitable for various evaluation problems where relatively accurate quantitative judgments can be made regarding the relative importance of adjacent evaluation indexes.

3 Common Objective Weighting Methods

Objective weighting methods determine index weights entirely through quantitative analysis of actual index data based on mathematical theories, ensuring absolute objectivity but imposing high requirements on sample data. However, these methods ignore subjective information such as human experience, potentially producing weight allocation results that contradict actual conditions, and they are domain-dependent with limited generalizability. Currently, major objective weighting methods include: variation coefficient method, multivariate statistical methods based on principal component analysis and factor analysis, vector similarity method, grey relational analysis method, entropy method, rough set method, and neural network method.

3.1 Variation Coefficient Method

The variation coefficient method determines index weights by calculating the degree of variation in measured data for each index. Greater internal data variation indicates stronger discriminative power of the index regarding evaluation objects, resulting in larger weight allocation values [15]. The mathematical foundation includes standard deviation and maximum deviation, where weights are obtained through calculation and normalization of standard deviations (or maximum deviations) within each index' s data.

The variation coefficient method has been widely applied in index system weighting. For example, early on, Shi Guangxin et al. [16] used this method to evaluate small watershed management benefits, obtaining index weights through dimensionless processing of index sample data and standard deviation calculations. The evaluation results objectively reflected actual conditions. In recent years, H. Zheng et al. [17] applied it to determine weights for wind farm economic operation evaluation indexes, processing large samples of actual operation and monitoring data from wind farms over nearly a decade for consistency and dimensionless treatment, then calculating standard deviations to determine weights. The evaluation system' s effectiveness was verified through comparative assessments of three wind farms.

The variation coefficient method offers three advantages: First, its calculation is relatively simple and practical. Second, it fully utilizes sample data, objectively reflecting the discriminative capacity of each index and ensuring absolute objectivity. Third, it imposes no restrictions on the number of evaluation indexes, providing broad applicability. However, the method also has limitations: First, evaluation results are highly correlated with sample selection, and different samples may produce different weight allocations. When sample size is small and lacks representativeness, the method' s accuracy is low. Second, it cannot handle outliers in sample data, and the presence of outliers introduces significant errors in weight determination. Third, it cannot reflect intrinsic relationships between indexes, analyzing each index independently. Fourth, it relies purely on objective calculation without incorporating decision-makers' understanding of index importance.

Therefore, this method is suitable for comprehensive evaluations where indexes are relatively independent, sample data is universal, relatively complete, and large-scale, and contains no outliers.

3.2 Multivariate Statistical Methods

Multivariate statistical methods determine index weights by applying multivariate statistical analysis to sample data, including principal component analysis and factor analysis.

3.2.1 Principal Component Analysis Principal component analysis [18] employs dimensionality reduction to transform a set of correlated indexes into another set of fewer uncorrelated comprehensive indexes (principal components) based on variance contribution rates, with further normalization to obtain index weights.

Since its emergence, principal component analysis has been widely applied. For example, early Chinese scholars Jin Xingri et al. [19] used this method for comprehensive evaluation of industrial enterprise economic benefits, standardizing sample data of major economic benefit indexes from Yanbian General Factory (1990-1995) and conducting principal component analysis to obtain four principal components and index weights. The final evaluation results aligned with those from ideal solution methods. In recent years, B. Prado et al. [20] employed this method to assess climate variables in the German city of Münster, using principal component analysis on relevant sample data from 2008-2012 to develop an evaluation model with one principal component explaining overall variables, producing results consistent with actual conditions.

Principal component analysis offers three main advantages: First, it replaces numerous correlated indexes with fewer independent ones, solving information overlap problems and simplifying index structure. Second, index weights are determined objectively from variance contribution rates of principal components, avoiding subjective influences and ensuring reasonableness. Third, it imposes no specific restrictions on the number of indexes or samples, providing wide applicability. However, the method has four limitations: First, its calculation process is relatively complex, and results are highly correlated with sample selection. Second, it loses some sample data information, potentially eliminating indexes with practical significance and creating deviations from actual conditions. Third, it assumes linear relationships between indexes, producing biases when applied to many real-world nonlinear systems. Fourth, it relies purely on objective data, ignoring subjective experience and potentially yielding results that contradict actual conditions.

Overall, principal component analysis is suitable for determining index weights in complex evaluation systems with relatively complete and representative sample data, certain correlations between indexes, and predominantly linear relationships.

3.2.2 Factor Analysis Factor analysis [21] shares similar basic concepts with principal component analysis, transforming correlated indexes into fewer uncorrelated indexes and determining weights based on variance contribution rates of each factor. The difference lies in that principal component analysis linearly combines original indexes, whereas factor analysis decomposes original indexes into common factors shared by all indexes and specific factors unique to each index, providing more explicit practical meaning.

Factor analysis has been widely applied in various comprehensive evaluation problems, particularly in socioeconomic domains. For example, early Chinese scholars Wan Jianqiang et al. [22] applied this method to evaluate listed company operating performance, comprehensively assessing 11 indexes from 13 representative companies in the building materials industry. The evaluation results aligned with actual economic rankings. In recent years, A. Bai et al. [23] used it for national economic ranking assessment, conducting factor analysis on 15 economic indicators from 20 countries using IMF dataset data. Three comprehensive factors explained and represented all indexes for evaluation, with final rankings nearly identical to world rankings, confirming the method' s reliability.

Factor analysis shares similar advantages and disadvantages with principal component analysis for index weighting. However, since the number of factors is smaller than original indexes (whereas principal components can equal original indexes), factor analysis generally loses more information and is less precise than principal component analysis, with more complex calculations. Additionally, factor analysis strictly requires correlations between indexes in the evaluation system. Nevertheless, factor analysis can explicitly explain the specific content of original indexes, interpret reasons for correlations, and provide deeper understanding of index content.

Overall, factor analysis is more suitable for complex evaluation problems requiring in-depth analysis of socioeconomic phenomena, with highly correlated indexes and large amounts of representative, complete sample data.

3.3 Vector Similarity Method

The vector similarity method [24] calculates weights by determining the similarity between feature vectors composed of each index' s sample data and a reference vector composed of ideal values for all indexes. The magnitude of vector similarity reflects each index' s contribution to achieving optimal system performance, with normalization yielding index weights.

The vector similarity method for index weighting was initially proposed by Chinese scholars Jiao Liming et al. [24] and subsequently applied by many domestic scholars in comprehensive evaluations. For example, Jiao Liming et al. [25]

used this method to evaluate air defense brigade system effectiveness, extracting six representative sample groups, processing data dimensionlessly, calculating similarity between each index data vector and the standardized ideal reference vector, and normalizing to obtain index weights for effective system assessment. Xie Ping et al. [26] applied it to lake eutrophication evaluation, using measured water quality data from 30 lakes nationwide as samples. After dimensionless processing of vectors composed of sample data and ideal reference vectors for each evaluation index, they calculated vector similarity and normalized to obtain index weights, producing results highly consistent with previous fuzzy and stochastic method evaluations.

The vector similarity method offers three main advantages: First, calculation is simple and straightforward, cleverly utilizing the fact that an ideal reference vector composed of all indexes becomes a unit vector with all elements equal to 1 after dimensionless processing, making similarity calculation between an index' s ideal value and its data equivalent and reducing computational steps. Second, results are easy to understand, considering relationships with optimal solutions and demonstrating strong practicality. Third, it fully utilizes sample data without human interference, ensuring strong objectivity. Fourth, it imposes no specific restrictions on index or sample quantities, providing broad applicability. However, the method also has limitations: First, it cannot solve information duplication problems caused by correlations between indexes, potentially inflating weights of correlated indexes through repeated calculations. Third, it relies purely on objective data, ignoring subjective experience and potentially producing results that contradict reality.

Overall, this method is suitable for comprehensive evaluation systems with moderate amounts of relatively complete, typical, and representative sample data, and relatively independent evaluation indexes.

3.4 Grey Relational Analysis Method

The grey relational analysis method [27] combines data comparison with geometric curve trend analysis to calculate weights, using the magnitude of relational degree between each solution and the optimal solution to determine index weights. Specifically, the method employs grey relational judgment matrices and correlation coefficients between each solution and the ideal solution to calculate each index' s contribution to achieving optimal system performance, with normalization yielding index weights.

Grey relational analysis has been widely applied in practical decision-making problems across many disciplines. As early as the 1980s, Chinese scholar Ma Zhiying et al. [28] applied this method to cotton variety evaluation, using trait data from seven cotton varieties in the Yellow River cotton disease-resistant region in 1986 as samples. After dimensionless data processing, they calculated grey relational coefficients between each variety and the ideal variety, normalized to obtain index weights, with final evaluation results consistent with fuzzy comprehensive evaluation outcomes. Later, C. Ho [29] applied it to bank operating performance evaluation, using financial documents from three Taiwanese banks as sample data and following similar steps to calculate index weights. The evaluation model produced results consistent with financial statement analysis, demonstrating the method' s effectiveness.

Grey relational analysis offers four main advantages: First, its calculation process is relatively simple, considering relationships with ideal decision solutions and producing intuitive, easy-to-understand results. Second, it requires no large samples, only small amounts of representative data, with no restrictions on index quantity. Third, it has certain error tolerance capabilities, as correlation calculations use maximum and minimum differences, mitigating inaccuracies from partial data loss or human errors and yielding relatively reasonable results. Fourth, it relies on sample data, avoiding human interference and ensuring strong objectivity. However, the method also has limitations: First, the discrimination coefficient value in grey relational coefficient calculation is subjectively determined without fixed standards, and different values affect final weight allocation, reducing credibility. Second, the method' s accuracy is affected by sample selection, with different samples potentially leading to different evaluation results. Third, it cannot solve information overlap problems caused by index correlations. Fourth, it does not consider subjective experience, potentially producing results that contradict actual conditions.

Overall, this method has broad applicability with no restrictions on index or sample quantities, and is particularly suitable for comprehensive evaluation systems with relatively complete, representative, and typical sample data, and relatively independent indexes.

3.5 Entropy Method

The entropy method [30] determines weights from the perspective of index disorder—or entropy—reflecting each index' s discriminative power regarding evaluation objects. Smaller entropy values indicate more ordered sample data, greater differences among samples, stronger discriminative capacity, and consequently larger weights. The method first calculates each index' s entropy using an entropy function, then normalizes entropy values into index weights.

Since its proposal, the entropy method has been widely applied across many domains. For example, early on, Zhu Shunquan et al. [31] applied it to evaluate listed company financial conditions, using 15 evaluation index data from 20 companies in the 2000 *China Securities Journal* as samples. After dimensionless processing, they calculated each index' s entropy and normalized to obtain weights, then derived comprehensive evaluation values through simple weighting, producing reasonable results. Later, A. Gorgij et al. [32] used it for groundwater quality assessment, evaluating 21 groundwater samples from Iran' s Azarshahr Plain in 2016 through similar entropy method steps to calculate index weights and determine water quality grades using a comprehensive evaluation model, with results consistent with spatial autocorrelation coefficient evaluations.

The entropy method offers three advantages for index weighting: First, its calculation is relatively simple, determining weights from the perspective of index discriminative power, producing intuitive and easy-to-understand results with strong practicality. Second, it relies entirely on sample data, avoiding subjective interference and ensuring strong objectivity. Third, it imposes no restrictions on index quantity, providing broad applicability. However, the method has three shortcomings: First, its accuracy is affected by sample selection, with different samples potentially producing different weight allocations, and it requires high data completeness and sample size. Second, it cannot reflect correlations between indexes or solve information overlap problems. Third, it cannot incorporate decision-makers' understanding of index importance and may produce results that contradict facts.

Therefore, this method is suitable for comprehensive evaluations with large sample sizes, complete and universal data information, and relatively independent indexes.

3.6 Rough Set Method

The rough set method [33] for index weighting first classifies evaluation objects based on all indexes in the system. After removing one index at a time, it considers the degree of change in object classification compared to the original classification, with index importance proportional to the magnitude of change. Attribute (index) importance in rough sets can be defined through algebraic representation or information representation, further developing into multiple weighting sub-methods based on equivalence relations, dominance relations, and tolerance relations. These rough set weighting methods differ primarily in their classification criteria: equivalence relations partition all evaluation objects by equivalence, dominance relations partition by superiority degree on conditional attribute sets, and tolerance relations partition by object differences. Equivalence relations require discrete data, dominance relations can handle continuous data, and tolerance relations mainly address missing sample data.

The rough set method has been widely applied to index weighting in various evaluation problems. For example, I. P. W. C. et al. [34] applied it to water quality evaluation, using measured water quality data from April and May (1992-1997) in the Han River Basin as conditional attribute samples and equal-weight aggregated averages of index values as decision attributes. After discretizing sample data, they calculated index importance using algebraic rough set methods based on equivalence relations and normalized to obtain weights, with final comprehensive water quality evaluation grades matching actual conditions. Zou Bin et al. [35] used it to evaluate energy consumption in East China, using 2011 energy consumption data for eight East China provinces/municipalities from the China Energy Statistical Yearbook as samples. They conducted superiority class partitioning on attribute sets (without discretization) and calculated index weights based on algebraic attribute importance definitions, producing results consistent with and reliable as those from grey relational weighting methods.

The rough set method offers five advantages for index weighting: First, it can handle extensive data types, including discrete and continuous data, with effective processing of continuous data reducing information loss from discretization. Second, it has strong error tolerance, can process missing sample data, and effectively solves weighting problems with incomplete data. Third, information representation of importance can compensate for the defect of zero weight values that may occur in algebraic representation, improving method precision. Fourth, it imposes no restrictions on index quantity, providing broad applicability. Fifth, it relies entirely on objective data, avoiding subjective interference. However, the method also has limitations: First, its precision is affected by sample selection, with different samples potentially producing different decision results. Second, it cannot solve information overlap problems caused by index correlations. Third, as an objective weighting method without prior information, calculation results may contradict decision-makers' understanding.

Overall, the rough set method has the widest application range among objective weighting methods and is suitable for multi-attribute decision-making problems with relatively independent indexes, relatively universal data samples, and large sample sizes.

3.7 Neural Network Method

The most commonly used neural network method is the BP neural network algorithm. Its basic weighting concept [36] involves nonlinear parallel learning training on large data samples according to certain learning mechanisms. Specifically, based on error precision requirements, continuous iterative adjustment of differences between output data and known sample output data yields a satisfactory connection weight matrix from the input layer to the hidden layer. The absolute values of connection weights from each input layer node to all hidden layer nodes are summed and normalized to obtain index weights.

Neural network methods are increasingly used for index weight determination. For example, early on, J. Ch. et al. [37] applied this method to e-government website evaluation, using relevant data from 20 Ningbo e-government websites as training samples (including 11 evaluation indexes and expert evaluation results) to train and calculate index weights. The evaluation model was then applied to assess 10 additional e-government websites, producing results consistent with expert evaluations. Later, S. Silva et al. [38] used it to evaluate extra virgin olive oil stability, with training samples consisting of 18 olive oil varieties stored under dark and light conditions (including 11 evaluation indexes and evaluation results from experimental data). After training to calculate index weights, the evaluation model was applied to 10 olive oil stability assessments beyond the training samples, achieving over 90% consistency with experimental test classifications, demonstrating high accuracy.

The neural network method offers three main advantages: First, it can handle nonlinear complex system evaluation problems and conduct dynamic assessments with powerful analytical capabilities. Second, through sample learning and training, it can obtain scientifically reasonable and practically validated information on index relative importance, ensuring objectivity and practicality. Third, it imposes no restrictions on evaluation index quantity, providing wide application scope. However, the method also has limitations: First, being entirely training sample-based, it imposes high requirements on samples, which must be correct and comprehensive in coverage (emphasizing broad coverage types rather than sheer volume) to ensure result reliability. It also somewhat ignores subjective experience, potentially producing results that contradict decision-makers' preferences.

In summary, the neural network method has strong processing capabilities for complex system evaluation problems and is suitable for determining index weights in various complex systems with relatively large amounts of comprehensive sample data and relatively independent indexes.

4 Common Integrated Weighting Methods

Integrated weighting methods combine subjective and objective weighting methods through different preference coefficients to determine index weights. By incorporating information from expert experience and decision-maker preferences in subjective methods and information about intrinsic relationships between indexes and evaluation objects in objective methods, integrated methods achieve complementary advantages through mathematical operations. Currently, various integrated weighting methods based on different principles exist, but they can be broadly classified into four categories: integrated weighting methods based on additive or multiplicative synthesis normalization, integrated weighting methods based on deviation square sum, integrated weighting methods based on game theory, and integrated weighting methods based on objective optimization.

Integrated weighting methods based on additive or multiplicative synthesis normalization directly add or multiply the index weights obtained from subjective and objective weighting methods with equal preference, followed by normalization to obtain comprehensive weights. For example, L. Yang et al. [39] used this method in supply chain risk assessment, equally weighting and multiplying subjective weights from analytic hierarchy process with objective weights from variation coefficient method, then normalizing to obtain comprehensive index weights, achieving high-precision evaluation results.

Integrated weighting methods based on deviation square sum approach the problem from the perspective of solution discriminability, seeking subjective-objective weight allocation coefficients that maximize the dispersion of comprehensive evaluation values—specifically, maximizing the total deviation square sum among comprehensive evaluation values of different solutions. For example, B. Meng et al. [40] applied this method to bank credit risk assessment, calculating optimal adjustment coefficients for subjective and objective weights based on the principle of maximizing overall deviation square sum of evaluation results from different objects, then normalizing to obtain comprehensive index weights. The evaluation results showed higher accuracy compared to single subjective or objective weighting methods.

Integrated weighting methods based on game theory seek compromise or consensus between different subjective and objective weights, preserving original weight information as much as possible and minimizing deviations from subjective and objective weights. For example, C. Lu et al. [41] used this approach in educational informatization development level assessment, calculating weight allocation coefficients for subjective weights from analytic hierarchy process and objective weights from variation coefficient method, then normalizing to obtain integrated comprehensive weights. The model achieved good results in evaluating educational informatization development levels across 11 schools in Suzhou.

Integrated weighting methods based on objective optimization determine subjective-objective weight coefficient allocation based on principles of optimal comprehensive decision results, including specific solution methods for maximizing comprehensive target values or maximizing deviation from negative ideal solutions. For example, J. Yan et al. [42] used this method in learning city assessment, allocating subjective-objective weight coefficients based on the principle of maximizing overall comprehensive evaluation values from expert estimation and entropy method, then accurately evaluating four learning cities using this model.

In summary, various integrated weighting methods have theoretical foundations and employ mathematical concepts such as linear equations and matrix operations for specific solutions. Some methods involve simple integration calculations, while others introduce significant computational complexity to the evaluation process. However, no absolute superiority exists among these methods, and no consistent conclusion has been reached regarding which integrated weighting method to select for specific evaluation problems. Compared with single subjective or objective weighting methods, integrated methods generally produce more scientific and reasonable evaluation results, but they may also introduce larger random deviations that cause results to deviate from actual conditions. Integrated weighting methods cannot completely replace single weighting methods, and a rational understanding is necessary when selecting weighting methods for practical problem research.

This paper focuses on analyzing the fundamental concepts and principles of specific weight calculation methods among common subjective weighting methods, objective weighting methods, and integrated subjective-objective weighting methods. It compares the advantages and disadvantages of various methods and clarifies their applicable scopes. The study finds that each weighting method emphasizes different aspects of problems, possesses certain advantages and defects in weight calculation, and exhibits significant differences in applicable scopes.

Therefore, when selecting weighting methods for multi-factor evaluation indexes, it is essential to rationally understand and grasp each method' s strengths and weaknesses, conduct concrete analysis of concrete problems, and select appropriate weighting methods based on actual characteristics of evaluation objects and problems—such as data sample availability, sample representativeness, and index correlations—to ensure relatively scientific and reasonable evaluation results.

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Author Contributions

Liu Qiuyan: Responsible for data collection, analysis, and paper writing.
Wu Xinnian: Proposed the research topic and framework, and revised and improved the paper.

Note: Figure translations are in progress. See original paper for figures.