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Full Text

Preamble

Finite-size Effects for Dyonic Giant Magnons in CP^3

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Abstract

We study finite-size giant magnons in $AdS_4 \times CP^3$ background using the classical spectral curve constructed in this paper. We compute the finite-size corrections to the dispersion relations for giant magnons using our twisted algebraic curve, based on the method proposed in [1] for the RP^3 case, in which the authors computed the finite-size corrections of giant magnons in $AdS_4 \times CP^3$ by introducing a finite-size resolvent G_{finite}(x). We obtain exactly the same result as in [2], where a totally different approach was used.

1 Introduction

The gauge/gravity duality conjecture proposed in [3] is a striking and thought-inspiring idea, which relates a large N gauge theory to a certain string theory in such a non-trivial way that makes it very hard to prove. Since integrable structures were found on both sides in AdS_5/CFT_4 and AdS_4/CFT_3 dual pairs [4], [5], [6], [7], integrability techniques can help us to understand and test this conjecture qualitatively (see [8] for a review).

Investigating such dualities without or with less supersymmetry is very important. One approach is to make marginal deformations in the field theory side. The β-deformation of AdS_5/CFT_4 is such an example, which in the field theory corresponds to an exact marginal deformation [9], while the deformed background in string theory can be generated by TsT transformations [10]. People have also investigated the three-parameter deformation which is also called γ-deformation as a generalization of β-deformation [11], [12]. Another interesting example is the duality between type IIA string theory on $AdS_4 \times CP^3$ and γ-deformed ABJM theory [13]. This is a three-parameter deformation which breaks supersymmetry completely, while here we are interested in the β-deformed theory with $= 2$ supersymmetry.

In our recent paper [15], the integrable structure of γ-deformed ABJM theory was investigated in detail. The twist matrices were obtained in various bases. In this paper, we will try to calculate the finite-size effect for giant magnons in $AdS_4 \times CP^3$ using the classical spectral curve. In order to compare our results with [2], we had better use different charges from our previous choice in [15]. These charges are listed in Table 1 . These two choices can be easily related through non-singular linear transformations, so they are equivalent. The twist matrix still has the form (in the su(2) grading)

$$\begin{pmatrix} 2\delta_2 - \delta_1 + 2\delta_2 & -2\delta_2 - \delta_1 + 2\delta_2 + \delta_3 \\ -\delta_1 + 2\delta_2 + \delta_3 & \delta_1 + 2\delta_2 + \delta_3 \\ 4\delta_2 - 2\delta_1 & -2\delta_2 + \delta_3 \\ 2\delta_2 - 2\delta_1 + 4\delta_2 + 2\delta_3 & \end{pmatrix} \tag{1.1}$$

but with the relation between δ_i's and γ_i's different:

$$\gamma_3 = \delta_3 - \delta_1. \tag{1.2}$$

We always assume that the deformation parameters are real, since otherwise generically integrability will be broken [14].

Table 1: Charges of fields under three U(1) generators of SU(4)__R.

Fields	U(1)__ 1	U(1)__ 2	U(1)__
Y^1	+1	0	0
Y^2	0	+1	0
Y^3	0	0	+1
Y^4	-1	-1	-1
Ψ_1	+1	0	0
Ψ_2	0	+1	0
Ψ_3	0	0	+1
Ψ_4	-1	-1	-1

We make the natural assumption that the type IIA string theory on $AdS_4 \times CP^3$ is integrable based on previous results on the field theory side. Its classical integrability can be proven similarly to the studies of type IIA in [17]. Although we did not make explicit calculations, we expect that the γ-deformed $AdS_4 \times CP^3$ is equivalent to strings in the undeformed theory with twisted boundary conditions˜__i(2π) =˜__i(0) + 2π(m__i - γj {ijk}J__k) based on a general analysis of TsT transformation, where __i's are the three U(1) directions $_1$, $_2$, listed in Table 1, and J__i's are the corresponding conserved angular momenta. For giant magnons, these m__i's are not necessarily integers.

The dispersion relation for RP^3 giant magnons in $AdS_4 \times CP^3$ has been calculated in [18], and the finite-size corrections were obtained in [2] by searching for the needed classical string solution directly. In this paper, we use the classical spectral curve method to calculate these quantities and compare our results to those of [2]. Finally, we find they match perfectly.

2 γ-deformed ABA at strong coupling

In this section we briefly review the γ-deformed AdS_4/CFT_3 asymptotic Bethe ansatz (ABA) equations and introduce some notations which will be useful in the following sections. We use the same notation as in our previous paper [15].

Let us first define some useful functions as

$$R_l = \prod_{j=1}^{K_l} \frac{x(u) - x_{l,j}^+}{x(u) - x_{l,j}^-}, \quad B_l = \prod_{j=1}^{K_l} \frac{1 - \frac{x_{l,j}^+}{x(u)}}{1 - \frac{x_{l,j}^-}{x(u)}}, \quad S_l = \prod_{j=1}^{K_l} \sigma_{BES}(x(u), x_{l,j}), \quad (2.1)$$

and the functions with no subscript mean a product of type-4 and type-$\bar{4}$ ones: $R = R_4 R_{\bar{4}}$, $B = B_4 B_{\bar{4}}$.

We also introduce the excitation number vector K = (L, K_1, K_2, K_3, K_4, $K_{\bar{4}}$) as before, then the γ-deformed ABA can be written as (in the su(2) grading)

$$\begin{aligned}
e^{-2\pi i (AK)_3} &= \frac{B^{(+)}}{B^{(-)}}\bigg|_{u=u_{1,k}} \frac{Q_1^{--}}{Q_1^{++}}, \\
e^{-2\pi i (AK)_2} &= \frac{R^{(-)}}{R^{(+)}}\bigg|_{u=u_{2,k}} \frac{Q_2^{++}}{Q_2^{--}}, \\
e^{-2\pi i (AK)_1} &= \frac{R^{(-)}}{R^{(+)}}\bigg|_{u=u_{3,k}} \frac{Q_3^{++}}{Q_3^{--}}, \\
e^{-ip(u_{4,k})L} &= \frac{B^{(-)}}{B^{(+)}}\bigg|_{u=u_{4,k}} \frac{Q_4^{++}}{Q_4^{--}} S_{4S_{\bar{4}}}, \\
e^{-ip(u_{\bar{4},k})L} &= \frac{B^{(+)}}{B^{(-)}}\bigg|_{u=u_{\bar{4},k}} \frac{Q_{\bar{4}}^{++}}{Q_{\bar{4}}^{--}} S_{4S_{\bar{4}}}, \\
e^{-2\pi i (AK)_{\bar{4}}} &= \frac{R^{(-)}}{R^{(+)}}\bigg|_{u=u_{\bar{4},k}} \frac{Q_{\bar{4}}^{++}}{Q_{\bar{4}}^{--}},
\end{aligned} \tag{2.2}$$

where the x^± are Zhukowsky variables,

$$x^{\pm} + \frac{1}{x^{\pm}} = \frac{u}{h(\lambda)} \pm \frac{i}{2}, \tag{2.3}$$

and we have already used the general notation f^{±} ≡ f(u ± i/2), f^{±±} ≡ f(u ± i), f^{[± a]} ≡ f(u ± ia/2).

The spectrum of all conserved charges is given by the momentum-carrying roots u_4 and $u_{\bar{4}}$ alone:

$$Q_n = \sum_{j=1}^{K_4} q_n(u_{4,j}) + \sum_{j=1}^{K_{\bar{4}}} q_n(u_{\bar{4},j}), \quad q_n = \frac{h(\lambda)}{i(n-1)} \left(\frac{1}{(x^+)^{n-1}} - \frac{1}{(x^-)^{n-1}} \right). \tag{2.4}$$

The conserved momentum reads

$$p = \tilde{p} + 2\pi (AK)_0, \tag{2.5}$$

where

$$\tilde{p} = \frac{1}{i} \sum_{j=1}^{K_4} \log \frac{x_{4,j}^+}{x_{4,j}^-} + \frac{1}{i} \sum_{j=1}^{K_{\bar{4}}} \log \frac{x_{\bar{4},j}^+}{x_{\bar{4},j}^-}. \tag{2.6}$$

At weak coupling, h(λ) behaves like h(λ) √(λ/2), and at strong coupling, h(λ) λ/2. The BES dressing phase σ_{BES}(u_k, u_j) satisfies

$$\sigma_{BES}(u_k, u_j) \simeq \exp\left[i\frac{(u_j - u_k)}{(x_k^+ - x_k^-)(x_j^+ - x_j^-)}\log\sigma_{BES}(u,v)\right], \tag{2.7}$$

when λ 1, we find

$$\sigma_{BES}(u,v) \simeq \exp\left[i\frac{(u_j - u_k)}{\alpha(x_k) + \alpha(x_j)}\right], \tag{2.8}$$

where α(x) = λ/8. It is convenient to introduce the resolvents:

$$G_a(x) = \sum_{j=1}^{K_a} \frac{\alpha(x_{a,j})}{x - x_{a,j}}, \quad H_a(x) = \sum_{j=1}^{K_a} \frac{\alpha(x_{a,j})}{x_{a,j}^2 - 1}\frac{1}{x - x_{a,j}}, \tag{2.9}$$

and

$$\bar{G}_a(x) = G_a(1/x), \quad \bar{H}_a(x) = H_a(1/x). \tag{2.10}$$

We can recover the conserved charges Q_n from these resolvents:

$$G_4(x) + \bar{G}_4(x) = \sum_{n=0}^{\infty} \frac{Q_{n+1}}{x^n}. \tag{2.11}$$

3 The classical spectral curve of γ-deformed AdS_4/CFT_3

In this section, we try to obtain the classical spectral curve from the generating functional [19], [20]. In the context of γ-deformation, the twisted generating functional is needed. The twisted generating functional for γ-deformed AdS_4/CFT_3 has been constructed in [15] and reads (we have used a different gauge from there)

$$\Lambda(x) = \tau_0^{1/2}\frac{B^{(-)} - Q_1^+}{B^{(+)} - Q_1^-}\frac{D_3}{D_4}\frac{R^{(-)} + Q_3^-}{R^{(+)} + Q_3^+}\frac{D_1}{D_2}, \tag{3.1}$$

where τ_1 = e^{-πi((δ_3-δ_1)L + (δ_1-δ_3)K_1 + (δ_1+2δ_2+δ_3)(K_4-K_4^-))}, τ_2 = 1.

In the scaling limit, we have the following expansions

$$
\begin{aligned}
B^{(-)} - Q_1^{+} &\simeq \exp\left[\frac{i}{2}(H_1 + \bar{H}_4 + \bar{H}_{\bar{4}})\right], \\
B^{(+)} - Q_1^{-} &\simeq \exp\left[\frac{i}{2}(\bar{H}_1 + H_2 + \bar{H}_2)\right], \\
R^{(-)} + Q_3^{-} &\simeq \exp\left[\frac{i}{2}(\bar{H}_2 + H_3 + \bar{H}_3)\right], \\
R^{(+)} + Q_3^{+} &\simeq \exp\left[\frac{i}{2}(H_1 + \bar{H}_3 + H_4 + \bar{H}_{\bar{4}})\right].
\end{aligned}
\tag{3.2}
$$

At strong coupling, the generating functional becomes

$$
\Lambda(x) \simeq \exp\left[i\frac{Lx}{2g} + i\sum_{a=1}^{3}(\bar{H}_a - H_a)\right] D, \tag{3.3}
$$

where D = (1 - λ_1d)(1 - λ_2d)(1 - λ_3d)(1 - λ_4d) and λ_a = e^{-iq_a}, with q_a being the so-called quasi-momenta. We have refined the formal expansion parameter in the following way:

$$
d = \exp\left[i\left(\frac{Lx}{2g} + \sum_{a=1}^{3}(\bar{H}_a - H_a)\right)\right] D. \tag{3.4}
$$

The AdS$_4$ × CP^3 classical string algebraic curve is a ten-sheet Riemann surface parameterized by e^{iq_a(x)}, e^{-iq_a(x)}. After some computations, we find

$$
\begin{aligned}
q_1(x) &= \frac{Lx}{2g} + \bar{H}_3 + \bar{H}_4 + \bar{H}_{\bar{4}} + \phi_3, \\
q_2(x) &= \frac{Lx}{2g} + H_1 + \bar{H}_2 + \phi_2, \\
q_3(x) &= \frac{Lx}{2g} + \bar{H}_2 + \bar{H}_1 + \phi_1, \\
q_4(x) &= \frac{Lx}{2g} + H_4 - \bar{H}_{\bar{4}} + \phi_4, \\
q_5(x) &= \frac{Lx}{2g} + H_{\bar{4}} + \bar{H}_4 - \bar{H}_3 + \phi_5,
\end{aligned}
\tag{3.5}
$$

where ϕ_3 = i log τ_1 + 2π(AK)*0, ϕ_4 = i log τ_1, $\phi_1 = \phi_2$ = 0, and q*{11-i} = q_i for i = 1, 2, ..., 5. As discussed in [15], we must also introduce the twist phases

$$
\tau_0 = e^{\pi i(-2\delta_2 L - (\delta_1+\delta_3)K_1 - (\delta_1+2\delta_2+\delta_3)(K_3-K_4-K_4))}, \quad \bar{\tau}_0 = e^{\pi i(2\delta_2 L + (\delta_1+\delta_3)K_1 + (\delta_1+2\delta_2+\delta_3)(K_3-K_4-K_4))}, \tag{3.6}
$$

such that $\phi_5 = \mathrm{i}\log\tau_0$. The relations between the conserved angular momentum and the excitation numbers read

$$
\begin{aligned}
J_{\phi_1} &= \frac{1}{2}(K_4 + K_{\bar{4}}) - p_1 + q + p_2,\\
J_{\phi_2} &= \frac{1}{2}(K_4 + K_{\bar{4}}) = K_4 - p_2,
\end{aligned}
\tag{3.7}
$$

where we denote the SU(4) Dynkin labels as $[\mathrm{p}_1, \mathrm{q}, \mathrm{p}_2]$. See Appendix B for more information. Using the above relations, we can write the twists AK appearing in the ABA equations as

$$
\begin{aligned}
(AK)_1 &= \frac{\gamma_1 - \gamma_2}{2} J_1 - \frac{\gamma_3}{2} J_3 + \frac{\gamma_3}{2}(J_2 - J_1),\\
(AK)_2 &= \frac{\gamma_1 + \gamma_2}{2}(J_1 + J_2) - \frac{\gamma_3}{2}(J_3 - J_1),\\
(AK)_3 &= \frac{\gamma_1 + \gamma_2}{2}(J_1 + J_2) + \frac{\gamma_3}{2}(2J_2 + J_3),\\
(AK)_4 &= \frac{\gamma_1 - \gamma_2}{2}(J_1 - J_2) + \frac{\gamma_2}{2} J_3,\\
(AK)_{\bar{4}} &= \frac{\gamma_1 - \gamma_2}{2}(J_1 - J_2) + \frac{\gamma_2}{2}(2J_1 - J_3).
\end{aligned}
\tag{3.8}
$$

With the help of the formulas given in Appendix A, it is very easy to show that the twisted ABA equations (2.2) are equivalent to the following equations in the scaling limit:

$$
\begin{aligned}
q_3(x + i0) - q_4(x - i0) &= Q_1 + x(H_1 + \bar{H}_2 - \bar{H}_3),\\
q_2(x + i0) - q_4(x - i0) &= Q_1 + x(\bar{H}_1 + H_2 - H_3),\\
q_1(x + i0) - q_4(x - i0) &= \bar{H}_2 + \bar{H}_4 + \bar{H}_{\bar{4}} + \phi_3 - \bar{H}_3 + \phi_2 - \bar{H}_1 + 2\pi n_1,\\
q_4(x + i0) - q_5(x - i0) &= \frac{Lx}{2g} + H_2 + H_4 + H_{\bar{4}} - \bar{H}_3 + \phi_1 - H_3 + 2\pi n_2,\\
q_4(x + i0) + q_5(x - i0) &= \frac{Lx}{2g} + \bar{H}_4 + \bar{H}_{\bar{4}} + \phi_4 - \bar{H}_{\bar{4}} + \bar{H}_4 + \phi_4 + \phi_5 = 2\pi n_3,
\end{aligned}
\tag{3.9}
$$

where x C_1, C_2, C_3, C_4 respectively.

4 Spectral curve for giant magnons in CP^3

In this section and the following, we will work with the β-deformation of $\mathrm{AdS}_4/\mathrm{CFT}_3$, where we take $\gamma_1 = \gamma_2 = 0$, γ_3 β, and we label the Dynkin nodes as (r, u, v) instead of (3, 4, $\bar{4}$). We use the ansatz

$$
\begin{aligned}
&q_1(x) = G_u(0) + \bar{G}_u(x) + \phi_1 + \Delta\phi_1, \\
&q_2(x) = G_v(0) + \bar{G}_v(x) + \phi_2 + \Delta\phi_2, \\
&q_3(x) = G_r(x) + G_r(0) + \bar{G}_r(x) + \phi_3 + \Delta\phi_3, \\
&q_4(x) = G_v(x) + G_r(x) + \bar{G}_v(x) + \phi_4 + \Delta\phi_4, \\
&q_5(x) = G_u(x) + G_u(0) + \bar{G}_u(x) + G_v(x) + G_v(0) + \bar{G}_r(x) + \phi_5 + \Delta\phi_5,
\end{aligned}
\tag{4.1}
$$

where the twists ϕ_i and $\Delta\phi$_i added here are to be determined. Let us make some comments on these two kinds of twists. The giant magnons are open string solutions with endpoints located at different places. If we want to treat them as closed strings, we have to consider Z_M orbifold of deformed ABJM theory. The $\Delta\phi$_i's incorporate this effect, while the existence of ϕ_i is due to the fact that we are considering the β-deformed theory.

The Dynkin labels of SU(4) are related to the excitation numbers by

$$
\begin{aligned}
p_1 + q + p_2 &= K_u + K_v - 2K_r, \\
p_2 - p_1 &= 2K_u + K_r, \\
q &= 2K_v + K_r.
\end{aligned}
\tag{4.2}
$$

While in this sector,

$$
[p_1, q, p_2] = [K_4 + K_{\bar{4}} - K_3, L - 2K_4 - 2K_{\bar{4}}, K_3]. \tag{4.3}
$$

In the last section, we calculated the twists ϕ_1, …, ϕ_5. With the help of eq. (3.9), we can write them in terms of the angular momentum. For the case at hand, we have

$$
\phi_1 = \phi_2 = 0, \quad \phi_3 = \pi\beta(J_1 - J_2), \quad \phi_4 = \pi\beta(J_1 + J_2), \quad \phi_5 = 0. \tag{4.4}
$$

4.1 Fix the twists from orbifolding

In this section we will mainly discuss the RP^3 giant magnon, which is the dyonic generalization of the RP^2 giant magnon. We closely follow the treatment of [21] (early work on treating giant magnons as closed strings on orbifolds includes [22]-[24]). Let us temporarily put aside the deformation. As mentioned above, giant magnons are open string solutions with non-periodic boundary conditions.

For the RP^3 magnon, we have

$$\begin{aligned}
Y^1(\sigma = 2\pi) &= Y^1(\sigma = 0)\exp(ip/2),\\
Y^2(\sigma = 2\pi) &= Y^2(\sigma = 0),\\
Y^3(\sigma = 2\pi) &= Y^3(\sigma = 0),\\
Y^4(\sigma = 2\pi) &= Y^4(\sigma = 0)\exp(-ip/2).
\end{aligned} \tag{4.5}$$

Formally identifying this open string as a closed string leads us to consider Z_M orbifolding of ABJM theory. This theory appears when we consider the low-energy effective theory of N M2-branes placed at a $C^4/(Z\_M \times Z\_k)$ orbifold singularity [25]-[28] with Z_M acting on C^4 as

$$(Y^1, Y^2, Y^3, Y^4) \to (e^{2\pi im/M}Y^1, Y^2, Y^3, e^{-2\pi im/M}Y^4). \tag{4.6}$$

Notice that this Z_M is inside the SU(4) R-symmetry group of ABJM theory. It is easy to see that we should identify p/2 with 2πm/M. Then we have to set the charges of Y^I's as

$$s_I = (m, 0, 0, -m). \tag{4.7}$$

From the analysis in [29], we have

$$s_I = (t_2, t_1 - t_2, t_1 + t_3, -t_1 - t_2 - t_3), \tag{4.8}$$

where t_I's are parameters of the orbifold. Comparing these two results, we find $t_1 = t_2 = t_3 = m$. Then the charges appearing in the su(2)-grading orbifolding ABA equations are

$$q^+ = (m, 0, t_1, 2t_1 - m, m, m). \tag{4.9}$$

The phases from the orbifold are

$$\exp(2\pi iq^+/M) = (e^{ip}, e^{ip/2}, 1, e^{-ip/2}, e^{ip/2}, e^{ip/2}). \tag{4.10}$$

Notice that in the ABA equations, the phase due to deformation $\exp(2\pi i(AK)\_j)$ should be multiplied by the phase due to new orbifolding $\exp(2\pi i\ q\hat{}+\_j/M)$, so the change of ϕ's due to this orbifolding should satisfy

$$\begin{aligned}
\Delta\phi_2 - \Delta\phi_3 - \Delta\phi_1 &= 0,\\
\Delta\phi_2 - \Delta\phi_4 &= p/2,\\
\Delta\phi_1 - \Delta\phi_5 &= -p/2,\\
\Delta\phi_4 - \Delta\phi_4 + \Delta\phi_5 &= p.
\end{aligned} \tag{4.11}$$

This leads to

$$\Delta\phi_1 = \Delta\phi_2 = \Delta\phi_5 = 0, \quad \Delta\phi_3 = \Delta\phi_4 = p/2, \tag{4.12}$$

exactly the same as the results given in [30] (see also [31]).

Thus the asymptotic behavior of the quasi-momenta when x $\to \infty$ is

$$\begin{aligned} q_3(x) &\simeq \frac{Lx}{2g} + \pi\beta(J_1 - J_2) + \frac{\Delta + S}{2g} + \mathcal{O}(1/x), \\ q_4(x) &\simeq \frac{Lx}{2g} + \pi\beta(J_1 + J_2) + \frac{\Delta - S}{2g} + \mathcal{O}(1/x). \end{aligned} \tag{4.13}$$

Comparing our ansatz eq. (4.1) with the asymptotic behaviors eq. (4.13), we conclude $\alpha = \Delta/2$g and S = 0.

The total momentum condition from the Bethe ansatz now reads

$$2\pi n = \tilde{p} + 2\pi(AK)_0 + p, \tag{4.14}$$

where n , p is the momentum of the giant magnon, and $\tilde{p}$ is defined as

$$\tilde{p} = \frac{1}{i}\sum_{j=1}^{K_4} \log\frac{x^+_{4,j}}{x^-_{4,j}} + \frac{1}{i}\sum_{j=1}^{K_{\bar{4}}} \log\frac{x^+_{\bar{4},j}}{x^-_{\bar{4},j}}. \tag{4.15}$$

For the β-deformed case, we have

$$(AK)_0 = 2\pi\beta J_1. \tag{4.16}$$

And the above result is consistent with the results from inversion symmetry:

$$2\pi n = q_3(1/x) + q_4(x) = \tilde{p} + \phi_3 + \Delta\phi_3 + \phi_4 + \Delta\phi_4 = \tilde{p} + 2\pi\beta J_1 - p. \tag{4.17}$$

For our convenience, we can set n = 0. In fact, any even n gives the correct result. This requirement comes from the ambiguity in the definition of $\tilde{p}$. Let us make this point more clear: $\tilde{p}$ is defined up to some integer multiple of 2π. For 'small giant magnons', any n gives the same result, but for RP^3 magnons, this integer must be an even number, because we have two copies of 'small giant magnons'.

Infinite size dispersion relation

Studying giant magnons using the classical spectral curve method first appears in [32], where the authors show that giant magnons correspond to logarithmic cuts in the algebraic curve language. The giant magnons in CP^3 have various forms, but essentially they belong to two different classes: the 'small' and 'big' giant magnons and their dyonic generalizations. They can all be constructed by setting some of the resolvents in ansatz eq. (4.1) to be

$$G_{mag}(x) = i \log \frac{x - X^+}{x - X^-}, \tag{4.18}$$

where (X$^{+)}$* = X^-.

As a warm-up exercise, let us first consider the 'small giant magnon' with

$$G_u(x) = G_{mag}(x), \quad G_v(x) = G_r(x) = 0. \tag{4.19}$$

The charges can be read from the asymptotic behavior of this curve:

$$\begin{aligned} J_1 &= ig(X^+ + X^-), \\ J_2 &= \Delta + ig(X^+ - X^-), \\ J_3 &= 2J_1. \end{aligned} \tag{4.20}$$

Then the dispersion relation can be found as

$$\Delta = \sqrt{1 + 16g^2 \sin^2 \frac{\tilde{p}}{2}}, \tag{4.21}$$

where $\tilde{p}$ = p - $2\pi\beta$ J__1.

Now we consider another kind of dyonic giant magnons in CP^3, which are also called a pair of small giant magnons or RP^3 magnons. They can be constructed by putting a 'small giant magnon' in each sector:

$$G_u(x) = G_v(x) = G_{mag}(x), \quad G_r(x) = 0. \tag{4.22}$$

From this setting, we obtain the charges as

$$\begin{aligned} J_1 &= 2ig(X^+ + X^-), \\ J_2 &= \Delta + 2ig(X^+ - X^-), \\ J_3 &= 0. \end{aligned} \tag{4.23}$$

Thus we find the dispersion relation

$$\Delta = \sqrt{1 + 64g^2 \sin^2 \frac{\tilde{p}}{4}}, \tag{4.24}$$

where $\tilde{p}$ = p - $2\pi\beta$ J_1 as before.

5 Finite-size corrections

The finite-size effects for giant magnons are usually computed through the Lüscher formula, which basically contains two different terms coming from two types of spacetime interpretation: the -term and the F-term, which correspond to the leading classical corrections and the first quantum corrections respectively. The F-term is easily computed from the classical spectral curve. In [1], the authors proposed a method to compute the -term from the algebraic curve (see also [33]). As we do not intend to construct the Drinfeld-Reshetikhin twist [34] of the AdS_4/CFT_3 S-matrix in this short note, we will try to compute the finite-size corrections using the twisted classical spectral curve proposed above.

The basic idea is that at finite size we can think of the giant magnon solutions as obtaining a small square-root cut tail at each end of the logarithmic cut. Considering this, we use the finite-size resolvent [1]

$$G_{finite}(x) = 2i \log \frac{\sqrt{x - X^+} + \sqrt{x - Y^+}}{\sqrt{x - X^-} + \sqrt{x - Y^-}}, \tag{5.1}$$

where Y^± are points shifted by some small amount away from X^±:

$$Y^{\pm} = X^{\pm}(1 \mp i\delta e^{\pm i\phi}). \tag{5.2}$$

As a simple check, when taking $\delta = 0$, this finite-size resolvent goes back to the infinite-size resolvent (4.18).

Now we will study the finite-size RP^3 dyonic giant magnons using the method mentioned above. For this purpose, we set

$$G_u(x) = G_v(x) = G_{finite}(x), \quad G_r(x) = 0. \tag{5.3}$$

We write

$$X^{\pm} = re^{\pm i\tilde{p}_0/4}. \tag{5.4}$$

The momentum of the giant magnon is

$$p = \tilde{p} + 2\pi\beta J_1, \tag{5.5}$$

where

$$\tilde{p} = 4i \log \frac{\sqrt{X^+} + \sqrt{Y^+}}{\sqrt{X^-} + \sqrt{Y^-}}. \tag{5.6}$$

The expansion of $\tilde{p}$ in small δ is

$$\tilde{p} = \tilde{p}_0 + 2\cos(\phi)\delta + \sin(2\phi)\delta^2 + \mathcal{O}(\delta^3). \tag{5.7}$$

The non-trivial large-x asymptotics of this curve are

$$\begin{aligned} q_3(x) &\simeq \pi\beta(J_1 - J_2) + \frac{\Delta + 2ig(X^+ - X^-)}{2g} + \mathcal{O}(1/x), \\ q_4(x) &\simeq \pi\beta(J_1 + J_2) + \frac{\Delta - 2ig(X^+ - X^-)}{2g} + \mathcal{O}(1/x), \end{aligned} \tag{5.8}$$

from which we solve

$$\begin{aligned} J_1 &= ig\left(\sqrt{\frac{X^-}{Y^-}} - \sqrt{\frac{X^+}{Y^+}} + \sqrt{\frac{Y^-}{X^-}} - \sqrt{\frac{Y^+}{X^+}}\right), \\ J_2 &= \Delta + ig\left(\sqrt{\frac{X^-}{Y^-}} - \sqrt{\frac{X^+}{Y^+}} - \sqrt{\frac{Y^-}{X^-}} + \sqrt{\frac{Y^+}{X^+}}\right). \end{aligned} \tag{5.9}$$

Then we also expand these two charges in δ up to O(δ^3) as

$$\begin{aligned} J_1 &= J_1^{(0)} + J_1^{(1)}\delta + J_1^{(2)}\delta^2 + \mathcal{O}(\delta^3), \\ J_2 &= J_2^{(0)} + J_2^{(1)}\delta + J_2^{(2)}\delta^2 + \mathcal{O}(\delta^3). \end{aligned} \tag{5.10}$$

The coefficients have already been computed in [35], and we report the results here. The result for J_1 begins at order δ^0:

$$\begin{aligned} J_1^{(0)} &= 4g\frac{r^2+1}{r}\sin\frac{\tilde{p}_0}{4}, \\ J_1^{(1)} &= 4g\cos\phi\left(\frac{r^2-1}{r}\sin\frac{\tilde{p}_0}{4} + \frac{r^2+1}{r}\cos\frac{\tilde{p}_0}{4}\right), \\ J_1^{(2)} &= 2g\cos(2\phi)\left(\frac{r^2+1}{r}\sin\frac{\tilde{p}_0}{4} - \frac{r^2-1}{r}\cos\frac{\tilde{p}_0}{4}\right) \\ &\quad + g\left(\frac{r^4+6r^2+1}{r^3}\sin\frac{\tilde{p}_0}{4} + \frac{r^4-1}{r^3}\cos\frac{\tilde{p}_0}{4}\right). \end{aligned} \tag{5.11}$$

A similar result for J_2 reads

$$J_2^{(0)} = \Delta = \sqrt{1 + 64g^2 \sin^2 \frac{\tilde{p}_0}{4}},$$
$$J_2^{(1)} = 0, \tag{5.12}$$
$$J_2^{(2)} = -\frac{64g^2 \sin^2(\tilde{p}_0/4)}{\sqrt{1 + 64g^2 \sin^2(\tilde{p}_0/4)}} \cos(2\phi).$$

Then we find the corrections to J_2:

$$J_2 - J_2^{(0)} = -\frac{64g^2 \sin^2(\tilde{p}_0/4)}{\sqrt{1 + 64g^2 \sin^2(\tilde{p}_0/4)}} \cos(2\phi)\delta^2 + \mathcal{O}(\delta^3). \tag{5.13}$$

We now need to find δ by solving the constraint

$$2\pi n_1 = q_4(x + i0) - q_7(x - i0) \in C_{47}, \tag{5.14}$$

where C_{47} is the square-root branch cut connecting the 4th and 7th sheets as we are considering the RP^3 magnons, for which [K_r, K_u, K_v] = [0, J_1, J_1]. We are interested in the leading finite-size corrections, therefore we can evaluate at x = X^+:

$$2\pi n = 2\alpha X^+ + 2G_{finite}(X^+ + i0) + 2G_{finite}(X^+ - i0) + \pi\beta(J_1 + J_2). \tag{5.15}$$

Solving this equation, we can fix δ as

$$\delta = \frac{64 \sin^2(\tilde{p}/4) e^{i\pi(n_1 - \beta J_2) - 2i\phi}}{i\Delta r/2g} \exp\left(\frac{ei\tilde{p}/4r^2 - e^{-i\tilde{p}/4}}{r^2 + 1}\right). \tag{5.16}$$

To ensure the energy corrections are real numbers, we must impose δ to be real (for generic cases the contributions at O(δ) order may be non-vanishing). Then the real part of δ^2 is

$$(\delta^2) = \frac{64 \sin^2(\tilde{p}/4)}{\sqrt{1 + 64g^2 \sin^2(\tilde{p}/4)}} \exp\left[-\frac{\Delta r(r^2 + 1)}{2g((r^2 - 1)^2 + 4r^2 \sin^2(\tilde{p}/4))}\right]. \tag{5.17}$$

Using the relation r = √(1 + 64g^2 $\sin^2(\tilde{p}/4)$), we obtain

$$(\delta^2) = \frac{64\sin^2(\tilde{p}/4)}{\sqrt{1+64g^2\sin^2(\tilde{p}/4)}}\exp\left[-\frac{J_1\sqrt{1+64g^2\sin^4(\tilde{p}/4)}}{g\sqrt{1+64g^2\sin^2(\tilde{p}/4)}\sin(\tilde{p}/2)}\right]. \quad (5.18)$$

Inserting this expression into eq. (5.13), we find

$$J_2 - J_2^{(0)} = -\frac{256g^2\sin^4(\tilde{p}/4)}{1+64g^2\sin^2(\tilde{p}/4)}\cos(2\phi)\exp\left[-\frac{J_1\sqrt{1+64g^2\sin^4(\tilde{p}/4)}}{g\sqrt{1+64g^2\sin^2(\tilde{p}/4)}\sin(\tilde{p}/2)}\right]. \quad (5.19)$$

The condition that the O(δ) term vanishes gives the relation between the phase ϕ and the conserved charges:

$$2\phi = \pi n' - \frac{\pi\beta J_1\sin(\tilde{p}/2)}{\sqrt{1+64g^2\sin^4(\tilde{p}/4)}}, \quad (5.20)$$

where n' = n_1 for the plus sign in (5.16) while n' = n_1 - 1 for the minus sign in (5.16). Here we still have $\tilde{p}$ = p - $2\pi\beta J_1$ as before.

6 Conclusion

In this note, we have constructed the classical spectral curve of γ-deformed AdS_4/CFT_3, which is consistent with the twisted ABA equations at strong coupling. The γ-deformed algebraic curve is not very different from the original one as we only add some appropriate phases which can be easily obtained from the twisted generating functional and some other natural requirements. To check our proposal, we compute the finite-size corrections of the dyonic giant magnons in $AdS_4 \times CP^3$ using our twisted algebraic curve. Our result is the same as the one in [2]. We hope this can be further verified by constructing the twisted S-matrix and appropriate twisted boundary as in [34] for AdS_5/CFT_4, and using the generalized Lüscher formula [36] to compute the corrections to the dispersion relation.

Recently, the quantum spectral curve for γ-deformed AdS_5/CFT_4 has been constructed in [37]; it would also be very interesting to find it for AdS_4/CFT_3 with γ-deformation.

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A Useful formula in scaling limit

We give some useful formulas for quantities expanded in the scaling limit:

$$
\begin{aligned}
B^{(+)} + B^{(-)} &\simeq \exp\left[i \sum_{j=1}^{K_4} \frac{x_{4,j}^2}{(x - x_{4,j})(x^2-1)} \right] \simeq \exp\left[i(H_a(x) + \bar{H}_a(x)) \right], \\
B^{(+)} - B^{(-)} &\simeq \exp\left[i \sum_{j=1}^{K_4} \frac{x_{4,j}}{x^2-1} \right] \simeq \exp\left[i(\bar{G}_4(x) + \bar{G}_{\bar{4}}(x)) \right], \\
R^{(+)} + R^{(-)} &\simeq \exp\left[i \sum_{j=1}^{K_4} \frac{x_{4,j}}{x - x_{4,j}} \right] \simeq \exp\left[i(G_4(x) + G_{\bar{4}}(x)) \right], \\
R^{(+)} - R^{(-)} &\simeq \exp\left[i \sum_{j=1}^{K_4} \frac{x_{4,j}^2}{x(x - x_{4,j})} \right] \simeq \exp\left[i(H_4(x) + H_{\bar{4}}(x)) \right].
\end{aligned}
\tag{A.1}
$$

B Global charges

The Dynkin labels of a state can be read off from the Bethe equations as follows: we expand the right-hand side of the Bethe equations for each flavor of root at infinity while keeping other roots finite. In this way, one can obtain the Dynkin label r_j via comparing this expansion with *k (1 + ir_j/h(λ)x*{j,k}) (1 + 1/x_{j,k}^2). For su(2) grading discussed in the main text and the SU(4) Dynkin label [p_1, q, p_2], we have

$$
\begin{aligned}
q &= r_1 + r_2 + r_3 = K_4 + K_{\bar{4}} - K_3, \\
p_1 &= r_4 = L - 2K_4 - K_3, \\
p_2 &= \bar{r}_4 = L - 2K_{\bar{4}} - K_3.
\end{aligned}
\tag{B.1}
$$

In terms of excitation numbers, this becomes

$$
\begin{aligned}
q &= \eta K_3 - \eta K_2 - \eta K_1 - \eta \delta_D, \\
p_1 &= 2\eta K_2 - (1+\eta)K_4 - (1-\eta)K_{\bar{4}} + \eta K_3 + \eta \delta_D, \\
p_2 &= 2\eta K_1 - (1+\eta)K_{\bar{4}} - (1-\eta)K_4 + \eta K_3 + \eta \delta_D,
\end{aligned}
\tag{B.2}
$$

where   is a parameter related to the grading.

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