Evolutionary Timescales of Mass and Radius and Their Influence on Changes in Rotational Angular Velocity in T Tauri Stars During the Slow Gravitational Contraction Phase Under the Action of Mass Ejection and Gravitational Contraction

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Abstract

This study investigates, for gravitational contraction stars with gravitational energy sources and ejected matter (such as T Tauri stars) after entering the slow gravitational contraction phase, the changes in mass and radius under the combined action of ejected matter and gravitational contraction, and their effects on rotational angular velocity changes. A coupled system of differential equations for the evolution of mass and radius with time and its solution are presented. Using the solution, we calculate the evolutionary timescales of mass and radius of T Tauri stars, as well as the effects on rotational angular velocity changes. Numerical results are presented, and the theoretical and numerical results are discussed.

Full Text

Evolutionary Timescales of Mass and Radius and Their Influence on Changes in Rotational Angular Velocity in T Tauri Stars During the Slow Gravitational Contraction Phase Under the Action of Mass Ejection and Gravitational Contraction

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Abstract: This paper studies gravitationally contracting stars with gravitational energy sources and mass ejection, such as T Tauri stars after they enter the slow gravitational contraction phase, and the changes in mass and radius under the combined action of mass ejection and gravitational contraction, as well as their influence on changes in the rotational angular velocity. A coupled set of differential equations for the time evolution of mass and radius and its solution are given. The solution is used to calculate the evolutionary timescales of the mass and radius of T Tauri stars and their influence on changes in rotational angular velocity. Numerical results are presented, and the theoretical and numerical results are discussed.

Keywords: T Tauri stars; slow gravitational contraction and mass ejection; evolutionary timescales of mass and radius; evolution of rotational angular velocity

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After a protostar forms, it first enters the phase of rapid gravitational contraction of the interstellar cloud. When the internal pressure of the interstellar cloud gradually increases and hydrostatic equilibrium is reached, the rapid gravitational contraction phase of the interstellar cloud enters the slow gravitational contraction phase. Stars in the slow gravitational contraction phase exhibit irregular luminosity variations and eject large amounts of matter; T Tauri stars are stars in the slow gravitational contraction phase that eject matter. In the rapid gravitational contraction phase, gravitational contraction dominates and mass loss can be neglected, so the rotational spin-up is relatively fast. After entering the slow gravitational contraction phase, because mass loss due to matter ejection becomes dominant while gravitational contraction is secondary, the rotational angular velocity begins to slow down[1].

For the mass lost by T Tauri stars through ejection of matter, see Refs.[2–10]. Reference[2] gives a comparatively detailed study, but under the action of gravitational contraction and matter ejection, the evolutionary timescales of mass loss and radius, and their influence on changes in rotational angular velocity, are not given. In particular, under the action of the first two factors, the influence of mass loss and radius contraction on the rotational angular velocity of T Tauri stars is not discussed.

When a star undergoes gravitational contraction, its radius decreases, which accelerates the rotational angular velocity; when mass is lost, the rotational angular velocity slows down. The change in the rotational angular velocity of T Tauri stars is produced by the combined action of these two factors: the reduction of radius due to gravitational contraction and the reduction of mass caused by mass ejection. References[11–16] discuss the rotational changes of T Tauri stars, but relatively little connection is made with changes in radius. This paper carries out further study in the above respects. T Tauri stars are a necessary process in the early evolution after the birth of every star, and the typical T Tauri stars studied in this paper have the general properties and significance of other T Tauri-type stars.

1 Basic assumptions of the physical model for T Tauri stars in the slow gravitational contraction phase

(1) The energy source of a T Tauri star is solely the gravitational energy generated by gravitational contraction; there is as yet no nuclear energy

source. It is a star with a gravitational-energy-source model, and therefore the mass radiation of the star may be neglected. Thus,

$$1 - \beta = P_{\mathrm{R}}/P = 0, \qquad \text{hence } \beta = 1$$

where P_{R} is the radiation pressure and P is the total pressure.

(2) The mass loss of the star comes from matter ejection produced by some physical mechanism; mainly hydrogen particles are ejected. Assume that the velocity v of the ejected matter is treated as a constant, and that the density of the ejected hydrogen particles is also treated as a constant.

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(3) The change in the stellar radius arises mainly from changes produced by gravitational contraction, and secondarily is also related to the kinetic energy of the ejected material. However, the outermost envelope of the ejected material of a T Tauri star is expanded, whereas the radius R of the inner layer decreases because of gravitational contraction.

(4) The radiant energy (luminosity) of a T Tauri star depends mainly on the difference between the gravitational contraction energy released by its own gravitational contraction and the kinetic energy carried away by the ejected material.

(5) According to Ref. [17], a contracting star evolves vertically in the H-R diagram during the stage of gravitational contraction (the Hayashi track), i.e., the evolutionary track is approximately perpendicular to the abscissa (effective temperature T_{e}) in the H-R diagram. Therefore, during contraction the surface temperature T_{e} of the star may be approximately regarded as constant and does not vary with time.

(6) Since, during the gravitational-contraction stage, the contracting star transports energy only by convection (a convective star), for a convective star the polytropic exponent is taken as $n = 1.5$, $\gamma = 5/3$.

2 Evolution equations determining the mass and radius of a T Tauri star

A T Tauri star undergoing slow gravitational contraction and ejecting material loses mass through the ejection of material. The mass-loss rate can be determined by the following equation [2]:

$$\frac{dm}{dt} = -4\pi R^2 N_{\mathrm{H}} V_0 m_{\mathrm{H}}, \tag{1}$$

where R is the radius; V_0 is the ejection velocity of the material at the surface, which may be obtained from the observed line profiles; N_{H} is the number density of hydrogen particles; and m_{H} is the mass of a hydrogen atom. According to the first two assumptions, V_0, N_{H}, and m_{H} are constants, whereas m and R are time variables. This is the first evolution equation for m and R. Next, determine the second evolution equation for m and R. According to the theory of stellar structure [18–19], the potential energy E' of the star can be written as

$$E' = -\beta \frac{\gamma - \frac{4}{3}}{\gamma - 1} \Omega,$$

where

$$\Omega = -\frac{3}{5-n} G \frac{m^2}{R}.$$

The gravitational contraction energy of the star is

$$E_{\mathrm{G}} = -E' = \frac{\gamma - 4/3}{\gamma - 1} \beta \Omega = -\frac{\gamma - 4/3}{\gamma - 1} \beta \left(\frac{3}{5-n} \right) G \frac{m^2}{R}.$$

According to assumptions (1) and (6) in Section 1, $\beta = 1$, $n = 1.5$, and $\gamma = 5/3$. Therefore,

$$E_{\mathrm{G}} = -\frac{3}{7} G \frac{m^2}{R}. \tag{2}$$

Because contraction of the star can only cause a change in radius and does not affect the mass, the rate of change of the contraction energy with time is

$$\frac{dE_{\mathrm{G}}}{dt} = \frac{3}{7} G \frac{m^2}{R^2} \frac{dR}{dt}. \tag{3}$$

$$E_{\mathrm{K}} = \frac{1}{2} m V_0^2. \tag{4}$$

Thus the rate of change of the kinetic energy carried away is

$$\frac{dE_{\mathrm{K}}}{dt} = \frac{1}{2} V_0^2 \frac{dm}{dt}. \tag{5}$$

According to assumption (4) in Section 1, the radiant energy E_{R} of a contracting star (T Tauri star) is the difference between the gravitational contraction energy E_{G} and the kinetic energy E_{K} carried away by the ejected material; that is,

$$E_{\mathrm{R}} = E_{\mathrm{G}} - E_{\mathrm{K}}.$$

Therefore,

$$\frac{dE_{\mathrm{R}}}{dt} = \frac{dE_{\mathrm{G}}}{dt} - \frac{dE_{\mathrm{K}}}{dt}.$$

But the rate of change of radiant energy, dE_{R}/dt, is equal to the luminosity L. Hence the luminosity L is equal to the gravitational contraction energy minus the kinetic energy carried away by the ejected material:

$$\frac{dE_{\mathrm{G}}}{dt} - \frac{dE_{\mathrm{K}}}{dt} = L. \tag{6}$$

Here the luminosity L can be written as

$$L = 4\pi R^2 \sigma T_{\mathrm{e}}^4. \tag{7}$$

Here σ is the Stefan constant, and T_{e} is the effective temperature of the stellar surface.

Substituting Eqs. (3), (5), and (7) into Eq. (6), one obtains the second evolution equation determining m and R:

$$\frac{3}{7} G \frac{m^2}{R^2} \frac{dR}{dt} - \frac{1}{2} V_0^2 \frac{dm}{dt} = 4\pi R^2 \sigma T_{\mathrm{e}}^4. \tag{8}$$

According to the assumption in Eq. (5) of Section 1, the evolutionary track of a contracting star on the H-R diagram is approximately perpendicular to the abscissa (effective temperature T_{e}); hence T_{e} on the right-hand side of Eq. (8) may be approximately regarded as a constant. According to the assumption in Eq. (2), the velocity V of the ejected material may also be regarded as a constant. Therefore, the system of differential equations formed by Eqs. (1) and (8) for the evolution of the mass m and radius R with time is

$$\frac{\mathrm{d}m}{\mathrm{d}t} = -4\pi R^2 N_{\mathrm{H}} V_0 m_{\mathrm{H}},$$

$$\frac{3}{7} \frac{Gm^2}{R^2} \frac{\mathrm{d}R}{\mathrm{d}t} - \frac{1}{2} V_0^2 \frac{\mathrm{d}m}{\mathrm{d}t} = 4\pi R^2 \sigma T_{\mathrm{e}}^4.$$

Substituting Eq. (1) into Eq. (2) gives the evolutionary equations for the mass and radius of a T Tauri star under gravitational contraction and ejection of material:

$$\frac{\mathrm{d}m}{\mathrm{d}t} = -4\pi R^2 N_{\mathrm{H}} m_{\mathrm{H}} V_0. \tag{9}$$

$$\frac{\mathrm{d}R}{\mathrm{d}t} = \frac{14\pi R^4}{3Gm^2}\left(N_{\mathrm{H}} m_{\mathrm{H}} V_0^3 + 2\sigma T_{\mathrm{e}}^4\right). \tag{10}$$

3 Evolutionary timescales of mass and radius (the simultaneous evolutionary relation of mass and radius)

The system of evolutionary equations (9)-(10) is integrable, and the evolutionary timescales of the mass and radius can be derived. To obtain the laws governing the variation of the mass m and radius R with time, numerical integration is required. The evolutionary timescales of the mass and radius obtained by the integrable method are given below.

Eliminating the time $\mathrm{d}t$ from Eqs. (9) and (10), one first obtains the relation between mass and radius during the evolutionary process:

$$\left(\frac{m}{R}\right)^2 \frac{\mathrm{d}R}{\mathrm{d}m} = \frac{7}{6G}\left(\frac{2\sigma T_{\mathrm{e}}^4}{N_{\mathrm{H}} m_{\mathrm{H}} V_0} + V_0^2\right),$$

after separation of variables and integration:

$$\int_{R_0}^{R} \frac{\mathrm{d}R}{R^2} = K_1 \int_{m_0}^{m} \frac{\mathrm{d}m}{m^2},$$

the result is

$$R = m/(K_1 + K_2 m). \tag{11}$$

or

$$m = K_1 R/(1 - K_2 R). \tag{12}$$

where

$$K_1 = \frac{7}{6G}\left(\frac{2\sigma T_{\mathrm{e}}^4}{N_{\mathrm{H}} m_{\mathrm{H}} V_0} + V_0^2\right) = \text{constant}. \tag{13}$$

$$K_2 = \frac{1}{R_0} - \frac{K_1}{m_0} = \text{constant}. \tag{14}$$

Equations (11)-(12) are the relations between mass and radius during the evolutionary process. First, solve for the evolutionary timescale of the mass. Substituting R in Eq. (11) into the right-hand side of Eq. (1), we have

$$\frac{\mathrm{d}m}{\mathrm{d}t} = -4\pi \left(\frac{m}{K_1 + K_2 m}\right)^2 N_{\mathrm{H}} m_{\mathrm{H}} V_0 = -K_3 \left(\frac{m}{K_1 + K_2 m}\right)^2$$

$$-K_3 \mathrm{d}t = \left(\frac{K_1 + K_2 m}{m}\right)^2 \mathrm{d}m = \frac{K_1^2 + 2K_1 K_2 m + K_2^2 m^2}{m^2} \mathrm{d}m.$$

After integration, one obtains:

$$-K_3(t - t_0) = -K_1^2 \left(\frac{1}{m} - \frac{1}{m_0}\right) + 2K_1 K_1 \ln\left(\frac{m}{m_0}\right) + K_2^2(m - m_0)$$

Taking $t_0 = 0$, the evolutionary timescale of the mass is

$$K_3 t = (m_0 - m)\left[K_2^2 + \frac{K_1^2}{m m_0}\right] + 2K_1 K_2 \ln\left(\frac{m_0}{m}\right). \tag{15}$$

where

$$K_3 = 4\pi N_{\mathrm{H}} m_{\mathrm{H}} V_0 = \text{constant} \tag{16}$$

Next, to solve for the evolutionary timescale of the radius, integrate Eq. (10), giving:

$$K_4 = \frac{14\pi}{3G}\left(N_H m_H V_0^3 + 2\sigma T_e^4\right) = \text{constant}. \tag{17}$$

Thus,

$$\frac{\mathrm{d}R}{\mathrm{d}t} = K_4 \frac{R^4}{m^2}.$$

Substituting m from Eq. (12) into the above gives

$$\frac{\mathrm{d}R}{\mathrm{d}t} = K_4 \frac{(1 - K_2 R)^2 R^4}{K_1^2 R^2} = K_5 (1 - K_2 R)^2 R^2.$$

Integrating gives

$$K_5 \int_0^t \mathrm{d}t = \int_{R_0}^{R} \frac{\mathrm{d}R}{(1-K_2R)^2R^2}.$$

The integral on the right-hand side can be evaluated by decomposing the rational expression into partial fractions; hence,

$$K_5 t = \int_{R_0}^{R} \frac{\mathrm{d}R}{R^2} + \int_{R_0}^{R} \frac{2K_2\,\mathrm{d}R}{R} + \int_{R_0}^{R} \frac{K_2^2\,\mathrm{d}R}{(1-K_2R)^2} + \int_{R_0}^{R} \frac{2K_2\,\mathrm{d}R}{1-K_2R}.$$

Therefore, the timescale for radius evolution is

$$K_5 t = \left(\frac{1}{R_0} - \frac{1}{R}\right) + \frac{K_2^2(R_0-R)}{(1-K_2R)(1-K_2R_0)} + 2K_2 \ln \frac{(1-K_2R_0)R}{(1-K_2R)R_0}. \tag{18}$$

where

$$K_5 = K_4/K_1^2. \tag{19}$$

4 Influence of the mass-loss rate and radius-contraction rate on changes in rotational angular velocity

When a star contracts gravitationally, its radius decreases and its rotational angular velocity increases; when mass is lost, however, the rotational angular velocity decreases. The change in the rotational angular velocity of a T Tauri star arises from the combined effects of the decrease in radius due to gravitational contraction and the decrease in mass caused by ejection of material.

When the star ejects matter, in addition to losing mass it also loses angular momentum; the angular momentum lost is the angular momentum carried away by the ejected material. Therefore, the angular momentum lost when a T Tauri star ejects material should be equal to the angular momentum carried away by the ejected material. Let the rotational angular velocity of the T Tauri star be ω, the radius of gyration be K_sR, and K_dR be the radius of gyration of the ejected mass $\mathrm{d}m$. If the ejection of material is isotropic (spherically symmetric), by conservation of angular momentum there is[^20]

$$\mathrm{d}\left(K_sR^2m\omega\right) = K_dR^2\omega\,\mathrm{d}m. \tag{20-1}$$

Written in terms of angular-momentum and mass-loss rates:

$$\frac{\mathrm{d}\left(K_s R^2 m\omega\right)}{\mathrm{d}t} = K_d R^2 \omega \frac{\mathrm{d}m}{\mathrm{d}t}. \tag{20-2}$$

After differentiation,

$$\left(\frac{K_s - K_d}{K_s}\right)\frac{1}{m}\frac{\mathrm{d}m}{\mathrm{d}t} + \frac{2}{R}\frac{\mathrm{d}R}{\mathrm{d}t} + \frac{1}{\omega}\frac{\mathrm{d}\omega}{\mathrm{d}t} = 0.$$

Here, K_s has different values for polytropic model stars with different indices n. When a star enters the main sequence, K_s is very small, and generally $K_s = 0.1$ is adopted; however, in the pre-main-sequence stage, such as for T Tauri stars, the value of K_s is larger than 1. According to Ref. [20], for a polytropic index $n = 1.5$ (the value adopted in this paper is the one corresponding to Eq. (6) in Section 2), by consulting the table one obtains $K_s = 1/5 = 0.2$. Because the ejection of material is isotropic, $K_d = 2/3$ is adopted. Thus, from the above equation the relative rate of change of angular velocity is

$$\dot{\omega}/\omega = \frac{7}{3}\frac{\dot{m}}{m} - 2\frac{\dot{R}}{R}. \tag{21-1}$$

If the present relative rate of change of angular velocity at $t = 0$ is considered, it can be written as

$$\dot{\omega}(0)/\omega(0) = \frac{7}{3}\frac{\dot{m}(0)}{m(0)} - 2\frac{\dot{R}(0)}{R(0)}. \tag{21-2}$$

Here, $\omega(0)$, $m(0)$, and $R(0)$ are respectively the present ($t = 0$) angular velocity, mass, and radius, while $\dot{\omega}(0)$, $\dot{m}(0)$, and $\dot{R}(0)$ are respectively the present ($t = 0$) rate of change of angular velocity, mass-loss rate, and radius-contraction rate. If the long-term change in angular velocity is considered, integrating Eq. (21-1) gives

$$\frac{\mathrm{d}}{\mathrm{d}t}(\ln\omega) = \frac{7}{3}\frac{\mathrm{d}}{\mathrm{d}t}(\ln m) - 2\frac{\mathrm{d}}{\mathrm{d}t}(\ln R)$$

Integrating,

$$\int_{\omega_0}^{\omega} \mathrm{d}\ln\omega = \frac{7}{3}\int_{m_0}^{m} \mathrm{d}\ln m - 2\int_{R_0}^{R} \mathrm{d}\ln R$$

$$\ln\left(\frac{\omega}{\omega_0}\right) = \ln\left(\frac{m}{m_0}\right)^{7/3} - \ln\left(\frac{R}{R_0}\right)^2 = \ln\left(\frac{m}{m_0}\right)^{7/3}\left(\frac{R_0}{R}\right)^2$$

$$\therefore \omega/\omega_0 = \left(\frac{m}{m_0}\right)^{7/3} \left(\frac{R_0}{R}\right)^2,$$

$$\omega = \omega_0 \, (m/m_0)^{7/3} (R_0/R)^2. \tag{22}$$

Here, ω_0, m_0, and R_0 are the initial values at $t = 0$. m and R vary with time t, being determined by Eqs. (9)-(10), or by Eqs. (15) and (18); therefore the angular velocity ω is also a function of time t and varies with time. If R from Eq. (11) is substituted into Eq. (22), the angular velocity ω can be obtained as a function of the stellar mass, namely

$$\omega = \omega_0 \left(\frac{m}{m_0}\right)^{1/3} \left(\frac{K_1 + K_2 m}{k_1 + K_2 m_0}\right)^2. \tag{23}$$

5 Numerical Calculation of the Long-Term Evolution of T Tauri Stars from the Theoretical Results

Using the theoretical results obtained in the preceding section, we carry out a numerical calculation of the evolution with time of the mass, radius, and rotational angular velocity of T Tauri stars. First, using the physical parameters of a T Tauri star given in Ref. [2]:

$$m_0 = 0.60 m_\odot = 1.1934 \times 10^{33} \text{ g}, \quad R_0 = 4.56 R_\odot = 3.1737 \times 10^{11} \text{ cm},$$

$$T_e(\text{K}) = 4100 \text{ K}, \quad N_\text{H} = 1.15 \times 10^{10} \text{ atoms/cm}^3, \quad V_0 = 225 \text{ km/s}$$

and, according to physical-constant data,

$$M_\text{H} = 1.6735 \times 10^{-24} \text{ g}, \quad \sigma = 5.6695 \times 10^{-5} \text{ erg/cm}^2 \text{ deg}^4 \text{ s}, \quad G = 6.67 \times 10^{-8} \quad \text{(cgs)},$$

substitution of these data into Eqs. (13), (14), (16), and (17) gives

$$K_1 = 1.28580 \times 10^{24} \quad \text{(cgs)}, \quad K_2 = -1.08875 \times 10^{-9} \quad \text{(cgs)}, \quad K_3 = 5.4414 \times 10^{-6} \quad \text{(cgs)},$$

$$K_4 = 7.09070 \times 10^{18} \quad \text{(cgs)}, \quad K_5 = 4.17560 \times 10^{-30} \quad \text{(cgs)}.$$

On the basis of the above data and the numerical values calculated for the constants K, the theoretical formulae can be used to compute the numerical values corresponding to different evolutionary stages of a T Tauri star: the long-term evolution of its mass, radius, and rotational angular velocity.

First, five numerical values for the evolution of mass loss are given, namely $m = 0.999999999m_0$, $0.99999990m_0$, $0.99999900m_0$, $0.999999000m_0$, and $0.999990000m_0$ (m_0 is the present mass). Substituting these, together with the numerical values of K_1 and K_2, into Eq. (11) yields the radius-evolution values for the five evolutionary stages. Then substituting the mass and radius evolution values for the five evolutionary stages, as well as the initial values m_0 and R_0, into Eq. (22) gives the evolutionary values of the rotational angular velocity ω for the five evolutionary stages. Finally, substituting the initial mass m_0, the five mass-loss values for the evolutionary stages, and the numerical values of K_1, K_2, and K_3 into Eq. (15), the values of the evolutionary timescales corresponding to the mass, radius, and rotational angular velocity in the five evolutionary stages are calculated. The results are shown in Table 1.

Table 1 Evolution with age of the mass, radius, and rotational angular velocity of a T Tauri star during the stage of slow gravitational contraction

Table 1 In the step of the slow gravitational contraction the evolution of mass, radius and rotational angular velocity of T Tauri with age

m/m_0	R/R_0	ω/ω_0	t (yr)
0.999999999	−0.2794795	12.80625	3.2502×10^4
0.999999990	−0.2794798	12.80623	3.2602×10^5
0.999999900	−0.2794822	12.80622	3.2610×10^6
0.999999000	−0.2795062	12.80017	3.2609×10^7
0.999990000	−0.27974687	12.77789	3.2611×10^8
Mass gradually decreases	Radius contraction gradually increases	Rotational angular velocity gradually decreases slowly	Timescale

6 Discussion

(1) After a protostar is born, it begins to enter a stage of rapid gravitational contraction. In this stage, gravitational contraction is dominant, mass loss can be neglected, and the rotation accelerates. When ejected matter enters the stage of slow gravitational contraction, mass loss becomes dominant, gravitational contraction becomes secondary, and the rotational angular velocity gradually decreases. This paper studies the evolutionary process of T Tauri stars after the evolution of rapid gravitational contraction has ended and the stars have begun to enter the stage of slow gravitational contraction. The numerical values given in Table 1 are the evolutionary values for five stages from the start time of rapid gravitational contraction,

$t = 0$, to the start time of slow gravitational contraction, $t = 3.2502 \times 10^4$ yr, and thereafter to $t = 3.26 \times 10^8$.

(2) The coupled evolutionary equations for mass m and radius R were derived under the conditions that the surface effective temperature T_e and the velocity V of the ejected matter are constants. Treating the surface effective temperature as a constant is based on Section 1, Eq. (5), assuming that before entering the main sequence the contracting star evolves along a path perpendicular to the horizontal coordinate of the H–R diagram (the effective temperature on the spectral-type coordinate), i.e., the Hayashi evolutionary track [17]. However, this is only approximately vertical and is restricted to the evolutionary stage before entering the main sequence. When the contracting star is about to enter the main sequence, the evolutionary track bends leftward and is no longer perpendicular to the spectral-type coordinate of the H–R diagram; at this stage, taking T_e as a constant is no longer applicable. In addition, the velocity V of matter ejected by T Tauri stars is regarded as unchanged. In the stage of slow gravitational contraction, the mass loss of T Tauri stars comes mainly from the ejected matter; because the energy source depends only on gravitational contraction and not on nuclear energy, mass loss caused by photon radiation (mass radiation) can therefore also be ignored.

(3) The numerical values given in the table are theoretically reasonable. The values in the first row represent the evolutionary mass, radius, and initial slowing rate of a T Tauri star at the end of the rapid gravitational-contraction stage and at the beginning of the slow gravitational-contraction stage, $t = 3.25 \times 10^4$ yr. The first column shows that the mass gradually decreases year by year because of the ejection of matter; the second column shows that the radius gradually contracts year by year, with the scale of contraction increasing (the negative sign indicates contraction of the radius); the third column shows that the rotational angular velocity gradually slows year by year; and the fourth column gives the evolutionary timescales of the mass and radius. According to the observations of the rotation of 28 T Tauri stars in Ref. [12], the values of the rotational angular velocity decrease, but only slightly. The changes in the slowing of the rotational angular velocity in this paper' s table are also very small over the five stages, which indicates agreement with the observations. The timescales in Table 1 are thus the evolutionary timescales for the mass beginning from $0.999999999m_0$, and also for the radius beginning from $-0.2794795R_0$.

7 Conclusion

The T Tauri star studied in this paper is a typical star among other types of T Tauri stars, and it represents the general properties of most T Tauri stars. In the stage of slow gravitational contraction, ejected matter causes the mass to decrease gradually; gravitational contraction causes the radius to contract gradually, with the scale of contraction increasing; and the rotational angular velocity gradually decreases. These changes are the results of theory and calculation and remain to be verified by observation. However, at present the acceleration of the rotational angular velocity of T Tauri stars has not yet been observed. As discussed in the preceding section, according to Ref. [17], observations of the rotation of 28 T Tauri stars show that the rotational angular velocity is decreasing.

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Evolutional Time Scale of Mass and Radius of T Tauri and Its Influence on the Rotation under the Action of the Ejected Material and Gravitational Contraction in the Step of the Slow Gravitational Contraction

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Abstract: The mass loss and radius contraction of T Tauri star under the role of the ejected material and gravitational contraction in the step of the slow gravitational contraction are studied. The evolutionary equations determining the variation of mass and radius with time are established. The evolutional time scales of mass and radius are given. The influences of mass loss and radius contraction on the variation of the rotational angular velocity are given too. As example, the numerical effects of mass loss and radius contraction on the rotational relative angular velocity are estimated in Table. In addition the obtained results are discussed and concluded.

Key words: T Tauri star; Slow gravitational contraction and ejected material; Evolution scales of mass and radius; Change of rotational angular velocity