Postprint of the Refined Laser Ranging Equation for Field Measurements

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Abstract

A novel laser ranging equation is derived, taking into account factors such as telescope tracking random jitter and spot jitter induced by atmospheric turbulence. Through numerical simulation methodologies, the effects of laser divergence angle, telescope tracking random jitter, and atmospheric turbulence-induced spot jitter on echo intensity are investigated and analyzed, concluding that telescope tracking jitter and laser divergence angle are important factors affecting ranging system performance.

Full Text

Improvement of Laser Ranging Equation in Practical Use

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Abstract

Considering factors such as random telescope tracking jitter and spot jitter caused by atmospheric turbulence, we derive a new laser ranging equation. Using numerical simulation methods, we analyze the effects of laser divergence angle, telescope tracking random jitter, and spot jitter induced by atmospheric turbulence on echo intensity. Our analysis indicates that telescope tracking jitter and laser divergence angle are crucial factors affecting the performance of ranging systems.

Keywords: Telescope tracking jitter; Atmospheric turbulence; Laser ranging equation; Laser divergence angle

The laser ranging equation serves as the fundamental theoretical basis for studying the detection capability of laser ranging systems. Analysis of the laser ranging equation provides theoretical support for optimal selection of parameters such as laser energy and divergence angle in ranging systems. Traditional derivations of the laser ranging equation assume a uniform spatial distribution of laser pulse energy and only consider atmospheric attenuation during laser propagation through the atmosphere. However, in actual laser ranging systems, the distribution pattern of laser energy and telescope tracking jitter during measurements both cause variations in echo intensity. This paper considers both the inherent distribution of the laser beam and the effects of atmospheric turbulence on laser transmission, deriving a new expression for the laser ranging equation based on traditional formulations.

1. Gaussian Distribution of Laser Energy

The lasers used in satellite laser ranging systems emit fundamental mode radiation fields from their optical resonators with Gaussian amplitude and energy distributions in the transverse cross-section. When a laser beam propagates a distance z, the relationship between the energy distribution E and the cross-sectional radius $\rho = \sqrt{x^2 + y^2}$ is given by:

$$E = Ae^{-\frac{2\rho^2}{\omega^2(z)}}$$

where A is a coefficient, $\omega(z)$ characterizes the beam radius at propagation distance z, ω_0 is the beam waist radius, and λ is the laser wavelength. Assuming the beam radius corresponding to $1/e$ of the central energy is $\rho = \omega(z)$, the beam radius at distance z is:

$$\omega(z) = \omega_0 \sqrt{1 + \left(\frac{\lambda z}{\pi \omega_0^2}\right)^2}$$

Integrating this expression yields the relationship between the laser beam energy distribution E and the total pulse energy E_{total}:

$$E_{\text{total}} = \frac{\pi \omega^2(z)}{2} A$$

When a space debris target is located at the center of the Gaussian beam, the reflected echo intensity is strongest. Deviations from the beam center result in reduced echo intensity. Figure 1 schematically illustrates the Gaussian distribution of laser energy.

[Figure 1: see original paper]

2. Probability Distribution of Target Deviation Due to Telescope Tracking Jitter

Assuming telescope tracking jitter $\vec{\phi}$ is a random variable following a Gaussian probability distribution, $\vec{\phi}$ can be expressed as:

$$p(\vec{\phi}) = \frac{1}{\sqrt{(2\pi)^2|\Sigma_1|}} e^{-\frac{1}{2}(\vec{\phi}-\vec{\mu})^T \Sigma_1^{-1}(\vec{\phi}-\vec{\mu})}$$

where $\vec{\phi} = (\phi_x, \phi_y)^T$, with ϕ_x and ϕ_y representing the components of telescope tracking jitter $\vec{\phi}$ in the azimuth and elevation axes respectively; $\vec{\mu} = (\mu_x, \mu_y)^T$ represents the fixed pointing bias, with μ_x and μ_y being the fixed pointing biases in the azimuth and elevation axes; and T denotes the transpose operator. Σ_1 is the covariance matrix of $\vec{\phi}$.

When no fixed pointing bias exists, the expression simplifies to:

$$p(\vec{\phi}) = \frac{1}{\sqrt{(2\pi)^2|\Sigma_1|}} e^{-\frac{1}{2}\vec{\phi}^T \Sigma_1^{-1}\vec{\phi}}$$

For uncorrelated tracking random errors between elevation and azimuth axes:

$$\Sigma_1 = \begin{pmatrix} \sigma_x^2 & 0 \\ 0 & \sigma_y^2 \end{pmatrix}$$

where σ_x and σ_y represent the telescope tracking precision in the azimuth and elevation axes, respectively.

Assuming telescope tracking jitter follows a Gaussian distribution and considering only random tracking errors, the distance deviation $\vec{\rho}$ of a space target from the beam center caused by tracking jitter is also a Gaussian random variable. The relationship between $\vec{\rho}$ and $\vec{\phi}$ is:

$$\vec{\rho} = \vec{\phi}R, \quad \vec{\phi} = \vec{\rho}/R$$

The probability distribution becomes:

$$p(\vec{\rho}) = \frac{1}{\sqrt{(2\pi)^2|\Sigma_{12}|}} e^{-\frac{1}{2}\vec{\rho}^T \Sigma_{12}^{-1}\vec{\rho}}$$

where $\Sigma_{12} = R^2\Sigma_1$.

3. Probability Distribution of Target Deviation Due to Atmospheric Turbulence

Atmospheric turbulence causes the laser beam to continuously and randomly change its characteristics during propagation, manifesting as beam expansion, spot drift, and pulse broadening. Spot drift causes the relative distance $\vec{\rho}$ of space debris from the beam center to vary randomly. $\vec{\rho}$ follows a Gaussian probability distribution:

$$g(\vec{\rho}) = \frac{1}{\sqrt{(2\pi)^2|\Sigma_2|}} e^{-\frac{1}{2}\vec{\rho}^T \Sigma_2^{-1} \vec{\rho}}$$

where Σ_2 is the covariance matrix of $\vec{\rho}$, and $\vec{\rho} = (\rho_x, \rho_y)^T$ represents the components in the telescope' s azimuth and elevation axes.

For uncorrelated spot drift between azimuth and elevation axes:

$$\Sigma_2 = \begin{pmatrix} \sigma_{2x}^2 & 0 \\ 0 & \sigma_{2y}^2 \end{pmatrix}$$

where σ_{2x}^2 and σ_{2y}^2 are the spot drift variances in the azimuth and elevation axes, respectively.

The root-mean-square deviation of beam center drift caused by atmospheric turbulence is:

$$\sigma_{2x} = \sigma_{2y} = \sqrt{\langle \rho^2 \rangle} = 0.44\lambda \sqrt{\left(\frac{z}{r_0}\right)^3}$$

where r_0 is the atmospheric coherence length and λ is the laser wavelength. When spot drift variances are equal in both axes:

$$\sigma_{2x}^2 = \sigma_{2y}^2 = 0.44^2 \lambda^2 \left(\frac{z}{r_0}\right)^3$$

4. Derivation of Laser Ranging Equation Considering Both Random Effects

The deviation distance of a diffuse reflection target relative to the beam center results from the combined effects of telescope tracking jitter and atmospheric turbulence. According to probability theory, the combined effect of two independent physical processes on the same parameter can be expressed as the convolution of two functions:

$$p_{\text{total}}(\vec{\rho}) = \int p(\vec{\rho} - \vec{\rho}')g(\vec{\rho}')d\vec{\rho}'$$

Since the two processes are independent, the convolution yields:

$$p_{\text{total}}(\vec{\rho}) = \frac{1}{\sqrt{(2\pi)^2|\Sigma_{\text{total}}|}} e^{-\frac{1}{2}\vec{\rho}^T \Sigma_{\text{total}}^{-1} \vec{\rho}}$$

where $\Sigma_{\text{total}} = \Sigma_{12} + \Sigma_2$.

The mean laser energy at distance z under the combined effects can be obtained as:

$$E_{\text{mean}} = \int E(\vec{\rho}) p_{\text{total}}(\vec{\rho}) d\vec{\rho}$$

Assuming total laser pulse energy E_0, atmospheric transmittance T_a, detector quantum efficiency η, laser divergence half-angle θ, transmit optical transmittance T_t, target distance R, receive optical transmittance T_r, effective reflection area S, reflectivity ρ_h, Planck constant h, and light frequency ν, the number of received photoelectrons n_e for a uniformly diffuse target is:

$$n_e = \frac{E_0 T_a \eta T_t T_r S \rho_h \lambda}{\pi R^2 h \nu \omega^2(z)}$$

Substituting the mean energy expression yields the space debris diffuse reflection laser ranging equation:

$$n_e = \frac{E_0 T_a \eta T_t T_r S \rho_h \lambda}{\pi h \nu (R^2\theta^2 + \sigma_x^2 R^2 + \sigma_y^2 + \sigma_{2x}^2 + \sigma_{2y}^2)}$$

Assuming the mean beam expansion due to atmospheric turbulence is $\omega_{\text{turb}}^2 = \sigma_{2x}^2 + \sigma_{2y}^2$, the equation becomes:

$$n_e = \frac{E_0 T_a \eta T_t T_r S \rho_h \lambda}{\pi h \nu [R^2\theta^2 + R^2(\sigma_x^2 + \sigma_y^2) + \omega_{\text{turb}}^2]}$$

5. Numerical Simulation and Analysis

5.1 Effect of Telescope Tracking Precision on Echo Intensity at Different Laser Divergence Angles Figure 2 shows the relationship between the echo energy ratio and telescope tracking precision when atmospheric turbulence effects are neglected, for a 1.05 m telescope aperture and target distance of 1500 km. The ratio of echo energy at divergence angles of 1 , 5 , and 10 to that at 10 is plotted against tracking precision. When telescope tracking precision is better than 5 , using a high-magnification beam expansion system to reduce laser divergence angle significantly improves echo intensity. However, as tracking performance degrades, reducing divergence angle no longer effectively increases received echo intensity.

[Figure 2: see original paper]

5.2 Effect of Telescope Tracking Precision on Echo Intensity Under Different Atmospheric Coherence Lengths Figure 3 illustrates the relationship between received echo energy ratio and atmospheric coherence length for a laser divergence angle of 10 and tracking precision of 1 , 5 , and 10 . The echo energy ratio increases with atmospheric coherence length for all tracking precision values. At a coherence length of 5 cm, the echo intensity ratio between 1 and 10 tracking precision is approximately 6.5. As coherence length increases, this ratio improves further. When atmospheric coherence length is 10 cm, the echo energy ratio for 1 tracking precision is about 10 times higher than for 10 precision, representing a nearly tenfold improvement. When coherence length reaches 20 cm, the echo intensity ratio between 1 and 10 tracking precision increases to 18, with further improvement potential as coherence length continues to increase.

[Figure 3: see original paper]

5.3 Effect of Atmospheric Turbulence Intensity on Echo Intensity at Different Laser Divergence Angles Figure 4 shows the variation of echo intensity ratio with atmospheric coherence length, neglecting turbulence effects, for laser divergence angles of 1 , 5 , and 10 compared to 10 . At a coherence length of 1 cm, the ratio of echo intensity between 1 and 10 divergence is approximately 3. As coherence length increases, this ratio improves. At 10 cm coherence length, the ratio is about 10, and with further increase in coherence length, the echo intensity continues to improve.

[Figure 4: see original paper]

5.4 Effect of Atmospheric Coherence Length on Echo Intensity Under Different Telescope Tracking Precision Figure 5 presents the relationship between echo energy ratio and telescope tracking precision for a laser divergence angle of 10 , comparing echo energy at coherence lengths of 1 cm, 5 cm, 10 cm, and 20 cm to that at 20 cm. At 1 tracking precision, improving atmospheric coherence length from 1 cm, 5 cm, and 10 cm to 20 cm increases echo intensity by factors of 2.8, 2.1, and 1.6 respectively. However, as tracking precision degrades, improvements in atmospheric turbulence conditions do not effectively increase echo intensity. At 10 tracking precision, the echo intensity ratios corresponding to coherence lengths of 1 cm, 5 cm, and 10 cm relative to 20 cm are 1.9, 1.5, and 1.2 respectively.

[Figure 5: see original paper]

6. Conclusion

The echo intensity of laser ranging systems is jointly affected by laser divergence angle, telescope tracking precision, and atmospheric turbulence intensity. When ranging low-orbit targets with atmospheric coherence length no less than 10 cm at the station, spot jitter caused by atmospheric turbulence has minimal impact on detection performance. Under the premise of ensuring telescope tracking precision, reducing laser divergence angle can significantly improve detection performance. However, after the laser beam quality is determined, reducing divergence angle requires increasing telescope aperture, which demands higher performance from servo control and drive systems for large-aperture telescopes. Under current technical conditions, using the ranging system's imaging terminal for closed-loop target tracking and real-time control of a high-speed piezoelectric steering mirror to correct laser emission direction can effectively reduce the impact of telescope tracking precision on ranging system performance.

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Note: Figure translations are in progress. See original paper for figures.