

Semiclassical spectrum for BMN string in $Sch_5 \times S^5$ postprint

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Full Text

Preamble

Semiclassical spectrum for BMN string in $Sch_5 \times S^5$

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Abstract

We investigate the algebraic curve for BMN strings in $Sch_5 \times S^5$. We compute the semiclassical spectrum from the algebraic curve and compare our results with the anomalous dimensions in the $sl(2)$ sector of the null dipole deformation of $N=4$ super Yang-Mills theory.

Introduction

The spectrum of superstrings in $AdS_5 \times S^5$ is related to the spectrum of scaling dimensions in planar $N=4$ supersymmetric Yang-Mills theory via the AdS/CFT duality [1–3]. Integrability on both sides of the duality helps us dramatically

in finding and understanding the AdS/CFT spectrum (for a comprehensive review, see [4]). In N=4 supersymmetric Yang-Mills theory, the planar anomalous dimension matrix of infinitely long composite operators corresponds to the Hamiltonian of an integrable spin chain [5]. This implies that the spectrum can be solved efficiently by the Bethe ansatz.

On the string side, classical integrability of superstrings in $\text{AdS}_5 \times \text{S}^5$ follows from the existence of an infinite number of conserved charges [6] generated by the monodromy matrix of the Lax connection. The algebraic curve for classical solutions of superstrings in $\text{AdS}_5 \times \text{S}^5$ can be obtained from the Lax connection [7–11]. It plays an important role in studying semiclassical strings in $\text{AdS}_5 \times \text{S}^5$.

In recent years, much attention has been devoted to integrable deformations of AdS/CFT. One intriguing example is the Schrödinger spacetime [12–14]. Schrödinger spacetime can be obtained from AdS background by an appropriate TsT (T-duality-shift-T-duality) transformation [15] or null Melvin twist and has been shown to be classically integrable [16–18]. String theories in Schrödinger spacetime are dual to null dipole deformed field theories [19] (see also [20–22]). It is interesting to study the spectrum on both sides of the Schrödinger/dipole CFT duality using the methods of integrability.

Integrability in the null dipole deformation of N=4 super Yang-Mills was discussed in detail in [23]. The dipole deformation can be described as a Jordanian Drinfeld-Reshetikhin twist [24, 25] in the spin chain picture. The traditional Bethe ansatz is inapplicable due to the absence of a vacuum state. The one-loop spectrum of the nontrivial twisted $\text{sl}(2)$ sector was instead obtained from the Baxter equation. In the large J limit, the anomalous dimension of the ground state perfectly matches the classical energy of the BMN string at order J^{-1} .

The purpose of this paper is to study the Schrödinger/dipole CFT duality by comparing the semiclassical spectrum around classical string solutions to the anomalous dimensions of operators in the $\text{sl}(2)$ sector at order J^{-2} in the large J limit. One reason to study the order J^{-2} terms is that in the well-studied $\text{AdS}_5/\text{CFT}_4$ correspondence, the gauge theory and string results match at order J^{-2} in the BMN limit [26]. One can expect that the null dipole deformation preserves this matching. Another reason is that at order J^{-2} we should consider one-loop quantum string theory corrections to the string energy, while the previous test at order J^{-1} involved purely bosonic classical string energies.

We compute fluctuation energies of the excitations and the one-loop shift of the ground state energy from the algebraic curve. We show that the semiclassical spectrum around the BMN string solution perfectly matches the spin chain prediction.

This paper is organized as follows. In Section 2 we discuss the $\text{Sch}_5 \times \text{S}^5$ background and TsT transformation in detail. We discuss the algebraic curve for strings in this background and obtain the quasi-momenta for the BMN string.

In Section 3, we review the algebraic curve method for computing the fluctuation energies around classical string solutions. Then we compute the semiclassical spectrum for the BMN strings. In Section 4, we compare string theory results obtained in Section 3 with the one-loop spectrum in the $sl(2)$ sector of the null dipole deformation of $N=4$ super Yang-Mills theory.

2 Algebraic curve for strings in $Sch_5 \times S^5$ from TsT transformation

2.1 $Sch_5 \times S^5$ background

Schrödinger spacetime can be constructed by applying TsT transformations to the AdS background. The $Sch_5 \times S^5$ solution is obtained by acting with a TsT transformation on $AdS_5 \times S^5$ [12–14, 19]. In this paper we are interested in a particular case of $Sch_5 \times S^5$. We begin with the $AdS_5 \times S^5$ solution of type IIB supergravity:

$$ds^2 = R^2 \left[\frac{-2dx^+ dx^- + (dx^1)^2 + (dx^2)^2 + dz^2}{z^2} + (d\psi + A)^2 + ds_{CP^2}^2 \right]$$

where

$$ds_{CP^2}^2 = d\phi_1^2 + \sin^2 \phi_1 (d\theta_1 + \cos \phi_2 d\theta_2)^2 + d\phi_2^2 + \sin^2 \phi_2 d\theta_2^2$$

and the five-form field strength is given by

$$F_5 = 4R^4 \left(\frac{dx^+ \wedge dx^- \wedge dx^1 \wedge dx^2 \wedge dz}{z^5} + \text{vol}(S^5) \right).$$

We perform a TsT transformation on this geometry. We make a first T-duality along x^- , followed by a shift $x^- \rightarrow x^- + \frac{dx^+}{z}$, and then apply a second T-duality along the z coordinate. After this TsT transformation, the solution reads:

$$ds^2 = R^2 \left[-\frac{\mu^2 (dx^+)^2}{z^4} - 2dx^+ d\tilde{x}^- + \frac{(dx^1)^2 + (dx^2)^2 + dz^2}{z^2} + (d\tilde{\psi} + A)^2 + ds_{CP^2}^2 \right]$$

$$B_2 = \mu R^2 (d\tilde{\psi} + A) \wedge \frac{dx^+}{z^2}.$$

The five-form F_5 is invariant under the transformation. The TsT transformation preserves the symmetries of $SO(4,2)$ that commute with \hat{P} and \hat{J} . The symmetries that commute with \hat{J} and \hat{P} generate the Schrödinger group. The Schrödinger group contains the Galilean group as a subgroup and has two more generators corresponding to a non-relativistic scale transformation $D + M$ and a special conformal transformation K . The energy of the string is defined as the global charge associated with the symmetry $(\hat{P} + K)/\sqrt{2}$, which is related to

the non-relativistic scale transformation $D + M$ by a similarity transformation. Holography enables one to compute the non-relativistic conformal dimensions of operators at strong coupling as the energies of strings.

¹A more general class of Schrödinger deformations of $AdS_5 \times X_5$ has been studied in [27].

The relations between the original and dual coordinates are:

$$d\psi = d\tilde{\psi} + \mu \frac{dx^+}{z^2}, \quad dx^- = d\tilde{x}^- + \mu(d\tilde{\psi} + A).$$

We consider closed strings on the deformed background. The dual coordinates satisfy periodic boundary conditions. The original coordinates have the following twisted boundary conditions:

$$x^-(2\pi) - x^-(0) = LJ, \quad \psi(2\pi) - \psi(0) = 2\pi m_1 - L \int_0^{2\pi} d\sigma (\partial_\tau \psi + A_\tau),$$

where $L = 2\pi/\sqrt{\lambda}$ is the deformation parameter in the dual field theory, m_1 , and $\sqrt{\lambda} = R^2/\alpha'$ is the square root of the 't Hooft coupling. J and P are global charges associated with symmetries \hat{J} and \hat{P} .

2.2 Flat connection

Integrability of the string sigma model is preserved by the TsT transformation, so strings in Schrödinger spacetime are integrable [16–18] (see also [28–42]).

We now construct the Lax connection for the IIB superstring in $Sch_5 \times S^5$. The type IIB superstring in $AdS_5 \times S^5$ can be described by a sigma-model in the supercoset space of the supergroup $SU(2,2|4)$ over $SO(5)$ [43]. To describe strings in Poincaré coordinates, we choose the coset representative:

$$g(x^\mu, z, \psi, \theta_i, \phi_i, \Theta) = B(x^\mu, z, \psi, \theta_i, \phi_i) e^{F(\Theta)},$$

where

$$B(x^\mu, z, \psi, \theta_i, \phi_i) = e^{ix^+ \hat{P}_+ + ix^- \hat{P}_- + ix^1 \hat{P}_1 + ix^2 \hat{P}_2} e^{-\frac{i}{2} \log(z) \hat{D}} e^{i\psi \hat{J}} B_1(\theta_i, \phi_i),$$

and Θ represents the fermionic coordinates. We use a matrix representation such that:

$$\hat{P}_+ = \frac{1}{\sqrt{2}} \text{diag}(1, 1, 0, 0, 0, 0), \quad \hat{P}_- = \frac{1}{\sqrt{2}} \text{diag}(0, 0, 1, 1, 0, 0).$$

The current associated with g can be defined as:

$$J = g^{-1}dg = e^{-F} \left(dx^\mu \hat{P}_\mu + \frac{idz}{z} \hat{D} + id\psi \hat{J} + B_1^{-1} dB_1 \right) e^F + e^{-F} de^F.$$

The Lie superalgebra $\mathfrak{su}(2,2|4)$ has a \mathbb{Z}_4 grading structure associated with a \mathbb{Z}_4 automorphism Ω . The automorphism Ω in this matrix representation is defined by:

$$\Omega(M) = KM^{st}K^{-1}, \quad K = \text{diag}(1, 1, 1, 1, 1, 1, -1, -1).$$

We can decompose the current as $J = J^{\wedge\{0\}} + J^{\wedge\{1\}} + J^{\wedge\{2\}} + J^{\wedge\{3\}}$, where $\Omega(J^{\wedge\{k\}}) = i^k J^{\wedge\{k\}}$. Then the equation of motion of the string in $\text{AdS}_5 \times S^5$ is equivalent to the conservation of the Noether current $J^{\wedge\alpha} = 0$, and the flatness condition $dJ + J^2 = 0$.

The Lax connection $L(x)$ for the superstring in $\text{AdS}_5 \times S^5$ has been derived in [6]:

$$L(x) = J^{(0)} + x^2 J^{(2)} + \frac{1}{x^2} J^{(2)*} + x J^{(1)} + \frac{1}{x} J^{(3)}.$$

If the current satisfies the equation of motion, the Lax connection is flat: $dL + L^2 = 0$. Using this flat connection, one can define the monodromy matrix:

$$T(x) = \mathcal{P} \exp \left(\int_0^{2\pi} L_\sigma(x) \right).$$

The eigenvalues of $T(x)$ do not depend on τ and generate an infinite number of conserved quantities.

The current components $J^{\wedge\alpha}$ in the undeformed case do not have explicit dependence on x^- and σ . The Lax connection for strings in $\text{Sch}_5 \times S^5$ can be derived from the undeformed case and used to obtain quasi-momenta. The quasi-momenta $p_i(x)$ are functions defined from the eigenvalues of the monodromy matrix $T(x)$. They are generating functions of conserved physical quantities. For instance, we can read the conserved global charges from the behavior at large x . Large- x asymptotic properties of the quasi-momenta for strings with twisted boundary conditions are more complex than those for closed strings. Below we analyze the asymptotic behavior of the quasi-momenta.

Expanding the Lax connection, we get:

$$L(x) = \frac{1}{x} J_\tau + O\left(\frac{1}{x^2}\right).$$

Expanding the monodromy matrix, we obtain:

$$g(\tau, 0)T(x)g(\tau, 0)^{-1} = g(\tau, 0)g(\tau, 2\pi)^{-1} \exp\left(\frac{1}{x} \int_0^{2\pi} d\sigma J_\tau + O\left(\frac{1}{x^2}\right)\right) = e^{iL(P_- \hat{J} - J \hat{P}_-)} \exp\left(\frac{1}{x} \int_0^{2\pi} d\sigma J_\tau + O\left(\frac{1}{x^2}\right)\right)$$

In our representation, this takes the form:

$$\exp\left(i\sqrt{2}LJ\right) \exp\left(\frac{i}{x} \text{diag}(1, 1, 1, Q) + O\left(\frac{1}{x^2}\right)\right),$$

where Q satisfies $Q\hat{1} = 2\pi i\sqrt{2} P_-$. The behavior of the quasi-momenta at large x is:

$$\hat{p}_1 = \frac{\pi L J P_-}{\Delta + S} + O(x^{-1}), \quad \hat{p}_2 = \frac{\pi L J P_-}{\Delta + S + J_1 + J_2} + O(x^{-1}),$$

$$\hat{p}_3 = \frac{-\pi L J P_-}{\Delta + S - J_1 + J_2} + O(x^{-1}), \quad \hat{p}_4 = \frac{-\pi L J P_-}{\Delta + S + J_1 + J_2 + J_3} + O(x^{-1}).$$

The quasi-momenta \hat{p}_1 and \hat{p}_4 are connected by a square root cut which has a branch point at infinity. The quasi-momenta satisfy:

$$\int dx (\tilde{p}_1^2 + \tilde{p}_2^2 + \tilde{p}_3^2 + \tilde{p}_4^2) = 0,$$

so we still have the constraint between length and filling fractions (see [9, 11]).

2.3 BMN string

We now exemplify the discussion above with the BMN string solution in $Sch_5 \times S^5$ presented in [23]:

$$x^+ = \tau, \quad x^1 = x^2 = 0, \quad z = \text{constant},$$

$$\phi_i = \theta_i = 0, \quad \psi = \mu m \sigma + \omega \tau, \quad x^- = \mu \omega \sigma + \frac{\tan(\kappa \tau)}{\kappa}.$$

The Virasoro constraint gives:

$$\kappa^2 + \omega^2 = m^2.$$

The conserved global charges are:

$$\Delta = \sqrt{\lambda} \kappa, \quad P_- \equiv -\Delta_- = \sqrt{\lambda} \omega, \quad J = \sqrt{\lambda} m,$$

where we denote Δ as the global energy of the string. Then the classical energy of the BMN string is given by:

$$\Delta_{\text{cl}} = \sqrt{J^2 + \mu^2 M^2}.$$

The quasi-momenta of the BMN string are:

$$\begin{aligned} \hat{p}_1 = \hat{p}_4 &= \frac{2\pi\kappa x \sqrt{x^2 - \sin^2 2\gamma}}{\sqrt{x^2 - \sin^2 2\gamma} + x \sin 2\gamma}, \\ \hat{p}_2 &= \frac{\pi\omega(2x - (1 + x^2) \tan \gamma)}{\sqrt{x^2 - \sin^2 2\gamma}}, \quad \hat{p}_3 = \frac{\pi\omega(2x + (1 + x^2) \tan \gamma)}{\sqrt{x^2 - \sin^2 2\gamma}}, \\ \tilde{p}_1 = \tilde{p}_2 &= \frac{2\pi\kappa \sqrt{x^2 - \sin^2 2\gamma}}{\sqrt{x^2 - \sin^2 2\gamma} + x \sin 2\gamma}, \quad \tilde{p}_3 = \tilde{p}_4 = \frac{\pi\omega(2x + (3 - x^2) \tan \gamma)}{\sqrt{x^2 - \sin^2 2\gamma}}, \end{aligned}$$

where

$$\sin 2\gamma = \frac{4\pi\sqrt{\lambda}JLM}{4\pi^2 J^2 + \lambda L^2 M^2}, \quad \tan \gamma = \frac{2\pi\sqrt{\lambda}LM}{2\pi J}.$$

We find an expected square root cut $[\csc 2\gamma, \infty]$ connecting \hat{p}_1 and \hat{p}_4 .

3 Semi-classical quantization of the BMN string

A powerful method for computing the semiclassical spectrum around string solutions was proposed in [44]. Here we begin with a review of this method for the reader's convenience.

The semiclassical spectrum around the BMN string is given by:

$$\Delta = \Delta_{\text{cl}} + \Delta_{\text{1-loop}} + \delta\Delta,$$

where $\Delta_{\text{cl}} + \Delta_{\text{1-loop}}$ is the ground state energy, and $\delta\Delta$ is the energy of the excitations. To compute $\delta\Delta$, we add a perturbation $\delta p(x)$ to $p(x)$ associated with the classical solution. The perturbation $\delta p(x)$ has a single pole at x_n . The position x_n is determined by:

$$p_i(x_n^{ij}) - p_j(x_n^{ij}) = 2\pi n.$$

The residues at the poles are:

$$\text{res}_{x=x_n^{ij}} \delta\hat{p}_k = (\delta_{ik} - \delta_{jk})\alpha(x_n^{ij})N_n^{ij},$$

$$\text{res}_{x=x_n^{ij}} \delta \tilde{p}_k = (\delta_{jk} - \delta_{ik}) \alpha(x_n^{ij}) N_n^{ij},$$

where $i < j$ and $N_n^{\hat{ij}}$ is the excitation number for excitation with polarizations (ij) and mode number n . The function $\alpha(x)$ is defined as:

$$\alpha(x) = \frac{1}{\sqrt{\lambda}} \frac{x^2}{x^2 - 1}.$$

The residues at $x = \infty$ are synchronized as:

$$\{\hat{p}_1, \hat{p}_2, \hat{p}_3, \hat{p}_4 | \tilde{p}_1, \tilde{p}_2, \tilde{p}_3, \tilde{p}_4\} + \dots$$

For each cut ij connecting $p_i(x)$ and $p_j(x)$, the perturbation $\delta p(x)$ satisfies:

$$\delta p_i^+ + \delta p_j^- = 0,$$

where the superscript denotes above and below the cut. The poles result in an energy shift:

$$\delta \Delta = \sum_{n, ij} N_n^{ij} \Omega_{ij}(x_n^{ij}),$$

where Ω_{ij} are the off-shell fluctuation energies.

The asymptotic behavior of the perturbation $\delta p(x)$ is:

$$\begin{aligned} \delta \hat{p}_1 &= O(x^{-1/2}), \quad \delta \hat{p}_4 = O(x^{-1/2}), \\ \delta \hat{p}_2 &= \frac{2\delta\Delta + N_{\hat{2}\hat{3}} + N_{\hat{2}\hat{4}} + N_{\hat{2}\tilde{3}} + N_{\hat{2}\tilde{4}} - N_{\hat{1}\hat{3}} - N_{\hat{2}\tilde{3}} - N_{\hat{1}\tilde{3}}}{x} + O(x^{-2}), \\ \delta \hat{p}_3 &= \frac{-2\delta\Delta - N_{\hat{2}\hat{3}} - N_{\hat{1}\hat{3}} - N_{\hat{1}\hat{4}} - N_{\hat{2}\tilde{3}} - N_{\hat{1}\tilde{3}} - N_{\hat{2}\tilde{4}} - N_{\hat{1}\tilde{4}}}{x} + O(x^{-2}), \\ \delta \tilde{p}_1 &= \frac{N_{\tilde{2}\tilde{3}} + N_{\tilde{2}\tilde{4}} - N_{\tilde{1}\tilde{3}} - N_{\tilde{1}\tilde{4}}}{x} + O(x^{-2}), \\ \delta \tilde{p}_2 &= \frac{N_{\tilde{1}\tilde{3}} + N_{\tilde{1}\tilde{4}} - N_{\tilde{2}\tilde{3}} - N_{\tilde{2}\tilde{4}}}{x} + O(x^{-2}), \\ \delta \tilde{p}_3 &= \frac{N_{\tilde{1}\tilde{3}} + N_{\tilde{2}\tilde{3}} - N_{\tilde{1}\tilde{4}} - N_{\tilde{2}\tilde{4}}}{x} + O(x^{-2}), \\ \delta \tilde{p}_4 &= \frac{N_{\tilde{1}\tilde{4}} + N_{\tilde{2}\tilde{4}} - N_{\tilde{1}\tilde{3}} - N_{\tilde{2}\tilde{3}}}{x} + O(x^{-2}). \end{aligned}$$

The order $O(x^{\hat{1}/2})$ terms in $\delta \hat{p}_1$ and $\delta \hat{p}_4$ are determined by the constraint (2.30). Since the classical solution already obeys the constraint (2.30), one can show these filling fractions $N_n^{\hat{ij}}$ satisfy:

$$\sum_{n,ij} n N_n^{ij} = 0.$$

The one-loop shift is given by:

$$\Delta_{1\text{-loop}} = \frac{1}{2} \sum_{n,ij} (-1)^{F_{ij}} \Omega_{ij}(x_n^{ij}),$$

using the Riemann bilinear identity (see eqs. 3.38 and 3.44 in [11]).

We now determine the most general form of the perturbation $\delta p(x)$ for the BMN string. When small poles are added to $\delta \hat{p}_2(x)$ with a square root cut, the branch point will be slightly displaced so $\delta \hat{p}_2(x)$ behaves like $\{x_0\} \sqrt{x-x_0}$ near the branch point $x_0 = \sin 2\gamma$. The most general form for $\delta \hat{p}_2(x)$ is $f(x) + g(x)/K(x)$ where f and g are rational functions of x and $K(x) = \sqrt{x^2 - \sin^2 2\gamma}$. From (3.6) and inversion symmetry we get:

$$\begin{aligned} \delta \hat{p}_1 &= \frac{g(1/x)}{K(1/x)}, & \delta \hat{p}_2 &= f(x) + \frac{g(x)}{K(x)}, \\ \delta \hat{p}_3 &= f(x) - \frac{g(x)}{K(x)}, & \delta \hat{p}_4 &= -\frac{g(1/x)}{K(1/x)}. \end{aligned}$$

One can obtain all other off-shell fluctuation energies from the knowledge of $\Omega_{\{2\tilde{3}\}}$ and $\Omega_{\{2\tilde{3}\}}$ alone using the efficient method provided in [45]. From inversion symmetry we have:

$$\Omega_{\tilde{1}\tilde{4}}(y) = \Omega_{\tilde{2}\tilde{3}}(1/y) + \Omega_{\tilde{2}\tilde{3}}(0),$$

and the quasi-momenta for BMN string in $Sch_5 \times S^5$ are pairwise symmetric up to constant terms:

$$\hat{p}_2 = \tilde{p}_4 + 2\pi\kappa \sin \gamma, \quad \hat{p}_1 = \tilde{p}_1 = \tilde{p}_2 = \tilde{p}_3 - 2\pi\kappa \sin \gamma.$$

So the off-shell fluctuation energies satisfy:

$$\Omega_{ij}(y) = \frac{1}{2} (\Omega_{i'j'}(y) + \Omega_{j'j}(y)),$$

where $(\tilde{1}, 2, \tilde{1}, 2, 3, 4, 3, 4) = (4, 3, 4, 3, 2, 1, 2, 1)$.

3.1 Fluctuation energies of excitations

We first consider the excitation $\tilde{2}\tilde{3}$. We have $\delta \tilde{p}_i = 0$ for $i = 1, 2, 3, 4$, and therefore $f(x) = 0$. We take the following ansatz:

$$g(x) = \frac{\alpha(x_{\tilde{2}\tilde{3}}^n)N_{\tilde{2}\tilde{3}}^n}{x - x_{\tilde{2}\tilde{3}}^n} K(x_{\tilde{2}\tilde{3}}^n).$$

Large-x asymptotics of $\delta\hat{p}_{-1}$ gives:

$$\frac{\sqrt{\lambda}}{J} \sum_n N_{\tilde{2}\tilde{3}}^n = 0,$$

and the level matching condition is:

$$\sum_n n N_{\tilde{2}\tilde{3}}^n = 0.$$

We obtain:

$$\Omega_{\tilde{2}\tilde{3}}(x) = \frac{2x^2 + 2xK(x)}{x^2 - 1}.$$

We next consider the S^3 excitation $\tilde{2}\tilde{3}$. We start with the following ansatz:

$$g(x) = \frac{\alpha_1 K(1)}{x+1} + \frac{\alpha_2 K(1)}{x-1} - \frac{\alpha(x_{\tilde{2}\tilde{3}}^n)N_{\tilde{2}\tilde{3}}^n}{x - x_{\tilde{2}\tilde{3}}^n} K(x_{\tilde{2}\tilde{3}}^n).$$

Large-x asymptotics of $\delta\tilde{p}_{-1}$ and $\delta\tilde{p}_{-4}$ give:

$$\alpha_1 + \alpha_2 = \frac{\alpha(x_{\tilde{2}\tilde{3}}^n)N_{\tilde{2}\tilde{3}}^n}{x_{\tilde{2}\tilde{3}}^n},$$

$$\alpha_1 - \alpha_2 = \frac{\alpha(x_{\tilde{2}\tilde{3}}^n)N_{\tilde{2}\tilde{3}}^n}{x_{\tilde{2}\tilde{3}}^n}.$$

Large-x asymptotics of $\delta\hat{p}_{-1}$ give:

$$\alpha_1 K(1) + \alpha_2 K(-1) = 0.$$

The filling fractions satisfy:

$$\alpha_1(\cos \gamma - \sin \gamma) + \alpha_2(\cos \gamma + \sin \gamma) + \delta\Delta = 0.$$

Solving these equations, we obtain:

$$\sum_n N_{2\bar{3}}^n = 0, \quad \sum_n n N_{2\bar{3}}^n = 0,$$

and:

$$\Omega_{2\bar{3}}(x_{2\bar{3}}^n) = \frac{2 \cos \gamma}{x_{2\bar{3}}^n} - \frac{2 \sin \gamma}{K(x_{2\bar{3}}^n)}.$$

Finally, using (3.12) and (3.15) we find all the off-shell fluctuation energies:

$$\begin{aligned} \Omega_{2\bar{3}}(x) = \Omega_{\bar{1}\hat{4}}(x) &= \frac{2x^2 + 2x\sqrt{x^2 - \sin^2 2\gamma}}{x^2 - 1}, \\ \Omega_{\bar{1}\hat{3}}(x) = \Omega_{\hat{2}\bar{4}}(x) &= \frac{x^2 + x\sqrt{x^2 - \sin^2 2\gamma} + \sqrt{1 - x^2 \sin^2 2\gamma}}{x^2 - 1}, \\ \Omega_{2\bar{3}}(x) = \Omega_{\bar{1}\hat{4}}(x) &= \frac{2 \cos \gamma}{\sqrt{x^2 - \sin^2 2\gamma}} - \frac{2 \sin \gamma}{x}, \\ \Omega_{\bar{1}\hat{3}}(x) = \Omega_{\hat{2}\bar{4}}(x) &= \frac{\cos \gamma}{x} - \frac{\sin \gamma}{\sqrt{x^2 - \sin^2 2\gamma}} + \frac{\sqrt{1 - x^2 \sin^2 2\gamma}}{x^2 - 1}, \end{aligned}$$

$$\Omega_{2\bar{3}}(x) = \Omega_{\hat{2}\bar{4}}(x) = \Omega_{\bar{1}\hat{4}}(x) = \Omega_{\bar{1}\hat{3}}(x) = \Omega_{2\bar{3}}(x) = \Omega_{\hat{2}\bar{4}}(x) = \Omega_{\bar{1}\hat{4}}(x) = \Omega_{\bar{1}\hat{3}}(x) = \frac{\cos \gamma}{x} - \frac{\sin \gamma}{\sqrt{x^2 - \sin^2 2\gamma}}.$$

We now solve for the pole positions x_n . We choose the solution:

$$\begin{aligned} x_{2\bar{3}}^n &= \frac{J}{\sqrt{J^2 + \lambda L^2 M^2}} + \frac{2\pi n}{\sqrt{\lambda}} + O\left(\frac{1}{J}\right), \\ x_{\bar{1}\hat{4}}^n &= \frac{J}{\sqrt{J^2 + \lambda L^2 M^2}} + \frac{\pi\sqrt{\lambda}n^2}{2J} + O\left(\frac{1}{J^2}\right), \\ x_{2\bar{3}}^n &= \frac{\pi J}{\sqrt{\lambda}LM} + \frac{\pi n}{LM} + O\left(\frac{1}{J}\right), \\ x_{\bar{1}\hat{3}}^n &= \frac{\pi J}{\sqrt{\lambda}(LM + 4\pi n)} + \frac{\sqrt{\pi J n}}{\sqrt{\lambda}(LM + 4\pi n)} + O\left(\frac{1}{J}\right), \\ x_{\hat{2}\bar{4}}^n &= \frac{\pi J}{\sqrt{\lambda}(LM + 4\pi n)} + \frac{\sqrt{\pi J n}}{\sqrt{\lambda}(LM + 4\pi n)} + O\left(\frac{1}{J}\right). \end{aligned}$$

We have $x_{\underline{n}} > 1$ for small LM , which are physical poles.

The exact expressions of $x_{\underline{n}}$ are very complex, so we only consider the leading order terms in the large J expansion. When $LM = 0$, a finite number of $x_{\underline{n}}$ ($x_{\{\hat{1}\hat{4}\}^{\hat{n}}}$) will enter the cut connecting \hat{p}_{-1} and \hat{p}_{-4} .

Plugging $x_{\underline{n}}$ into the off-shell fluctuation energies, in the large J limit we get the on-shell fluctuation frequencies:

$$\Omega_{\hat{2}\hat{3}}(x_{\hat{2}\hat{3}}^n) = \Omega_{\hat{1}\hat{4}}(x_{\hat{1}\hat{4}}^n) = \frac{\lambda(\pi n^2 + LMn)}{2\pi J^2} + O\left(\frac{1}{J^3}\right),$$

$$\Omega_{\hat{1}\hat{3}}(x_{\hat{1}\hat{3}}^n) = \Omega_{\hat{2}\hat{4}}(x_{\hat{2}\hat{4}}^n) = \frac{2\pi n^2}{J} + \frac{LMn}{J^2} + O\left(\frac{1}{J^3}\right),$$

$$\Omega_{\tilde{2}\tilde{3}}(x_{\tilde{2}\tilde{3}}^n) = \Omega_{\tilde{1}\tilde{4}}(x_{\tilde{1}\tilde{4}}^n) = \frac{2\pi n^2}{J} + \frac{8\pi LMn}{J^2} + O\left(\frac{1}{J^3}\right),$$

$$\Omega_{\tilde{1}\tilde{3}}(x_{\tilde{1}\tilde{3}}^n) = \Omega_{\tilde{2}\tilde{4}}(x_{\tilde{2}\tilde{4}}^n) = \frac{16\pi^2 n^2}{J^2} + \frac{24\pi LMn + 5L^2 M^2}{32\pi^2 J^2} + O\left(\frac{1}{J^3}\right),$$

$$\Omega_{\hat{2}\hat{3}}(x_{\hat{2}\hat{3}}^n) = \Omega_{\hat{2}\hat{4}}(x_{\hat{2}\hat{4}}^n) = \Omega_{\hat{1}\hat{4}}(x_{\hat{1}\hat{4}}^n) = \Omega_{\hat{1}\hat{3}}(x_{\hat{1}\hat{3}}^n) = \frac{\lambda(4\pi n + LM)^2}{32\pi^2 J^2} + O\left(\frac{1}{J^3}\right),$$

$$\Omega_{\tilde{2}\tilde{3}}(x_{\tilde{2}\tilde{3}}^n) = \Omega_{\tilde{2}\tilde{4}}(x_{\tilde{2}\tilde{4}}^n) = \Omega_{\tilde{1}\tilde{4}}(x_{\tilde{1}\tilde{4}}^n) = \Omega_{\tilde{1}\tilde{3}}(x_{\tilde{1}\tilde{3}}^n) = \frac{\lambda(4\pi n - LM)^2}{32\pi^2 J^2} + O\left(\frac{1}{J^3}\right).$$

Then we obtain the energy shift $\delta\Delta$ given by (3.7).

3.2 One-loop shift

The one-loop shift is equal to one half of the graded sum of all fluctuation mode frequencies. Using zeta function regularization, we have:

$$\sum_{n=-\infty}^{\infty} \sqrt{(n+q)^2 + p} = \int_{-\infty}^{\infty} dk \sqrt{k^2 + p} + \sum_{n=1}^{\infty} (\sqrt{(n+q)^2 + p} + \sqrt{(n-q)^2 + p} - 2\sqrt{n^2}) = 0,$$

and

$$\sum_{n=1}^{\infty} (\sqrt{(n+q)^2 + p} - \sqrt{(n-q)^2 + p}) = q\sqrt{1+p/q^2} + \zeta(-1, 1+q) + \zeta(-1, 1-q) = 0.$$

Therefore, when we compute the one-loop shift energy at order J^{-2} , only the contribution from $\Omega_{-}\{\hat{1}4\}$ is nontrivial. Then we sum over the energies of the $\mathfrak{sl}(2)$ modes to get the one-loop shift:

$$\Delta_{1\text{-loop}} = \frac{1}{2} \sum_{k=1}^{\infty} \Omega_{\hat{1}4}(x_{\hat{1}4}^k) = \frac{\lambda L^2 M^2}{8\pi^2 J^2} - \frac{\lambda L^4 M^4}{96\pi^2 J^2} + \frac{\lambda L^6 M^6}{2880\pi^2 J^2} + O\left(\frac{L^8 M^8}{J^2}\right).$$

4 Comparison between the string and the gauge theory results

Comparison between the results obtained in gauge theory and string theory is possible in the large spin regime with $J \rightarrow \infty$ and λ/J^2 kept fixed and small (see e.g. [26, 46–48]). Type IIB superstring theory in $\text{Sch}_5 \times S^5$ is dual to null dipole deformed $N=4$ super Yang-Mills. The $\mathfrak{sl}(2)$ sector, which is nontrivially affected by the deformation, has been studied in [23]. The one-loop spectrum of the $\mathfrak{sl}(2)$ sector can be obtained from the Baxter equation. It was proposed in [23] that the Baxter equation takes the same form as in the undeformed case:

$$t(u)Q(u) = (u + i/2)^J Q(u + i) + (u - i/2)^J Q(u - i),$$

where

$$t(u) = 2u^J + \frac{LM}{J} u^{J-1} + \dots$$

The 1-loop energy is given by:

$$\Delta^{(1)} = \frac{\lambda}{8\pi^2} \partial_u \log Q(u + i/2) \Big|_{u=i/2}.$$

We now solve the Baxter equation in expansion in LM. At each order in LM, the Q-function is simply a polynomial. We write the ansatz:

$$Q(u) = \sum_{k=0}^{\infty} p_{k+m}(u) L^k M^k,$$

where $p_{\{k+m\}}(u)$ is a polynomial in u of degree $k + m$. The small LM expansion of Q can be interpreted as a function with a finite number of zeros near the Bethe roots in the undeformed limit and an infinite number of zeros of order L^{-1} . In the string picture, the zeros of order L^{-1} correspond to the cut connecting \hat{p}_1 and \hat{p}_4 .

Substituting the above ansatz into the Baxter equation, we can determine $Q(u)$ up to multiplication by a function in LM. We consider the following two solutions:

$$Q_0 = 1 + \frac{LM}{2(J+1)}u + \frac{L^2M^2}{2(J+2)}\left(\frac{u^2}{2(J+1)} - \frac{J}{4(J+1)}\right) + \frac{L^3M^3}{2(J+3)}\left(\frac{u^3}{6(J+1)(J+2)} - \frac{Ju}{4(J+1)(J+2)} + \frac{u}{2(J+1)}\right)$$

$$Q_1 = (u - u_n) + \frac{LM}{2(J+2)}\left(u^2 - u_n^2 + \frac{1}{4(J+1)}\right) + \frac{L^2M^2}{2(J+3)}\left(\frac{u^3 + u_n u^2 - u_n^3 - u_n}{4(J+1)(J+2)} + \frac{2Ju_n}{4(J+1)(J+2)}\right) + O(L^3M^3)$$

In the undeformed case, Q_0 and Q_1 correspond to the ground state and one-particle state respectively. Although not shown here, we have computed Q_0 and Q_1 to L^4M^4 order.

Then the energies of the ground state and one-particle state are:

$$\Delta_0^{(1)} = \frac{\lambda L^2 M^2}{8\pi^2(J+1)} - \frac{\lambda L^4 M^4}{96\pi^2(J+1)^2} + O(L^5 M^5),$$

$$\Delta_1^{(1)} = \frac{2\pi n^2}{J} + \frac{2\lambda L M u_n}{\pi^2(J+2)(4u_n^2+1)} + \frac{\lambda L^2 M^2}{8\pi^2(J+2)^3(J+3)}(4(J(J(J+10)+20)+24)u_n^2 + 4(J+2)^2(J(4J+7) -$$

where

$$c_4 = 96J^3, \quad c_2 = 4(J+2)^2(J^8 + 18J^7 + 495J^6 + 2474J^5 + 4520J^4 + 12304J^3 - 1808J^2 - 4656J - 35568),$$

$$c_0 = (J+2)^4(J+4)(J^5 + 6J^4 - 211J^3 - 738J^2 + 35568J^2 + 33696J + 17280).$$

In the large J limit, we get:

$$\Delta_0^{(1)} = \frac{\lambda L^2 M^2}{8\pi^2 J^2} - \frac{\lambda L^4 M^4}{96\pi^2 J^2} + O\left(\frac{L^5 M^5}{J^2}\right),$$

$$\Delta_1^{(1)} = \frac{2\pi n^2}{J} + \frac{\lambda L M n}{\pi J^2} + \frac{\lambda L^2 M^2}{8\pi^2 J^2} - \frac{\lambda L^4 M^4}{96\pi^2 J^2} \left(\frac{\pi^4 n^2}{6} + 1\right) + O\left(\frac{L^5 M^5}{J^2}\right) + O(J^{-3}).$$

Assuming that the interaction between particles can be neglected in the large J limit as in the undeformed case, the energy of an excited state above the ground state energy is:

$$\delta\Delta = \sum_n N_n \left(\frac{2\pi n^2}{J} + \frac{\lambda L^2 M^2}{4\pi^2 J^2} - \frac{\lambda L^4 M^4}{16\pi^4 J^2 n^2} + O\left(\frac{L^5 M^5}{J^2}\right) \right) + O(J^{-3}),$$

where N_n is the excitation number for mode number n , and we assume that the total momentum is zero.

To compare the spectral curve result with the spin chain result, we expand $\Omega_{\widehat{14}}(x_{\widehat{14}}^n)$ for small LM and obtain the energy shift of the $sl(2)$ sector:

$$\Omega_{\widehat{14}}(x_{\widehat{14}}^n) = \frac{2\pi n^2}{J} + \frac{\lambda L^2 M^2}{4\pi^2 J^2} - \frac{\lambda L^4 M^4}{16\pi^4 J^2 n^2} + O\left(\frac{L^5 M^5}{J^2}\right),$$

where we have used the level matching condition. The result agrees with (4.13).

The energy of the spin chain ground state to order $L^6 M^6$ has been computed in [23]:

$$\Delta_0^{(1)} = \frac{\lambda L^2 M^2}{4\pi^2} \left[\frac{1}{2(J+1)} - \frac{L^2 M^2}{24(J+1)^2} + \frac{L^4 M^4 (J^2 + J + 2)}{720(J+1)^3 (J+2)} \right] + O(L^8 M^8).$$

The order J^{-1} term matches the classical quantity $\Delta_{\{cl\}} - J$, and the order J^{-2} terms perfectly match the one-loop shift $\Delta_{1\text{-loop}}$ given in (3.33).

5 Conclusion and discussion

In this paper we study the algebraic curve for superstrings in $Sch_5 \times S^5$ and its application to the spectral problem. The asymptotic properties of the quasi-momenta for strings in $Sch_5 \times S^5$ are nontrivial. The point at infinity is a branch point of a cut connecting two Riemann sheets. We compute the semiclassical spectrum of the BMN string. Remarkably, we show that in the large J limit the string results match the gauge field results obtained from the Baxter equation. We provide a detailed test of the Schrödinger/dipole CFT duality.

Our results encourage further exploration of integrability in the Schrödinger/dipole CFT duality. It would be nice to derive the full quantum spectral curve of null dipole deformed $N=4$ super Yang-Mills theory, because the quasi-momenta are related to the quantum spectral curve in the strong coupling limit. It is also worth trying to obtain higher-order corrections on the field theory side to get a precise match with string theory predictions. One can also study the three-dimensional counterpart of Sch_5 , the warped AdS_3 . We hope that integrability will be a powerful tool for the spectral problem of warped AdS_3 /dipole CFT duality [49].

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