Observational Analysis of Temporal Variations in the Power-Law Distribution of Lower Chromospheric Bright Points in Solar Active Regions (Postprint)

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Abstract

The power-law distribution of solar flare and microflare energies is a typical characteristic of solar activity. Investigating their statistical characteristics and temporal variation patterns is of great significance for studying the transport and dissipation of solar energy and the heating mechanism of the upper atmosphere. To study the statistical laws of energy release in the solar lower atmosphere and their evolution, we use a two-dimensional region labeling algorithm to analyze the frequency characteristics representing the energy distribution of bright points and their variation patterns on the lower chromosphere Ca II H d3 968.5 monochromatic image data of NOAA 10930 active region observed by the Solar Optical Telescope (SOT) aboard the Hinode satellite. In a time series with a time span of 124 h and a field of view of 202” .4 × 85” .3, a total of 2.99x1 护 bright points were obtained, with an average of 24. 15 1 6. 34 bright points per square arcminute per frame. The main analysis results are as follows: (1) The instantaneous distribution of bright point sizes (L= 瓜万) basically conforms to a power-law distribution and satisfies self-similarity; (2) The occurrence rate and signal-to-noise ratio of bright points decrease with increasing size. The continuous occurrence of numerous small-scale bright points may be a stable and considerable energy source for heating the upper atmosphere; for example, bright points with sizes smaller than 4” account for 53.23% of the total flux in the sample; (3) The instantaneous number density of small-scale bright points decreases with the decay of the active region;}4) The sizes of the bright point set follow a power-law distribution, with a coefficient of variation o-(妇 of only 4%. The power-law exponent y of this observed sample is 1.97, and that of the low-noise sample (L } g}}) is 2.12; (5) However, y does not converge within the observation period. The y of the sample set is a process that exhibits sawtooth-like variation during observation accumulation: it slowly increases during quiet periods and suddenly decreases at active moments. (6) The y of instantaneous bright point subsets is inversely proportional to their total flux. After removing large-scale individuals (L > 8”) from the sample, small- and medium-scale bright points still follow the above pattern. Solar activity not only produces medium- and large-scale brightening regions, but also has a global impact on the frequency distribution of bright points across all size ranges, causing a decrease in instantaneous y.

Full Text

Preamble

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Observational Analysis of Temporal Variations in the Power-Law Distribution of Low-Chromospheric Bright Points in a Solar Active Region

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Abstract

The power-law distribution of solar flare and microflare energies represents a fundamental characteristic of solar activity. Investigating its statistical properties and temporal evolution is crucial for understanding energy transport and dissipation in the solar atmosphere and the heating mechanisms of the upper layers. This study examines the statistical patterns of energy release in the solar lower atmosphere and their evolution using a two-dimensional region-labeling algorithm applied to Ca II H monochromatic images from the Hinode Solar Optical Telescope (SOT). A total of 29,968 bright points were identified in the low chromosphere of NOAA Active Region 10930. The main findings are: (1) The instantaneous frequency distribution of bright point scales follows a power law and exhibits self-similarity; (2) The production rate and signal-to-noise ratio of bright points decrease with increasing scale, suggesting that numerous small-scale bright points may provide a stable and substantial energy source for heating the upper atmosphere—for instance, bright points smaller than 4 contribute approximately 53.23% of the total light flux; (3) The instantaneous number density of small-scale bright points decreases as the active region decays; (4) The power-law index γ of the observed sample is 1.97, while that of the low-noise sample (excluding large-scale events) is 2.12; (5) However, γ does not converge during the observation period, instead following a sawtooth pattern: gradually increasing during quiet periods and dropping sharply during active moments; (6) The instantaneous power-law index γ is inversely proportional to the total light flux of the subset. This relationship holds even after removing large-scale individuals, indicating that solar activity not only produces medium- and large-scale brightenings but also globally affects the frequency distribution across all scales, causing a reduction in the instantaneous γ.

Keywords: power-law distribution; bright point; solar flare; solar chromosphere; region-labeling method

1. Introduction

A typical statistical feature of solar activity is the power-law distribution of flare energies across X-ray [1–4], hard X-ray [5–8], and extreme ultraviolet [9–10] wavelengths. The peak flux, scale, and lifetime of flares and microflares generally follow power-law distributions [11], with the fundamental form $N(X) = \alpha X^{-\gamma}$, where X represents a measured quantity related to flare energy. Studies show that the photon flux of hard X-ray flares follows a power-law relationship with photon energy E [5], demonstrating self-similarity in solar activity. Although different active regions exhibit varying eruption mechanisms, frequencies, and intensities, their power-law indices γ remain similar. The power-law distributions of bright points in individual active regions and those obtained from long-term statistical studies of full-disk solar activity are fundamentally similar in both pattern and mechanism [12–13], representing intrinsic properties of solar activity.

As a scale-free complex network, the overall power-law distribution of solar activity represents an integration of numerous scale-free distributions. In scale-free complex networks, structures of vastly different scales but with identical evolutionary mechanisms often exhibit self-similarity. This phenomenon is common in experimental, space, and simulated plasmas [14]. The power-law distribution of bright points across various scales may be independent of solar activity cycles. While coronal bright points show spatiotemporal periodicity, with eruption-prone regions aligning with strong magnetic field distributions [15–16], and solar activity exhibits both long-term (11-year) and short-term periodicities [3,17] that affect Earth' s ionosphere [18], studies indicate that the power-law index γ shows no clear correlation with the solar cycle [3,5,11,20]. Statistical analysis of over 20,000 flares from the Geostationary Operational Environmental Satellites (GOES) between 1976–2000 revealed nearly identical power-law indices for X-ray peak flux during solar maximum and minimum [19], suggesting that the flare power-law distribution is a statistical law relatively independent of activity level, with underlying physical mechanisms that remain constant across the solar cycle.

However, some studies report γ varies with the solar cycle, being larger during solar maximum [21]. Observed frequency distributions often deviate from power laws at the low end, with measured frequencies falling below the power-law prediction. This bias largely stems from instrumental limitations—detection thresholds restrict identification of low-energy events above background noise. Lowering instrumental thresholds significantly increases the sample size of small-scale events. For instance, the UCSD X-ray detector on OSO-7 showed a plateau in frequency for peak fluxes of 1-10 photons $\mathrm{cm}^{-2}\ \mathrm{s}^{-1}\ \mathrm{keV}^{-1}$, with a low-end threshold of ~0.3 photons $\mathrm{cm}^{-2}\ \mathrm{s}^{-1}\ \mathrm{keV}^{-1}$ [5]. In contrast, detectors with thresholds an order of magnitude lower (0.01-0.1 photons $\mathrm{cm}^{-2}\ \mathrm{s}^{-1}\ \mathrm{keV}^{-1}$) produced smooth power-law distributions across 0.5-20 keV [6].

Small-scale events are also under-sampled due to limited temporal and spatial resolution. Numerous short-lived events with lifetimes comparable to or shorter than the cadence are missed, while neighboring structures below the diffraction limit may be blended. Comparative studies using the New Solar Telescope (NST, 77 km diffraction limit) and the Solar Optical Telescope (SOT, 157 km diffraction limit) show that 98.6% of photospheric bright points have lifetimes shorter than SOT' s cadence, making them undetectable by SOT [21]. Similar high miss rates affect low-chromospheric bright points [22] and interplanetary magnetic flux ropes [23].

Power-law distributions cannot extend to zero indefinitely. If bright points strictly followed $N(E) = \alpha E^{-\gamma}$, the total energy would diverge [24]. In reality, the total energy must converge, requiring the complete frequency distribution to have a low-end turnover point where frequencies drop below the power law. This turnover may represent the minimum bright point scale. When instrumental resolution lies within the power-law segment, sample size increases dramatically with improved resolution. However, below this turnover point, further resolution improvements yield diminishing returns in total energy recovery.

The solar atmosphere is an open system with continuous energy input and dissipation. Magnetic fields generated in the solar interior [25] rise through the convection zone as large flux tubes [26], bringing magnetic energy and complexity into the upper atmosphere. The resulting network is a complex system with growth, diverse nodes, and spatiotemporal complexity. New magnetic flux emerges preferentially near existing active regions [30-31], following a preferential attachment mechanism where new nodes connect to high-degree nodes, amplifying degree differences and evolving into a scale-invariant state with power-law degree distributions [27-29].

The power-law distribution of solar activity is crucial for understanding coronal heating. Since magnetically sensitive lines are scarce, direct observation of magnetic field evolution during energy release is impossible. Statistical analysis of bright point distributions provides valuable information. If $\gamma > 2$, large flares dominate the energy budget; if $\gamma < 2$, small-scale events may be more important. Low-chromospheric oscillations could propagate MHD waves upward, causing turbulent dissipation and heating [33-34]. For individual active regions with limited energy, whether eruptions strictly follow power laws remains uncertain [24,32], requiring analysis of distribution characteristics across evolutionary stages.

Notably, photospheric bright points in quiet Sun regions follow log-normal distributions for scale [21,36] and area [37], unlike the power-law behavior in active regions. Our previous work [22] using 3D region-labeling found that low-chromospheric bright points in active regions follow power-law distributions at intermediate timescales but exponential distributions at long timescales, with large bright points showing Poisson-distributed waiting times.

This study uses low-chromospheric observations to analyze temporal variations in instantaneous bright point distributions, addressing: (1) What laws govern instantaneous frequency distributions at short timescales? (2) How do these distributions evolve with active region development? (3) What correlations exist between distribution parameters?

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2. Data and Methods

2.1 Observational Data and Preprocessing

We selected NOAA Active Region 10930, a magnetically complex region in the southern hemisphere (S05) that appeared on December 9, 2006. The main sunspot had a radius of ~90 × 100 , with multiple flux emergence sites producing over twenty small pores and micro-pores. The region evolved from β to β-γ-σ magnetic configuration, generating one X-class flare, three M-class flares, and dozens of C-class flares during its disk passage .

We used Ca II H $ $3968.5 Å monochromatic image sequences from the Broadband Filter Imager (BFI) aboard the Hinode Solar Optical Telescope (SOT). BFI provides 217 .1 × 108 .5 field-of-view images with 0 .106 × 0 .106 pixel size and 120 s cadence. The observation period (December 9, 20:02 to December 14, 23:54 UT) covered 124 hours with a 202 .4 × 85 .3 field of view. Brightening scales ranged from sub-arcseconds to tens of arcseconds, with shapes varying from elliptical to irregular. We collectively term these diverse brightenings “bright points” (BPs) for statistical purposes.

While Hinode’ s Extreme-Ultraviolet Imaging Spectrometer (EIS) and X-Ray Telescope (XRT) also detect bright points, their higher atmospheric layers and coarser resolution (~2) provide less detail. A cluster of dozens of BPs in Ca II H may appear as single features in EUV/X-ray images. Therefore, we performed identification and statistics on Ca II H images.

2.2 Bright Point Identification

We developed a 2D region-labeling algorithm for data compression and BP identification. The procedure involved: 1. Removing corrupted images and correcting bad pixels 2. Aligning images using fast Fourier transform cross-correlation 3. Selecting continuous observation periods 4. Applying brightness-based thresholds to mark high-intensity pixel clusters 5. Using erosion-dilation operations to isolate and smooth BP regions 6. Measuring scale L, area A, and light flux ϕ for each BP

Scale L is defined as the square root of the minimum bounding rectangle area. The BFI diffraction limit of 0 .2 allows detection of micro-BPs. The processed time series contained 29,968 BPs with scales ranging from 0 .21 to 76 .11, showing total light flux variations spanning orders of magnitude at an average density of 24.15 ± 6.34 BPs per square arcminute.

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3. Analysis of Instantaneous Bright Point Distribution Characteristics

3.1 Power-Law Characteristics of Instantaneous Distributions

The instantaneous frequency distribution of bright point scales follows a power law. Despite order-of-magnitude variations in activity level, the dispersion coefficient $\sigma(\delta)$ of each frame' s distribution remains limited. In log-log coordinates, the distribution aligns well with linear regression results [FIGURE:2(a)]. The dispersion coefficients $\sigma(\delta)$ and $\sigma(\zeta)$ follow normal distributions, indicating that instantaneous frequency distributions obey power laws within error margins.

This self-similarity manifests as approximately parallel fitted lines across frames. The power-law index γ shows only ~0.80% variation between active and quiet moments, with an approximately normal distribution [FIGURE:2(b)]. Even when total light flux differs by 7% between active and quiet states, the frequency distributions are parallel curves with similar γ values (0.814 vs. 0.876) [FIGURE:1(c)].

The instantaneous sample capacity n correlates with total light flux Φ [FIGURE:2(b)], though large flares cause deviations. Table 2 summarizes power-law parameters for various samples.

3.2 Stability of Small-Scale Bright Point Production

A key feature of power-law distributions is rapidly decreasing frequency with increasing scale, while production rate fluctuations increase with scale. We quantified stability using four metrics for scale-grouped BPs: 1. Average number density d 2. Fraction of total sample capacity R_n 3. Instantaneous occurrence rate R_{occ} (fraction of frames containing BPs of given scale) 4. Signal-to-noise ratio $R_{S/N}$ (mean/standard deviation in 1-hour windows)

Table 3 shows that BPs with $L \leq 1$ appear in 92.16% of frames with $R_{S/N} = 9.92$, while larger BPs ($L \geq 10$) have $R_{S/N} = 0.03$–0.07. Small-scale BPs produce stable, continuous energy input, contributing 53.23% of total light flux despite their small individual sizes. Considering BFI' s low temporal resolution and high miss rate for short-lived events [22], the actual contribution may be higher.

3.3 Temporal Evolution of Bright Point Distributions

Small-scale BPs show clear decay trends. The relative growth rate R_g (slope of number density vs. time) is negative for all scales $L < 4$ [FIGURE:3(a)], consistent with the overall active region decay. This may be partially due to limb darkening but aligns with the region' s declining activity.

The active region's decay manifests as a monotonic decrease in the instantaneous scaling index α [FIGURE:4(a)], consistent with decreasing sample capacity. The power-law index γ shows a sawtooth pattern: gradually increasing during quiet periods but dropping sharply during major eruptions [FIGURE:4(b)]. This occurs because large-scale BPs temporarily dominate the distribution.

3.4 Correlations Between Instantaneous Distribution Parameters

Analysis reveals intrinsic relationships between distribution parameters: - γ is inversely proportional to instantaneous total light flux Φ [FIGURE:5(c)] - γ correlates linearly with sample capacity n - α and γ are independent parameters - These relationships persist in low-noise samples (excluding $L > 8$ BPs), confirming they are not statistical artifacts

Solar activity not only produces medium- and large-scale BPs but also globally alters the distribution pattern across all scales, causing the observed γ reduction during active periods.

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4. Cumulative Sample Distribution and Evolution

The complete bright point sample follows a power-law distribution with dispersion coefficient $\sigma(\zeta) = 4\%$ [FIGURE:6(a)]. The observed and low-noise samples have $\gamma = 1.97$ and 2.12, respectively . However, γ does not converge with observation duration D [Figure 7: see original paper]. Instead, it exhibits sawtooth variations: gradual increases during quiet periods interrupted by sharp drops when large events occur.

Using low-noise samples reduces these fluctuations, yielding more stable γ values less sensitive to observation period selection [FIGURE:7(b)]. This suggests that while cumulative γ has limited meaning for the full sample, the low-noise sample provides more reliable characterization of the underlying distribution.

The area A and light flux ϕ distributions follow binomial rather than power-law forms [FIGURE:6(b,d)], with systematic deviations at both distribution ends. The right-end deviations (large BPs) stem from limited sample sizes and possible exponential rather than power-law behavior. The left-end deviations (small BPs) reflect instrumental under-sampling due to the 120 s cadence and resolution limits.

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5. Discussion and Future Work

5.1 Instrumental and Physical Effects

Distribution deviations arise from both physical and observational factors: - **Large-BP end**: Limited statistics of major eruptions in single active regions; potential blending of adjacent features; flux underestimation due to saturation - **Small-BP end**: High miss rates for short-lived events; resolution limitations; "large-flare effect" where nearby small BPs are obscured

Higher temporal/spatial resolution observations are needed to accurately characterize small-scale BP distributions and their contribution to coronal heating.

5.2 Future Directions

1. **High-resolution observations**: Use data with higher cadence and smaller diffraction limits to reduce under-sampling of small-scale events
2. **Magnetic field comparison**: Correlate BP distributions with photospheric magnetic field evolution to understand energy transport
3. **Multi-wavelength studies**: Compare BP distributions across atmospheric layers to reveal how energy release mechanisms vary with height
4. **Full-disk statistics**: Extend analysis to full-disk, long-term observations to study solar cycle dependence
5. **Quiet Sun comparison**: Investigate why quiet Sun photospheric BPs follow log-normal distributions while active region chromospheric BPs follow power laws

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6. Conclusions

Using a 2D region-labeling algorithm on Hinode/SOT Ca II H observations of NOAA AR 10930, we identified 29,968 low-chromospheric bright points. Key findings:

1. **Power-law behavior**: Bright point scales follow power-law distributions ($\gamma = 1.97$ for full sample, $\gamma = 2.12$ for low-noise sample) with self-similarity across instantaneous subsets
2. **Energy contribution**: Small-scale BPs (< 4) contribute 53.23% of total light flux, representing a stable energy source for upper atmosphere heating
3. **Temporal evolution**: The power-law index γ shows sawtooth variations, increasing during quiet periods and dropping during eruptions
4. **Scale dependence**: Production rates and signal-to-noise ratios decrease with increasing BP scale
5. **Global influence**: Solar activity affects the entire distribution pattern, not just producing large-scale events

The inverse relationship between γ and total light flux is intrinsic to small- and medium-scale BPs, not a statistical artifact of large-event injection. These results provide observational constraints for coronal heating mechanisms and demonstrate the need for high-resolution, multi-wavelength studies to fully understand energy release in the solar atmosphere.

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Note: Figure translations are in progress. See original paper for figures.