Motion of photons in a background of gravitational wave (postprint)

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Abstract

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Full Text

Preamble

Photon Motion in a Gravitational Wave Background

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Abstract. The motion of photons in a Michelson interferometer is re-analyzed using both geometrical optics and wave optics. The classical photon paths in a gravitational wave background are derived from Fermat' s principle, which yields the same result as null geodesics in general relativity. The deformed Maxwell equations and wave equations for electric fields in a gravitational wave background are presented within the flat-space approximation. Both methods demonstrate that the response of an interferometer depends on the frequency of the gravitational wave, yet is almost independent of the frequency of mirror vibrations. This implies that vibrating mirrors cannot adequately mimic gravitational waves.

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1. Introduction

The transient gravitational wave events GW150914 and GW151226 were detected during the first observing run (O1, from September 12, 2015 to January 19, 2016) of Advanced LIGO [?, ?]. These events were interpreted as resulting from the coalescence of binary black hole systems: the first from black holes of masses $36M_\odot$ and $29M_\odot$ merging into a $62M_\odot$ black hole, and the second from black holes of approximately $14M_\odot$ and $8M_\odot$ forming a $\sim 21M_\odot$ black hole.

The gravitational wave signals are chirps with frequency ranges of 35–150 Hz and 35–700 Hz, respectively. These signals were extracted from the noise background using identical methods (see, e.g., refs. [?, ?]), and the interpretations have been widely accepted.

A LIGO detector is essentially a Michelson interferometer with its arms replaced by Fabry-Pérot cavities [?]. One purpose of these cavities is to increase the interaction time between photons and gravitational waves. On average, a photon completes approximately 140 round trips in a Fabry-Pérot cavity, remaining in the cavity for about $280L/c \approx 3.73$ ms, where $L \approx 4$ km is the arm length. During 3.73 ms, a gravitational wave with frequency near 268 Hz propagates through the detector by one wavelength, suggesting that a photon cannot "feel" the length variation of the arms [?, ?]. However, it is widely accepted that the response of a Michelson-Fabry-Pérot interferometer is approximately $2N$ times that of a Michelson interferometer of the same size, where N is the average number of round trips (see, e.g., [?, ?]). Previous studies on instrumentation and calibration [?, ?, ?, ?, ?, ?, ?] indicate that the detector response to a gravitational wave is frequency-independent for GW frequencies below 1 kHz. What is the origin of this apparent inconsistency?

In fact, nearly all previous studies of detector response to gravitational waves have employed mirror swings [?, ?, ?, ?, ?], seemingly supported by theoretical analyses of GW response [?]. These analyses utilize Laplace transformations, whose validity remains questionable[1]. In this paper, we re-analyze the detector

[1] To our knowledge, the first application of Laplace transformations in interferometer response studies appears in [?] and [?]. These references employ the equation $\mathcal{L}\left(\int_0^L h(t)dt\right) = \frac{1-e^{-pL}}{p}\mathcal{L}(h(t))$, where $\mathcal{L}$ denotes Laplace transformation and p is the Laplace domain variable. However, for $h(t) = \sin(\omega t)$, direct calculation shows $\int_0^L \sin(\omega t)dt = \frac{1}{\omega}(\cos(\omega(t-L)) - \cos(\omega t))$,

response to gravitational waves without using Laplace transformations, focusing specifically on the Michelson interferometer.

Photon motion in a detector is governed by general relativity, meaning photons should follow null geodesics in spacetime. Two alternative descriptions exist. The first is classical geometrical optics, where photon paths in three-dimensional space are determined by Fermat' s principle. In this approach, the metric is treated as a position-, direction-, and time-dependent refractive-index tensor for a special transparent medium in flat space. The second is wave optics, based on the electric field wave equation derived from Maxwell' s equations. The first aim of this paper is to study photon motion in a single arm using both methods. Although realistic detectors require two arms, we may consider just one since photon motion in each arm follows the same law but with opposite polarity due to the quadrupole nature of gravitational waves. We show that paths determined by Fermat' s principle coincide with those from null geodesics even in a GW background. We also demonstrate that a gravitational wave can act as a time-dependent medium in flat spacetime and present the deformed Maxwell equations and electric field wave equations in the flat-spacetime approximation. Using both methods, we obtain the same conclusion as from null geodesic analyses [**?**, **?**]: the detector response depends on the gravitational wave frequency.

The key objective of an interferometric gravitational wave detector is to measure variations in the phase difference between two light beams, which arise from GW-induced variations in the proper lengths of the two arms. Since GW signals are submerged in noise, methods must be developed to extract them. Additionally, detectors require calibration to determine their sensitivity. Because both gravitational waves and mirror swings cause variations in arm length, a naive assumption is that GW effects can be mimicked by mirror vibrations, and many detector investigations have proceeded accordingly [**?**, **?**, **?**, **?**, **?**]. The other purpose of this paper is to show that photon motion in a GW background differs fundamentally from that in a vibrating arm. The detector response depends on GW frequency, with zero response at certain specific frequencies. In contrast, the detector response to vibrations is nearly independent of frequency.

The remainder of this paper is organized as follows. Section 2 examines photon motion in both flat spacetime and a GW background within the framework of geometrical optics. Photon trajectories derived from Fermat' s principle are shown to coincide with null geodesics in general relativity. The round-trip time difference in a GW background is demonstrated to depend on GW frequency. Section 3 investigates electromagnetic wave propagation using wave optics. The deformed Maxwell equations and electric field wave equations are presented in the GW background within the flat-spacetime approximation, yielding identical time differences as in geometrical optics. Section 4 compares photon motion in

while $\frac{1-e^{-pL}}{p^2+\omega^2}(p\cos(\omega L)+\omega\sin(\omega L)-p) \neq \frac{1-e^{-pL}}{p^2+\omega^2}$. The equation should be revised as $\mathcal{L}\left(\int_0^L h(t)dt\right) = \frac{1-e^{-pL}}{p}\mathcal{L}(h(t)) - e^{-pL}\mathcal{L}\left(\int_0^\tau h(t)dt\right)$.

a GW background with that in a vibrating arm. Conclusions and remarks are presented in Section 5. Throughout the paper, we use units where $c = 1$, so $x^0 = t$.

2. Photon Motion in a Gravitational Wave Background

We consider Minkowski spacetime and spacetime with a plus-polarized, linear, normally incident plane gravitational wave. In a time-orthogonal coordinate system, the metrics for both cases can be written as

$$ds^2 = -dt^2 + g_{ij}dx^i dx^j, \quad i, j = 1, 2, 3.$$

The distance between two nearby points on a simultaneous hypersurface is measured by

$$dl^2 = g_{ij}dx^i dx^j.$$

Consider a photon traveling from point A to point B in space. There are infinitely many possible classical paths. The length of each path is given by

$$L = \int_A^B \sqrt{g_{ij}\frac{dx^i}{dt}\frac{dx^j}{dt}}\, dt,$$

where dx^i/dt is the photon' s coordinate velocity.

In geometrical optics, the photon path is determined by Fermat' s principle:

$$0 = \delta L = \delta \int_A^B \sqrt{g_{ij}\frac{dx^i}{dt}\frac{dx^j}{dt}}\, dt.$$

Only isochronal variation is considered, so $\delta t = 0$. The total-derivative term vanishes because the integration endpoints are fixed. From this expression, we obtain

$$g_{ij,k}\frac{dx^i}{dl}\frac{dx^j}{dl} - 2\frac{d}{dl}\left(g_{ik}\frac{dx^i}{dl}\right) = 0.$$

For convenience, we take l as the path parameter. Then the equation becomes

$$g_{ij,k}\frac{dx^i}{dl}\frac{dx^j}{dl} - 2\frac{d}{dl}\left(g_{ik}\frac{dx^i}{dl}\right) = 0,$$

which can be rewritten as

$$\frac{d^2x^i}{dl^2} + \Gamma^i_{\mu\nu}\frac{dx^\mu}{dl}\frac{dx^\nu}{dl} = 0, \quad \mu, \nu = 0, 1, 2, 3.$$

In terms of Christoffel symbols, this can be recast into the spatial part of the 4-dimensional geodesic equation. It should be noted that this equation differs, in general, from the geodesic equation in 3-dimensional space. The difference arises from the time dependence of g_{ij}, or equivalently, from the time-dependent "refractive index."

Since light travels along a null curve and the path length takes an extremal value, the travel time from A to B,

$$T = \int_A^B \sqrt{g_{ij}\frac{dx^i}{dt}\frac{dx^j}{dt}}^{\,-1} dt,$$

should also take an extremal value for a fixed path. That is,

$$0 = \delta T = -\int_A^B \left(g_{ij}\frac{dx^i}{dt}\frac{dx^j}{dt}\right)^{-3/2} g_{ij,0}\frac{dx^i}{dt}\frac{dx^j}{dt}\delta t\, dl.$$

Here, only time variation is considered ($\delta x^i = 0$). The total derivative also vanishes, yielding

$$g_{ij,0}\frac{dx^i}{dl}\frac{dx^j}{dl} = 0.$$

In terms of Christoffel symbols, this reduces to the temporal part of the four-dimensional geodesic equation:

$$\frac{d^2t}{dl^2} + \Gamma^0_{\mu\nu}\frac{dx^\mu}{dl}\frac{dx^\nu}{dl} = 0.$$

These results show that in the framework of geometrical optics, a photon travels along a null geodesic of 4-dimensional spacetime in both flat spacetime and a GW background, as expected in general relativity. The arc length l along a path serves as the affine parameter of the null geodesic. The photon trajectories are straight lines in 3-space, just as in the absence of a GW. However, the distance traveled, which is the arc length of the geodesic $L = \int dl$, and thus the travel time, are influenced by the time-dependent metric $g_{\mu\nu}$ in the GW background.

We now calculate the travel time for a photon moving along the x-axis from mirror A at position $x_1 = 0$ to mirror B at $x_2 = L$ (henceforth, L denotes the coordinate length of the arm). In the absence of GWs and matter, the metric is $g_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1)$. For a monochromatic GW with plus polarization incident along the z-axis, the metric becomes

$$g_{\mu\nu} = \mathrm{diag}(-1, 1 + h_{11}, 1 - h_{11}, 1), \quad h_{11} = \Re[e_{11}e^{-i(2\pi f t+\phi)}],$$

where e_{11} is the GW amplitude, $\Re$ denotes the real part, and ϕ is the initial GW phase when the photon leaves mirror A.

It is well known that without a GW, the distance between two mirrors in an interferometer arm is

$$L = \int_0^L dx,$$

and the photon travel time from one mirror to the other is $T = L$ (since we use units where $c = 1$).

In a GW background, the travel time from one mirror to the other becomes T':

$$T' = \int_0^L (g_{11})^{-1/2} dx = \int_0^L \frac{dx}{\sqrt{1 + h_{11}}}.$$

Using Taylor expansion and treating the GW as an extremely small perturbation of flat space, we obtain

$$T' \approx L - \frac{1}{2} \int_0^L h_{11}(t) dx = T + \Delta T'.$$

Thus, a GW background modifies the interferometer arm length. A photon "feels" a distance difference $\Delta L = \Delta T' = \frac{1}{2} \int_0^L h_{11}(t) dx$ due to the GW, where h_{11} is a function of time t.

Now consider a round trip from mirror A to mirror B and back. For the $A \to B$ leg, the travel distance is

$$T'_{A \to B} = L + \frac{1}{2} \int_0^L h_{11}(t) dx = T + \Delta T'_{A \to B},$$

and for the $B \to A$ return leg,

$$T'_{B \to A} = L + \frac{1}{2} \int_L^0 h_{11}(t) dx = L + \frac{1}{2} \int_{t_0}^{t_0 + 2T} h_{11}(t) dt = T + \Delta T'_{B \to A},$$

where t_0 is the photon' s departure time from A. In the first line, dx is always negative. Higher-order terms have been neglected. Note that in principle $\Delta T'_{A \to B} \neq \Delta T'_{B \to A}$.

For a complete round trip, the total time difference is

$$\Delta T' = \frac{1}{2} \int_{t_0}^{t_0 + 2T} h_{11}(t) dt = \frac{e_{11}}{4\pi f} \sin(2\pi f T) \Re \left[e^{-i(\phi + 2\pi f (t_0 + T))} \right].$$

This shows that for specific GW frequencies $f = n/2L$ (with $n = 1, 2, 3, ...$), $\Delta T' = 0$ regardless of the initial phase ϕ. Thus, for GWs with frequency $f = n/2L$, traveling photons exhibit no measurable difference between the presence and absence of a GW. This result has been obtained previously in the literature [**?**, **?**].

Defining the dimensionless GW frequency $a = 2Lf$ (> 0), the round-trip travel time difference becomes

$$\Delta T' = \frac{e_{11}}{4\pi f} \Re \left[-i e^{-i(\phi + a\pi t_0 / L)} \left(1 - e^{-2ia\pi} \right) \right].$$

In particular, when $t_0 = 0$,

$$\Delta T' = \frac{e_{11}}{4\pi f} [\sin(2a\pi + \phi) - \sin(\phi)].$$

3. Propagation of Electromagnetic Waves in a Gravitational Wave Background

In wave optics, light is treated as an electromagnetic wave satisfying Maxwell' s equations. In vacuum curved spacetime, these read

$$(\sqrt{-g}F^{\mu\nu})_{,\nu}=0,\quad F_{\mu\nu,\lambda}+F_{\lambda\mu,\nu}+F_{\nu\lambda,\mu}=0,$$

where $F_{\mu\nu}=-F_{\nu\mu}=\partial_\mu A_\nu-\partial_\nu A_\mu$ is the electromagnetic field tensor and A_μ is the electromagnetic potential. The electric and magnetic fields observed by static observers ($U^\mu=(1,0,0,0)$) are

$$E_i=-F_{i\nu}U^\nu=-F_{i0}=\partial_0 A_i-\partial_i A_0,$$

$$B_i=-\frac{1}{2}\epsilon_{i\mu\lambda\sigma}U^\mu F^{\lambda\sigma}=\frac{1}{2}\epsilon_{i\lambda\sigma}F^{\lambda\sigma}=\frac{1}{2}g^{\alpha\lambda}g^{\beta\sigma}\epsilon_{i\alpha\beta}(\partial_\lambda A_\sigma-\partial_\sigma A_\lambda).$$

In the GW background described by metric (15), the explicit components of the magnetic field are

$$B_1=(1+h_+)(\partial_2A_3-\partial_3A_2),\quad B_2=(1-h_+)(\partial_3A_1-\partial_1A_3),\quad B_3=\partial_1A_2-\partial_2A_1.$$

The non-zero components of $F_{\mu\nu}$ and $F^{\mu\nu}$ are

$$F_{0i}=E_i,\quad F_{12}=B_3,\quad F_{23}=(1-h_+)B_1,\quad F_{31}=(1+h_+)B_2,$$

$$F^{01}=-(1-h_+)E_1,\quad F^{02}=-(1+h_+)E_2,\quad F^{03}=-E_3,\quad F^{12}=B_3,\quad F^{23}=B_1,\quad F^{31}=B_2.$$

In this GW background, Maxwell' s equations become

$$F^{\mu0}_{,0}+F^{\mu i}_{,i}=0,$$

which yield

$$(1-h_+)E_{1,1}-(1+h_+)E_{2,2}-E_{3,3}=0,$$
$$((1-h_+)E_1)_{,0}+B_{3,2}-B_{2,3}=0,$$
$$((1+h_+)E_2)_{,0}-B_{3,1}+B_{1,3}=0,$$
$$E_{3,0}+B_{2,1}-B_{1,2}=0.$$

The remaining Maxwell equations $F_{0i,j}+F_{j0,i}+F_{ij,0}=0$ and $F_{ij,k}+F_{ki,j}+F_{jk,i}=0$ give

$$E_{1,2}-E_{2,1}+B_{3,0}=0,$$
$$E_{1,3}-E_{3,1}-((1+h_+)B_2)_{,0}=0,$$
$$E_{2,3}-E_{3,2}+((1-h_+)B_1)_{,0}=0,$$
$$B_{3,3}+(1+h_+)B_{2,2}+(1-h_+)B_{1,1}=0.$$

These equations can be written compactly as

$$\begin{cases} \nabla \cdot \mathbf{E} = h_+(E_{1,1} - E_{2,2}), \\ \partial_t \mathbf{E} + \nabla \times \mathbf{B} = 0, \\ -\nabla \times \mathbf{E} + \partial_t \mathbf{B} = h_+(\mathbf{B}_{1,1} - \mathbf{B}_{2,2}), \\ \nabla \cdot \mathbf{B} = 0, \end{cases}$$

where $\mathbf{E}$ and $\mathbf{B}$ are column vectors and h is a square matrix:

$$h = \begin{pmatrix} 0 & -h_+ & 0 \\ -h_+ & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

These are the deformed Maxwell equations in a GW background within the flat-spacetime approximation.

We derive electromagnetic wave equations from these deformed Maxwell equations. Taking the time derivative of the second equation and using the third, we obtain

$$\partial_t^2 \mathbf{E} - \nabla^2 \mathbf{E} = \partial_t^2 (h \cdot \mathbf{E}) - \nabla(\nabla \cdot \mathbf{E}) - \nabla \times \partial_t (h \cdot \mathbf{B}).$$

Since the ratio of GW frequency to photon frequency is very small, it can serve as a perturbation parameter, yielding

$$\partial_t^2 \mathbf{E} - \nabla^2 \mathbf{E} = h \cdot \partial_t^2 \mathbf{E} - h_+ \nabla(E_{1,1} - E_{2,2}) - \nabla \times (h \cdot (\nabla \times \mathbf{E})).$$

This can be rewritten as

$$(1 - h) \cdot \partial_t^2 \mathbf{E} - \nabla^2 \mathbf{E} = -h_+ \nabla(E_{1,1} - E_{2,2}) - \nabla \times (h \cdot (\nabla \times \mathbf{E})).$$

In components, the deformed wave equations are

$$\begin{cases} (1 - h_+)\partial_t^2 E_1 - \nabla^2 E_1 = -h_+(E_{1,11} - E_{2,21}) - h_+(E_{1,33} - E_{3,13}), \\ (1 + h_+)\partial_t^2 E_2 - \nabla^2 E_2 = -h_+(E_{1,12} - E_{2,22}) - h_+(E_{3,23} - E_{2,33}), \\ \partial_t^2 E_3 - \nabla^2 E_3 = h_+(E_{3,22} - E_{3,11}). \end{cases}$$

Similar equations for $\mathbf{B}$ can be obtained analogously.

For electromagnetic waves propagating along the x-axis, $E_1 = 0$. The first equation becomes $0 = 0$ at leading order. The remaining two equations are

$$(1 + h_+)\partial_t^2 E_2 - \nabla^2 E_2 = -h_+(E_{3,23} - E_{2,33}),$$

$$(1 + h_+)\partial_t^2 E_3 - \nabla^2 E_3 = h_+(E_{3,22} - E_{3,11}).$$

Since the right-hand side of the first equation reduces to $h_+ E_{2,22} + h_+(\nabla^2 E_2 - E_{2,11})$, we obtain at leading order

$$\partial_t^2 E_2 - \nabla^2 E_2 = h_+(E_{2,22} - E_{2,11}).$$

For propagation along the x-axis, $E_{2,22} = E_{2,33} = E_{3,22} = E_{3,33} = 0$, giving

$$\partial_t^2 E_2 - (1 - h_+)E_{2,11} = 0,$$

and similarly

$$\partial_t^2 E_3 - (1 - h_+)E_{3,11} = 0.$$

These are the deformed wave equations for the electric field, yielding the coordinate speed of electromagnetic waves in a GW background:

$$v_x = \sqrt{1 - h_+(t)} \approx 1 - \frac{1}{2}h_+(t),$$

and similarly

$$v_y = \sqrt{1 + h_+(t)} \approx 1 + \frac{1}{2}h_+(t).$$

Thus, the photon' s coordinate speed varies with the spacetime perturbation caused by the GW, a result obtained by other methods (e.g., [**?**]).

Let T' and T be the propagation times from A to B and back in the presence and absence of a GW, respectively. The time difference $\Delta T'$ due to the GW satisfies

$$\int_{t_0}^{t_0+2T+\Delta T'} v\,dt = 2L.$$

Setting $f = a/2L$ and $t_0 = 0$, this yields

$$\Delta T' = \frac{e_{11}}{4\pi f}\left[\sin\left(\frac{a\pi}{L}(\Delta T' + 2L)\right) - \sin\phi\right].$$

The leading-order terms reproduce Eq. (23) because $\Delta T' \ll L$. Clearly, when the dimensionless frequency $a = 1, 2, 3, ...$, we have $\Delta T' = 0$, meaning the detector has zero response.

4. Gravitational Wave vs. End Mirror Swing

The above discussion considers ideal cases. In practice, interferometers experience various vibrations. Distinguishing GWs from mirror swings is crucial for gravitational wave detection. We now demonstrate the essential difference between GW effects and mirror vibrations.

For simplicity, we fix mirror A and let mirror B undergo harmonic oscillation $\Re[Ae^{-i(2\pi ft+\phi)}]$, where f is the vibration frequency and ϕ is the initial phase when light leaves mirror A. Let τ be the one-way travel time with mirror B oscillating. Then

$$2\tau - 2L = \Re\left[2Ae^{-i(2\pi ft+\phi)}\right].$$

Denoting the travel time difference with and without mirror swing by ΔT, and considering a photon leaving mirror A at $t = 0$ with initial phase ϕ, the leading approximation gives

$$\Delta T = \Re\left[Ae^{-i(2\pi fL+\phi)}\right].$$

[Figure 1: see original paper] Time difference for a photon round trip from A to B and back to A as a function of gravitational wave frequency. Parameters: $t_0 = 0$, $c = 3 \times 10^8$ m/s, $e_{11} = 10^{-20}$, $L = 4000$ m. The time difference depends on the GW initial phase. Different colored curves represent different initial phases.

The mirror swing' s initial phase affects the travel time difference. Since a continuous laser is used, initial phases are uniformly distributed. Consequently, regardless of the mirror' s vibration frequency, some photons always "feel" the swing. In contrast, photons cannot "feel" GWs with frequency $f = n/2L$.

Figure 1 plots $\Delta T'$ due to a GW from Eq. (23) (with $e_{11} = 10^{-20}$ and $L = 4000$ m). Different colors denote different GW initial phases ϕ, with curves corresponding to $\phi = 0, \pi/4, \pi/2, 3\pi/4, \pi, 5\pi/4, 3\pi/2, 7\pi/4$. The time difference always vanishes when the GW frequency is $f = n/2L$, meaning photons take the same round-trip time in these specific GW backgrounds as in flat spacetime. The figure also shows that the GW initial phase affects the measurement readout. The envelope has the form $\operatorname{sinc} x = \sin x/x$.

[Figure 2: see original paper] Initial phase dependence of GW measurement. The GW frequency is $f = a/2L$ with $a = 0.4, 0.6, 0.8, 1.0, 1.2, 1.4, 1.6$.

Note that the curve for $f = 1/2L$ (i.e., $a = 1$) coincides exactly with the abscissa, meaning photons cannot "feel" GWs with $f = 1/2L$ regardless of initial phase.

For mirror swing, the travel time difference is

$$\Delta T = A\cos(2\pi f L + \phi).$$

[Figure 3: see original paper] Frequency dependence of travel time difference caused by mirror vibration. Parameters: $L = 4000$ m, $c = 3 \times 10^8$ m/s, $A = 4 \times 10^{-17}$ m. Different colors show specific initial phases $\phi = 0, \pi/4, \pi/2, 3\pi/4, \pi, 5\pi/4, 3\pi/2, 7\pi/4$.

The interferometer response is nearly independent of vibration frequency. The travel time difference also depends on initial phase, as shown in Figure 4.

[Figure 4: see original paper] Initial phase dependence of detector response to mirror swing. Parameters: $L = 4000$ m, $c = 3 \times 10^8$ m/s, $A = 4 \times 10^{-17}$ m. Different colors represent different vibration frequencies.

5. Conclusions and Remarks

This paper re-analyzes photon motion in a gravitational wave background using both geometrical and wave optics. In geometrical optics, photon paths in 3-dimensional space are obtained via Fermat' s principle, reproducing the geodesic equations and confirming that photon paths are null geodesics of general relativity. The proper distance between mirrors and photon travel times vary with the GW. In wave optics, we present deformed Maxwell equations and electric field wave equations in the flat-spacetime approximation. The wave equations show that the electromagnetic wave' s coordinate speed $v(t)$ between mirrors is modulated by the GW. Both methods yield identical results for the GW-induced time difference, Eq. (23).

Both GWs and mirror swings cause variations in the proper distance between mirrors, but with different characteristics. For GWs, the amplitude of $\Delta T'$ varies with GW frequency, as shown by the envelope in Figure 1 (a property reported in [**?**, **?**, **?**]). For mirror swings, however, the amplitude of ΔT is frequency-independent. This difference arises because a swinging mirror only affects the light' s reflection point, not its propagation within the arm, whereas a GW affects light propagation everywhere along the arm.

Therefore, swinging mirrors cannot adequately mimic gravitational waves, contrary to their use in previous instrumental and calibration studies [**?**, **?**, **?**, **?**, **?**, **?**, **?**].

Our discussion focuses on photon motion in a single arm, but all conclusions apply to the other arm as well. The only difference is that phase variations in the two arms have opposite polarity due to the quadrupole nature of GWs. The total effect is the sum of contributions from both arms. If time differences vanish in both arms for specific GW frequencies, the detector has no response to those GWs. For swinging mirrors, zero response occurs only when both arms vibrate synchronously.

Finally, we have discussed time differences in a Michelson interferometer. Detectors like LIGO and Virgo are Michelson-Fabry-Pérot interferometers. Analysis of Michelson-Fabry-Pérot interferometers will be presented elsewhere.

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Note: Figure translations are in progress. See original paper for figures.