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Full Text

Note on the Butterfly Effect in Holographic Superconductor Models

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Abstract

In this note we remark that the butterfly effect can be used to diagnose the phase transition of superconductivity in a holographic framework. Specifically,

we compute the butterfly velocity in a charged black hole background as well as anisotropic backgrounds with Q-lattice structure. In both cases we find its derivative with respect to temperature is discontinuous at critical points. We also propose that the butterfly velocity can signalize the occurrence of thermal phase transition in general holographic models.

I. INTRODUCTION

Recently the quantum butterfly effect has become a hot spot of research that links gauge/gravity duality to quantum many-body theory and quantum information theory [?]. Diagnosed by out-of-time-order correlation (OTOC) functions, the butterfly effect describes information scrambling over a quantum chaotic system. On the gravity side, the butterfly effect is described by a shock wave geometry on the horizon that can be induced by an infalling particle which is exponentially accelerated. The butterfly effect ubiquitously exists in holographic theories due to its sole dependence on the near-horizon data of the gravitational bulk theory. In particular, the Lyapunov exponent λ_L is always characterized by the Hawking temperature of the black hole as $\lambda_L = 2\pi k_B T$, while the butterfly velocity is completely determined by the horizon geometry [?, ?, ?]. Moreover, a bound on chaos has been proposed as $\lambda_L \leq 2\pi k_B T$, and the saturation of this bound is viewed as the criterion for a quantum chaotic system to have a classical gravity dual description [?].

Stimulated by the above investigation in holographic approaches, many physicists in condensed matter and quantum information communities have made great efforts in measuring the OTOC in laboratory settings [?, ?]. This related progress is supposed to provide more practical tools to test proposals in holographic theories and, in turn, push forward the investigation of butterfly effects in quantum many-body systems.

In a recent paper [?], we investigated the butterfly effect in holographic models that exhibit metal-insulator transition (MIT) and found that the butterfly velocity v_B can diagnose quantum phase transitions (QPT). The key point is that the occurrence of QPT usually involves RG flows from UV to different IR fixed points [?]. On the other hand, the butterfly velocity v_B depends solely on the IR geometry. Therefore, the change of IR fixed points may be reflected by the distinct behavior of v_B. In this note, we intend to argue that the butterfly effect can exhibit attractive behavior during the course of thermal phase transition as well. This extension is natural, since based on Landau theory the occurrence of thermal phase transition is always accompanied by symmetry breaking characterized by some order parameter. While in the context of holography, spontaneous symmetry breaking is usually a reflection of the instability of the background in the bulk, signalized by the appearance of black hole hair which is supposed to deform the horizon—that is, the IR geometry—strongly.

Therefore, to provide evidence supporting this argument, we will investigate the temperature behavior of the butterfly velocity in holographic superconductor models. Specifically, we will demonstrate that the derivative of v_B with respect to temperature is discontinuous at critical points of phase transition.

We organize this paper as follows. In the next section, we first consider the butterfly effect in the simplest holographic model with superconductivity, which is constructed over a charged black hole. Then we turn to study this effect over more complicated backgrounds with lattice structure in subsection II C. A brief discussion about possible extensions and experimental prospects will be presented at the end of this note.

II. BUTTERFLY EFFECTS AND HOLOGRAPHIC SUPERCONDUCTIVITY

In this section we first introduce the butterfly velocity on anisotropic backgrounds. Using these anisotropic butterfly velocity results, we reveal that the butterfly velocity can diagnose superconductivity phase transitions.

A. Butterfly Velocity on Anisotropic Background Given a background with a black brane, we can compute the shockwave solution on the horizon generated by a particle released from the asymptotic AdS region. The butterfly effect is represented by this sort of shockwave geometry, from which the Lyapunov exponent and butterfly velocity can be read off [?, ?]. For a generic anisotropic black brane geometry,

$$ds^2 = \left[-\frac{(1-z)f(z)}{z^2}dt^2 + \frac{dz^2}{(1-z)f(z)z^2} + \frac{V_x(z)}{z^2}dx^2 + \frac{V_y(z)}{z^2}dy^2\right]$$

the butterfly velocity is also anisotropic, which can be written as [?]

$$\bar{v}_B(\theta) = v_B\sqrt{\frac{\sec^2(\theta)V_x(z)}{V_x(z)+\tan^2(\theta)V_y(z)}}\Bigg|_{z=1}$$

where θ is the polar angle. $v_B = \bar{v}_B(0)$ is the butterfly velocity along the x-direction, given by

$$v_B = \sqrt{\frac{-2\pi\hat{T}}{V_y(z)\left[V_x'(z)-2V_x(z)\right]+V_x(z)\left[V_y'(z)-2V_y(z)\right]}}\Bigg|_{z=1}$$

where the prime denotes the derivative with respect to the radial coordinate z, and $\hat{T}$ is the Hawking temperature of the black brane. The metric ansatz and these formulas are applicable for the two holographic models in the subsequent subsections.

B. Butterfly Effects in a Simple Holographic Superconductor The minimal ingredients to build a superconductor model in a holographic framework are provided by adding a charged complex scalar field into Einstein-Maxwell theory, with Lagrangian [?, ?]

$$\mathcal{L} = R + \frac{6}{L^2} - \frac{1}{4}F^2 - |D_\mu \Psi|^2 - M^2|\Psi|^2.$$

Notice that we have set the AdS radius $L = 1$. Here $F = dA$ is the curvature of the $U(1)$ gauge field A, Ψ is the charged complex scalar field with mass M and scaling dimension $\Delta_\Psi = \frac{3}{2} + \sqrt{\frac{9}{4} + M^2}$, and charge q. The covariant derivative is $D_\mu = \partial_\mu - iqA_\mu$.

We solve the equations of motion by taking the metric ansatz and

$$A = \mu(1 - z)a(z)dt, \quad \Psi = \Psi(z),$$

where $f(z) \equiv (1 + z + z^2 - \mu^2 z^3/4)S(z)$ and μ is the chemical potential in the dual field theory. The Hawking temperature is then given by

$$\hat{T} = \frac{(12 - \mu^2)S(1)}{16\pi}.$$

The corresponding dimensionless temperature is $T = \hat{T}/\mu$. Note that since the system is isotropic, the anisotropic metric can be reduced to the isotropic case, i.e., $V_x = V_y$. For simplicity, we set the mass and charge of the complex scalar field as $M^2 = -2$ and $q = 2$, such that its scaling dimension is $\Delta_\Psi = 2$ and its asymptotic behavior at UV is

$$\Psi = z\Psi_1 + z^2\Psi_2.$$

We treat Ψ_1 as the source and Ψ_2 as the expectation value of the dual operator, while setting $\Psi_1 = 0$ so that condensation occurs spontaneously. At high temperature, the solution to the equations of motion with the given ansatz is simply the Reissner-Nordström AdS (RN-AdS) black brane solution with $\Psi(z) = 0$ and $S(z) = a(z) = V_x(z) = V_y(z) = 1$. However, below the critical temperature, imposing regular boundary conditions on the horizon and requiring the scalar field to decay at UV as in Eq. (7), one can numerically find new black brane solutions with scalar hair, which are dual to a superconducting phase in the boundary theory.

In this simple isotropic model we have $V_x(z) = V_y(z)$, so the formula simplifies to $v_B = \sqrt{\pi T\mu/[2V_x(1) - V_x'(1)]}$. We now numerically compute v_B as a function of temperature T during the phase transition. Figure 1 shows $\partial_T v_B$ as a function of temperature T, with the inset plot showing v_B versus T. In this figure it is evident that the derivative $\partial_T v_B$ is discontinuous at the critical temperature T_c (the vertical red dashed line), which indicates that the butterfly velocity can be utilized as a new independent probe of the phase structure of the superconductor.

[Figure 1: see original paper]

C. Butterfly Effects in Holographic Q-Lattice Superconductor The second model we consider is the holographic superconductor on Q-lattices, which has been studied in [**?**]. Its Lagrangian reads

$$\mathcal{L} = R + \frac{6}{L^2} - \frac{1}{4}F^2 - |D_\mu \Psi|^2 - M^2|\Psi|^2 - |\nabla\Phi|^2 - m^2|\Phi|^2.$$

Compared with the Lagrangian in (4), an additional neutral complex scalar field Φ with mass m is introduced to break translational invariance [**?**]. We can solve this gravitational system by taking the metric (1) and

$$\Phi = e^{i\hat{k}x} z^{3-\Delta_\Phi}\phi(z), \quad A = \mu(1-z)a(z)dt, \quad \Psi = \Psi(z),$$

where $\Delta_\Phi = \frac{3}{2} + \sqrt{\frac{9}{4} + m^2}$ is the scaling dimension of Φ. Note that since we only introduce the lattice in the x-direction, our geometry is anisotropic. Each black brane solution is characterized by three scaling-invariant parameters: the Hawking temperature $T \equiv \hat{T}/\mu$, the lattice amplitude $\lambda \equiv \hat{\lambda}/\mu^{3-\Delta_\Phi}$ with $\hat{\lambda} = \phi(0)$, and the wave vector $k \equiv \hat{k}/\mu$.

For normal states ($\Psi = 0$), there exist MITs when adjusting λ or k [**?**]. A complete phase diagram can be found in [**?**, **?**]. The superconducting phase has been numerically found in [**?**], where it was shown that the lattice structure suppresses condensation and lowers the critical temperature compared with the situation when the lattice is absent. The phase structure in [**?**] demonstrates that the transition to the superconducting phase from the metallic phase is easier than from the insulating phase.

Our main purpose now is to compute the butterfly velocity $\bar{v}_B(\theta)$ for a given background and observe its behavior during the phase transition from a normal phase (which could be metallic or insulating) to a superconducting phase. First, we focus on the butterfly velocity along the x-direction, i.e., $\bar{v}_B(0) = v_B$. For simplicity, we set $\{M^2, m^2, q\} = \{-2, -2, 2\}$ and demonstrate the temperature behavior of v_B in two typical cases in Figure 2: one for metal-superconductor transition and the other for insulator-superconductor transition. In both cases we find that the first-order derivative of v_B with respect to temperature is discontinuous.

Next we examine the butterfly velocity $\bar{v}_B$ as a function of temperature T in different directions (see Figure 3). It is interesting to notice that at a given temperature, the butterfly velocity increases with increasing polar angle in the first quadrant, which implies the lattice structure suppresses the propagation of quantum information. We demonstrate the anisotropy of $\partial_T \bar{v}_B(\theta)$ more transparently in Figure 4, which arises from the introduction of the lattice structure. It can be seen from Figure 4 that $\partial_T \bar{v}_B(\theta)$ is discontinuous at $T_c \approx 0.0451$ in any direction. This reflects the fact that the emergence of condensation, which is responsible for the discontinuity of $\partial_T \bar{v}_B(\theta)$, depends solely on the temperature. Also, the period of $\partial_T \bar{v}_B(\theta)$ can be clearly read off as π, respecting the period of $\bar{v}_B(\theta)$. In summary, the butterfly velocity is a good diagnostic of the superconducting phase transition.

[Figure 2: see original paper]

[Figure 3: see original paper]

[Figure 4: see original paper]

III. DISCUSSION

In this note we have proposed that the butterfly effect should exhibit distinct behavior during the course of thermal phase transition. In our two holographic superconductor models we have explicitly demonstrated that the first derivative of v_B with respect to temperature is discontinuous.

Next we point out some possible generalizations of our work. First, it is desirable to analytically obtain the discontinuity of $\partial_T \bar{v}_B(\theta)$ in the two models we discussed. Second, based on the arguments presented in the introduction, it is natural to expect that the interesting phenomenon in this note can be observed in other holographic superconductor models. In addition, we also expect that the butterfly effect can capture the occurrence of other sorts of thermal phase transitions as well, for instance, the transition between RN black holes and dilatonic black holes [**?**], or the MIT induced by Charge Density Waves (CDW) [**?**].

Finally, we conjecture that the non-analytical behavior of the butterfly velocity at critical temperature could be diverse. For example, instead of the first-order discontinuity that we have observed in this note, zero-order discontinuity or higher-order discontinuity might be observed in other sorts of thermal phase transitions.

Furthermore, we expect that what we have observed for the butterfly effect in holographic contexts can be extended to characterize thermal phase transitions in realistic systems which may not have a gravity dual. The validity of such an extension could in principle be tested experimentally in light of recent progress on measurements of the OTOC [**?**, **?**].

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Note: Figure translations are in progress. See original paper for figures.