The e ective neutrino mass of neutrinoless double-beta decays:how possible to fall into a well postprint

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Abstract

The neutrinoless double-beta (0 2) decay is currently the only feasible process in particle and nuclear physics to probe whether massive neutrinos are the Majorana fermions. If they are of the Majorana nature and have a normal mass ordering, the effective neutrino mass term m ee of a 0 2 decay may suffer significant cancellations among its three components and thus sink into a decline,resulting in a "well" in the three-dimensional graph of | m ee| against the smallest neutrino mass m1 and the relevant Majorana phase . We present a new and complete analytical understanding of the fine issues inside such a well, and identify a novel threshold of | m ee| in terms of the neutrino masses and flavor mixing angles: | m ee|∗ = m3 sin2 13 in connection with tan 12 = m1/m2 and = . This threshold point, which links the local minimum and maximum of | m ee|, can be used to signify observability or sensitivity of the future 0 2 -decay experiments. Given current neutrino oscillation data, the possibility of | m ee| < | m ee|∗ is found to be very small.

Full Text

Preamble

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Abstract

The neutrinoless double-beta (0 2) decay is currently the only feasible process in particle and nuclear physics to probe whether massive neutrinos are Majorana fermions. If they are of Majorana nature and have a normal mass ordering, the effective neutrino mass term m _ee of a 0 2 decay may suffer significant cancellations among its three components and thus sink into a decline, resulting in a "well" in the three-dimensional graph of | m _ee| against the smallest neutrino mass m and the relevant Majorana phase . We present a new and complete analytical understanding of the fine structure inside such a well, and identify a novel threshold of masses and flavor mixing angles: = . This threshold point, which links the local minimum and maximum of | m _ee|, can be used to signify the observability or sensitivity of future 0 2 -decay experiments. Given current neutrino oscillation data, the possibility of | m _ee|* = m sin² in connection with tan = √(m /m) and = is found to be very small.

1. Introduction

Since Ettore Majorana first formulated the theory of a fermionic particle that is its own antiparticle in 1937 [1], a huge amount of attention has been paid to Majorana fermions in particle and nuclear physics and to Majorana zero modes in solid-state physics [2]. In particular, after the experimental discoveries of solar, atmospheric, reactor, and accelerator neutrino oscillations [3], the question of whether massive neutrinos are Majorana fermions has become an especially burning issue among a number of fundamentally important questions in neutrino physics and cosmology. If this is the case, then the neutrinoless double-beta (0 2) decays of some even-even nuclei are expected to take place [4], namely: N(A, Z) → N(A, Z+2) + 2e , where the lepton number is violated by two units. Given the fact that neutrino masses are so small that all lepton-number-violating processes must be desperately suppressed, currently the unique and only feasible way to demonstrate the Majorana nature of massive neutrinos is to observe 0 2 decays. In this respect, a number of ambitious experiments are either underway or in preparation [5].

In the standard scheme of three neutrino flavors, the rate of a 0 2 decay is proportional to the squared modulus of the effective Majorana neutrino mass term [6]

$$\langle m\rangle_{ee} = m_1|U_{e1}|^2 e^{2i\rho} + m_2|U_{e2}|^2 + m_3|U_{e3}|^2 e^{2i\sigma},$$

where m_i denotes the i-th neutrino mass (for i = 1, 2, 3), U_{ei} is the corresponding element of the 3×3 neutrino mixing matrix U [8], and and stand for the Majorana phases. One often chooses |U_{e1}| = cos cos , |U_{e2}| = sin cos , and |U_{e3}| = sin to parametrize the mixing matrix. The three mixing angles , , and have been determined to a good degree of accuracy from current neutrino oscillation data, as have the values of Δm² and the modulus of Δm^2 [3]. However, the sign of Δm^2 and the two phase parameters in Eq. (1) remain unknown, nor does the absolute neutrino mass scale. That is why | m _ee| is usually plotted as a function of m in the normal mass ordering (NMO) case ($\Delta m^2 > 0$) or m in the inverted mass ordering (IMO) case ($\Delta m^2 < 0$) by allowing and to vary from 0 to 2 [9]. In such a so-called Vissani graph, a two-dimensional "well" can appear in the NMO situation due to significant cancellation among the three components of m _ee [10], a disappointing possibility which is definitely consistent with present experimental data. The bottom of the well signifies the case of | m _ee| $\to 0$, which is located at the center of the well.

Two immediate questions arise: (1) How possible is it for the three neutrinos to have a normal mass ordering? (2) How possible is it for the actual value of | m _ee| to fall into the well and become unobservable in any realistic 0 2 experiments? A combination of current atmospheric (Super-Kamiokande [11]) and accelerator-based (T2K [12] and NO A [13]) neutrino oscillation data preliminarily favors the NMO at the 2 level. If this turns out to be the case, an answer to the second question will be highly desirable because it can help interpret the discovery or null result of a 0 2 experiment in the standard three-flavor scheme, although some kind of hypothetical (ad hoc) new physics may also contribute to | m _ee|.

The present work aims to answer the second question by providing a new and complete analytical understanding of the fine structure of the three-dimensional well of | m _ee| against m and , as illustrated in Fig. 1 [Figure 1: see original paper], where the best-fit values $\Delta m^2 = 2.47\times10^{3}$ eV², $\Delta m^2 = 7.54\times10$ eV², $\sin^2 = 0.308$, and $\sin^2 = 0.0234$ [14] have been taken as typical inputs. We identify a novel threshold of masses and flavor mixing angles: tan = √(m /m) and = . This threshold point links the local minimum and maximum of | m _ee|, and it can be used to signify the observability or sensitivity of future 0 2 -decay experiments. Given current neutrino oscillation data, the possibility of | m _ee|* = m $\sin^2$ is found to be very small.

The phase convention taken here is highly advantageous when considering the interesting and experimentally-allowed neutrino mass limit m $\to 0$ (or m $\to$ 0), in which (or) automatically disappears [7].

Figure 1 shows a three-dimensional illustration of the upper (orange) and lower (blue) bounds of | m _ee| as functions of m and in the NMO case, where the best-fit values $\Delta m^2 = 7.54\times10$ eV², $\Delta m^2 = 2.47\times10^{3}$ eV², $\sin^2 = 0.308$, and $\sin^2 = 0.0234$ [14] have been used as typical inputs. Fig. 1 shows that the depth of the well of | m _ee| is mainly sensitive to a narrow parameter space of m and , while the other Majorana phase plays an important role in shaping the bottom of the well [15]. The latter point can be seen analytically as follows. Taking tan = -(m sin)/(m cos + m $\tan^2$) so as to maximize or minimize | m _ee| for given values of m and , and substituting this into the expression of | m _ee| in Eq. (1), one arrives at the following upper ("U") and lower ("L") bounds:

$$|\langle m \rangle_{ee}|_{U,L} = \left| m_{12} \cos^2 \theta_{13} \pm m_3 \sin^2 \theta_{13} \right|,$$

where the sign "+" (or "-") corresponds to "U" (or "L"), and

$$m_{12} \equiv \sqrt{m_1^2 \cos^4 \theta_{12} + m_1 m_2 \sin^2 2\theta_{12} \cos \rho + m_2^2 \sin^4 \theta_{12}}.$$

It is easy to understand this result intuitively: for any given values of m and , the maximum of | m _ee| arises when the sum of the first two components of m _ee has the same phase as the third one (i.e.,), and the minimum arises when the difference between these two phases is equal to . The bottom of the well shown in Fig. 1 corresponds to | m _ee|_L = 0, or equivalently

$$m_{12} = m_3 \tan^2 \theta_{13}.$$

Given the expressions m = √(m ² + Δm²) and m = √(m ² + Δm²) in the NMO case, Eq. (5) allows us to determine how the two free parameters m and are correlated with each other. Using the same best-fit inputs of Δm² , Δm² , sin² , and sin² as those used in plotting Fig. 1, we illustrate the numerical correlation between m and dictated by Eq. (5) in Fig. 2 [Figure 2: see original paper]—the red curve. Such a correlation curve roughly looks like an ellipse, but a careful analytical check shows that it does not really obey the standard equation of an ellipse. Fig. 2 tells us that touching the bottom of the well (i.e., | m _ee| → 0) is not a highly probable event at all, because it requires m and to lie in the narrow regions 2 meV m 7 meV and 0.86 / 1.14, respectively [16].

Another salient feature of the well is the "bullet"-like structure of | m _ee|_L as shown in Fig. 1, corresponding to the parameter space of m m tan² . In other words, the surface of this bullet is described by

$$|\langle m \rangle_{ee}|_L = m_3 \sin^2 \theta_{13} - m_{12} \cos^2 \theta_{13}.$$

The extremum of | m _ee|_L in this inner region of the well is supposed to be located at a point fixed by the following two conditions:

$$\frac{\partial |\langle m \rangle_{ee}|_L}{\partial \rho} = 0, \quad \frac{\partial |\langle m \rangle_{ee}|_L}{\partial m_1} = 0.$$

These lead to:

$$m_1 m_2 \sin^2 2\theta_{12} \cos^2 \theta_{13} \sin^2 \theta_{13} - \sin \rho = 0,$$

$$\frac{\Delta m_{21}^2 \sin^2\theta_{12}(m_2\sin^2\theta_{12} + m_1\cos^2\theta_{12}\cos\rho)}{m_1 m_2 m_{12}\cos^2\theta_{13}} = 0.$$

The first condition definitely leads to = 0 or . But Fig. 2 clearly shows that should only take a value around inside the well, and thus it is appropriate to take = instead of = 0. In this case m = |m cos^2 - m sin^2 | holds, and the second condition in Eq. (7) is simplified to

$$\frac{\Delta m_{21}^2 \sin^2\theta_{12}}{m_1 m_2 |m_1\cos^2\theta_{12} - m_2\sin^2\theta_{12}|}\cos^2\theta_{13} \pm \left(\cos^2\theta_{12} - \sin^2\theta_{12}\right)\sin^2\theta_{13} = 0,$$

where "±" correspond to the prerequisites m < m tan^2 and m > m tan^2 , respectively. However, Eq. (8) can never be fulfilled since its second term is much larger than its first term as a result of (a) 2.50×10^{1} sin^2 2.50×10^{1} and 1.85×10^{2} sin^2 2.50×10^{2} at the 3 level [14] and (b) m /m m /m in the NMO case. Nevertheless, Eq. (8) can at least allow us to draw a conclusion that is absolutely consistent with current experimental data: | m _ee|_L increases when m < m tan^2 holds, and it decreases when m > m tan^2 holds. Hence there must be a local maximum for | m _ee|_L, denoted as | m _ee|_L*, at the position fixed by = and

$$m_1 = m_2\tan^2\theta_{12} = \sqrt{m_1^2 + \Delta m_{21}^2}\tan^2\theta_{12} = \sqrt{\frac{\Delta m_{21}^2\sin^2\theta_{12}}{\cos 2\theta_{12}}}.$$

In Fig. 1 this point is exactly the tip of the bullet inside the well! In other words, the local maximum | m _ee|_L* arises from Eq. (6) at m = 0. Given the best-fit values of Δm^2 , Δm^2 , sin^2 , and sin^2 that have been used in plotting Fig. 1, the numerical location of the tip of the bullet turns out to be (m , , | m _ee|_L*) = (4 meV, 180°, 1 meV).

The above analysis explains why the bottom of the well does not converge to a single point and why it is not flat either. In a similar way, one can understand why there is a local minimum for | m _ee|_U as shown in Fig. 1. The extremum of | m _ee|_U is expected to be located at a position determined by

$$\frac{\partial|\langle m\rangle_{ee}|_U}{\partial\rho} = 0, \quad \frac{\partial|\langle m\rangle_{ee}|_U}{\partial m_1} = 0,$$

which lead to:

$$m_1 m_2\sin^2 2\theta_{12}\cos^2\theta_{13}\sin^2\theta_{13} + \sin\rho = 0,$$

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$$\frac{\Delta m^2_{21} \sin^2\theta_{12}(m_2 \sin^2\theta_{12} + m_1 \cos^2\theta_{12} \cos\rho)}{m_1 m_2 m_{12} \cos^2\theta_{13}} = 0.$$

Of course, only = is allowed with respect to the first condition in Eq. (12). The second condition in Eq. (12) can never be satisfied for the same realistic reasons given below Eq. (8). An analogous and straightforward analysis tells us that the local minimum of | m _ee|_U exactly coincides with the local maximum of | m _ee|_L, and thus both of them are described by Eqs. (10) and (11). This interesting result explains why the upper (in orange) and lower (in blue) bounds of | m _ee| connect with each other in Fig. 1 when m = √(1 + Δm²) tan² and = hold. Note that the overlap of the local maximum of | m _ee|_U can also be understood from Eq. (3) itself. At m = √(1 + Δm²) tan² and = , one simply has | m _ee|_U = m sin² as a consequence of m = 0. So in the NMO case, | m _ee|* = m sin² | m _ee|_L* = 1 meV stands for a threshold of | m _ee|.

To visualize the steepness of the slope of | m _ee|_L around the well in Fig. 1, let us project its contour onto the m - plane by taking | m _ee|_L = n| m _ee|_L* (for n = 0, 1, 2, ···) in Fig. 2. It is especially interesting to compare between the contours of | m _ee|_L = 0 (the red curve) and | m _ee|_L = | m _ee|_L* (the blue curve and the black point). They clearly show how the well becomes narrower when the value of | m _ee|_L goes down. The profile of | m _ee|_L* is taken into account. Now that | m _ee|_L > | m _ee|_L* always holds outside the blue curve in Fig. 2, we argue that the parameter space of | m _ee|_L | m _ee|_L* (i.e., 0.4 meV | m _ee|_L 1 meV, 2 meV m 7 meV, and 0.86 / 1.14) is a simple measure of the chance for | m _ee| to fall into the well and become completely unobservable. In other words, | m _ee|_L will be partially open and thus lose its "well" feature as | m _ee|_L | m _ee|_L*.

In general, | m _ee| depends on all three unknown parameters m , , and . To illustrate how probable or improbable it is for | m _ee| to have a value smaller than the threshold | m _ee|_L* in a more explicit way, we plot the three-dimensional parameter space of m , , and in Fig. 3 [Figure 3: see original paper], where the best-fit values of Δm² , Δm² , sin² , and sin² used in plotting Figs. 1 and 2 have been input. For clarity, the intersecting surfaces on the - plane corresponding to m = 1, 2, 4, and 6 meV are specified in the figure. One can see that this parameter space is very small compared with the whole cubic space (i.e., the whole regions of m , , and allowed by current experimental constraints). In comparison with m and , the phase is only weakly constrained in Fig. 3. When the first two components of m _ee in Eq. (1) essentially cancel each other out (i.e., 2 meV m 7 meV and 0.86 / 1.14), a large part of the range of is allowed (e.g., the black intersecting surface corresponding to m = 4 meV in Fig. 3). But when the value of m decreases, the value of should approach , such as the green intersecting surface corresponding to m = 1 meV in Fig. 3. In this case the second component of m _ee can be cancelled by the other two components to a maximal level. For a similar reason, the value of   should approach 0 or 2 when the value of m increases (e.g., the blue intersecting surface corresponding to m = 6 meV in Fig. 3). In any case, we conclude that the possibility of | m _ee|_L* involves significant cancellations among its three components and is really small.

From an experimental point of view, the threshold | m _ee|_L* should signify an ultimate limit of | m _ee| in the foreseeable future. At present, the most sensitive 0 2 -decay experiments can only set an upper limit of around 165 meV [17], which depends on some theoretical uncertainties in calculating the relevant nuclear matrix elements [18]. The most ambitious next-generation high-sensitivity 0 2 -decay experiments (e.g., nEXO [19]) are likely to probe | m _ee| at the level of a few tens of meV [5], a sensitivity still much larger than the threshold 1 meV. In this sense, there would be no hope to observe any 0 2 -decay signal if | m _ee| were unfortunately around or below the value of | m _ee|_L* in the standard three-flavor scheme.

Before ending our discussions about m _ee and its possible parameter space in the NMO case, let us briefly comment on the relationship tan   = √(m /m) from a model-building point of view. This condition, together with = , allows for | m _ee|_L* = m $\sin^2$ as a remarkable threshold. It is well known that the Cabibbo angle _C of quark flavor mixing can be related to the ratio of quark masses m_d and m_s in a class of models [22]: tan _C √(m_d/m_s), which is consistent with experimental data to a good degree of accuracy. In comparison, the possibility of tan   √(m /m) is also interesting, particularly when the NMO is true for the three mass eigenstates of _e, _ , and _ neutrinos. For example, we find that an effective Majorana neutrino mass matrix of the form M_ can essentially predict | m _ee|_L* = m $\sin^2$ and tan   = √(m /m) together with   = /4, = /2, = , and = 0 in the standard parametrization of U. Because M_ possesses the exact - reflection symmetry, which can easily be simplified to the - permutation symmetry in the   → 0 limit, one may take it as a starting point to build a phenomenological neutrino mass model in this connection [23].

2. Summary

In summary, we have achieved new and important insights into the effective neutrino mass m _ee of 0 2 decays in the NMO case–a case which seems to be more likely than the IMO case according to today' s preliminary experimental data. Because | m _ee| depends not only on the unknown neutrino mass m but also on the free Majorana phases and , a novel three-dimensional presentation of | m _ee| against m and reveals an intriguing "well" structure in the NMO case. The present work provides a new and complete analytical understanding of the fine structure inside such a well. We find a particularly interesting threshold of | m _ee| in terms of the neutrino masses and flavor mixing angles: | m _ee|* = m $\sin^2$ in connection with tan   = √(m /m) and = . We suggest that this threshold point, which links the local minimum and maximum of | m _ee|, can be used to signify the observability or sensitivity of future 0 2 -decay experiments. In view of current neutrino oscillation data, we conclude that the possibility of | m _ee|_L* must be very small. In other words, it should be very promising to detect a signal of 0 2 decays and verify the Majorana nature of massive neutrinos in the foreseeable future, even if they have a normal mass spectrum.

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Notes: 1. The phase convention taken here is highly advantageous when considering the interesting and experimentally-allowed neutrino mass limit m → 0 (or m → 0), in which (or) automatically disappears [7]. 2. Note that the accuracy of a prediction for the experimental sensitivity crucially depends on our knowledge of the relevant nuclear physics. In the worst possible scenario, uncertainties from nuclear physics might even weaken the expected experimental sensitivities by a factor as large as 5 [5]. 3. In Ref. [20] a purely statistical analysis of the possibility of | m _ee| 1 meV has been done to see to what extent the Majorana phases and can be constrained for a given value of m . While in Ref. [21] the conditions for | m _ee| > 1 meV are analyzed in the special case of m → 0 or → 0.

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Note: Figure translations are in progress. See original paper for figures.