

## An Enhanced Positioning Method Using Velocity-Smoothed Elevation (Postprint)

**Authors:** Liu Cheng, Li Fang

**Date:** 2017-09-26T00:00:00+00:00

### Abstract

To enhance the positioning accuracy and continuity of Global Navigation Satellite Systems (GNSS), this study proposes a combined filter constructed by integrating the elevation coordinate component with the velocity component. The velocity measurements are employed to continuously smooth and correct the user elevation, yielding high-precision elevation coordinate values that subsequently establish elevation constraint conditions for the measurement equation set, thereby improving the three-dimensional positioning accuracy of GNSS as well as the receiver clock bias solution precision. Furthermore, owing to the high accuracy and favorable smoothness characteristics of the elevation value sequence processed by the filter, it is particularly amenable to fitting and extrapolation, thus providing enhanced assistance in non-complete constellation scenarios with satellite signal deficiencies and enabling continuous three-dimensional positioning. Finally, the methodology is analyzed through Global Positioning System (GPS) field experiments, validating its feasibility and practicality.

### Full Text

## An Enhanced Positioning Method Based on Height Smoothing with Velocity

**Liu Cheng<sup>1</sup>, Li Fang<sup>2</sup>**

<sup>1</sup>Beijing Institute of Tracking and Telecommunication Technology, Beijing 100094, China, Email: [liucheng@beidou.gov.cn](mailto:liucheng@beidou.gov.cn)

<sup>2</sup>Institute of Physics, Chinese Academy of Sciences, Beijing 100190, China

### Abstract

To improve the accuracy and continuity of Global Navigation Satellite System (GNSS) positioning, this paper proposes a combined filter designed using

both height coordinate components and velocity components. Velocity measurements are used to continuously smooth and correct user height, obtaining high-precision height coordinate values that form height constraints for the measurement equation system, thereby improving three-dimensional positioning accuracy and receiver clock bias solution precision. Meanwhile, due to the high precision and favorable smoothness of the height value sequence after filter processing, it is particularly suitable for fitting and extrapolation, thus better assisting incomplete constellation scenarios with missing satellite signals to achieve continuous three-dimensional positioning. Finally, the method's feasibility and practicality are analyzed and verified through GPS field experiments.

**Keywords:** Enhanced positioning method; Global Navigation Satellite System; Satellite navigation and positioning; Height constraint; Incomplete constellation

## 1. Introduction

Affected by satellite constellation geometry, GNSS positioning accuracy in the vertical direction is generally poor, with single-frequency coarse code positioning errors typically ranging from several meters to tens of meters, and even reaching tens of meters under signal occlusion or poor positioning conditions. As part of the same least-squares solution, the height solution correlates with other unknowns, and height coordinate errors also affect the two-dimensional horizontal coordinates and receiver clock bias solution precision. Poor height positioning accuracy has long been a major issue in satellite navigation systems.

To address this problem, scholars have conducted extensive research, hoping to obtain more accurate height values through other methods to improve height positioning errors, thereby forming height constraints for the measurement equation system to improve two-dimensional horizontal and receiver clock bias solution precision. In China's BeiDou-1 satellite positioning system and CAPS (China Area Positioning System), digital elevation models (DEM) and electronic maps were used to obtain user orthometric height [1-3]. In the China Area Positioning System, barometric altimetry was also used to assist the then-incomplete satellite constellation in achieving three-dimensional positioning [4-6]. Further proposals suggested applying barometric altimetry technology to ground mobile communication base stations, using barometric correction reference points and communication transmission links to deliver relevant measurement data to nearby users for correcting high-precision absolute height [7]. According to the high correlation between height coordinate components and receiver clock bias, reference station-based differential techniques were used to simulate and predict receiver clock bias, adding an auxiliary equation for receiver clock bias variation to the positioning equation system to reduce vertical dilution of precision and improve height positioning accuracy [8-9]. Reference [10-11] proposed using receiver clock bias prediction to assist height positioning.

Although these methods can improve height errors to varying degrees, they require additional facilities or means, increasing investment and cost while also

increasing system complexity. This paper proposes a method that effectively improves height positioning accuracy using GNSS's own positioning information without requiring any additional equipment. By introducing height direction velocity measurements, a filter of certain length is designed to continuously smooth user height in real-time using the latest measurements. Even when height coordinate component positioning errors are large, height direction velocity values can have high precision, so the filter can effectively obtain high-precision real-time user height values. This method can be used for both normal positioning and for incomplete constellation scenarios with brief satellite signal loss to help achieve three-dimensional positioning. Due to the high correlation between satellite positioning height coordinate components and receiver clock bias [11], when height component precision improves, receiver clock bias solution precision can also improve accordingly.

## 2. Principle and Method of Velocity Smoothing Height

In applications such as vehicle and ship positioning, user motion conforms to Newton's laws of inertia, and height position changes are generally slow and smooth. Treating each epoch's positioning process independently and using single-point positioning results as final outputs is clearly not optimal. Since height coordinate components can have high precision even when positioning errors are large, these two components can be combined: using height solution values to obtain accurate absolute starting values, and using velocity solution values to obtain accurate height variation quantities, thus forming a combined filter.

This concept is similar to carrier phase smoothing of pseudorange measurements in satellite navigation [12-13], where high-precision relative measurements correct less accurate absolute measurements to improve measurement precision.

**2.1 Velocity Smoothing Height Filter Design** For a user's height at the latest epoch  $n$ , the previous  $l$  height solution values  $H_n = 1, 2, \dots, l$  can be used to obtain  $l$  height estimates  $\hat{H}_n = 1, 2, \dots, l$ :

$$\hat{H}_n = H_n - \sum_{i=1}^l v_{n-i} \cdot T$$

where  $v_{n-i}$  is the height direction velocity component and  $T$  is the positioning time interval. Based on this series of estimates, the smoothed height value  $H_n$  at epoch  $n$  can be obtained:

$$H_n = \frac{1}{l} \sum_{i=1}^l \hat{H}_{n-i}$$

If considering the actual positioning solution result  $H_n$  at current epoch  $n$ , the above method can first be used to obtain  $\hat{H}_n$ , and then a weighted average method can be applied:

$$H_n = \frac{1}{l+1} \left[ H_n + \sum_{i=1}^l \hat{H}_{n-i} \right]$$

Here,  $l$  is the filter length. The larger the  $l$  value, the more  $H_n$  depends on velocity values, and the smoother  $H_n$  becomes. Equation (3) represents the general velocity smoothing height filter.

Regarding when to activate the filter, there are two cases: first, waiting until the number of positioning epochs reaches the filter length  $l$  to obtain accurate smoothed values  $H_n$  before normal filter operation begins; second, activating the filter from the beginning of positioning, initializing  $H_n$  values using normal positioning solutions, and continuously updating  $H_n$  values as epochs increase until reaching filter length  $l$ . The latter approach allows filter application from the start, but due to fewer epochs initially, larger error values may affect early results, causing some bias. The filter requires operation over a period and reaching the ideal design length to gradually stabilize and eliminate this bias.

**2.2 Filter Accuracy Analysis** From equation (3), velocity smoothing height filter errors mainly come from two aspects: first, remaining errors after averaging actual height solution values from each epoch; second, accumulated errors from velocity solution values within filter length  $l$ .

Assuming the mean square error of each height coordinate component is  $\sigma_H$ , and the mean square error of height velocity component is  $\sigma_v$ , according to error propagation law, the mean square error  $\sigma_{H_n}$  of  $H_n$  is:

$$\sigma_{H_n}^2 = \frac{1}{l} \sigma_H^2 + \frac{(l+1)(l+2)}{6} T^2 \sigma_v^2$$

Currently, GNSS height velocity error  $\sigma_v$  is generally less than 0.3 m/s, and receiver positioning interval  $T$  is about 0.1-1 s. Therefore, the influence of velocity error on the right side of equation (6) is very small and can be approximated as:

$$\sigma_{H_n}^2 \approx \frac{1}{l} \sigma_H^2$$

Thus, a velocity smoothing height filter of length  $l$  can reduce height coordinate component mean square error to approximately  $1/\sqrt{l}$  of the original value.

### 3. Velocity Smoothing Height Positioning Method

**3.1 Enhanced Positioning Method for Normal Constellation** When visible satellites are more than four, there are two positioning algorithms: the original algorithm treats height as an unknown, while the enhanced algorithm treats the smoothed height value  $H_n$  from the filter as a known quantity, reformulating the measurement equation system. The method flow is shown in [Figure 1: see original paper].

[Figure 1: see original paper]

The enhanced algorithm effectively reduces one unknown, forming a height constraint condition for the original measurement equations:

$$\sqrt{(x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2} + c\Delta t = \rho_i \quad (i = 1, 2, \dots, n)$$

where  $x, y, z$  and  $\Delta t$  are unknowns,  $h$  is user altitude,  $H_n$  is orthometric height after simple coordinate conversion,  $x_i, y_i, z_i$  are coordinates of satellite  $i$  in the Earth-centered Earth-fixed coordinate system, and  $\rho_i$  is the pseudorange observation from satellite  $i$  to the user. Specific solutions for equation (8) can be found in references [15-18].

**3.2 Assisted Positioning Method for Incomplete Constellation** When visible satellites are fewer than four, it is called an incomplete constellation scenario. Without other auxiliary means or information, users cannot complete three-dimensional positioning. However, the height positioning values from slightly earlier epochs can be extrapolated to obtain height prediction values for incomplete constellation scenarios, transforming the positioning solution problem into a two-dimensional one.

The main advantage of using smoothed height value sequences for extrapolation, rather than directly modeling and extrapolating height positioning values, is that filtered height values are more accurate: (1) Since the smoothed height sequence curve is more obvious, modeling and prediction can achieve higher accuracy; (2) Because each epoch's height values are smoothed through a sufficiently long window, gross errors from observation data mutations can be effectively avoided and overcome, significantly improving robustness; (3) Less prior data can be used in fitting and extrapolation, reducing computation.

The effectiveness of predicted height in assisting incomplete positioning largely depends on the user's motion environment. If height changes slowly, predicted height can assist incomplete positioning for a long time, and simple mean or weighted mean processing of prior height value sequences can achieve good positioning results. However, if height changes dramatically, predicted height will struggle to reflect real height changes, limiting method effectiveness.

#### 4. Experiments and Analysis

Using a satellite navigation receiver module developed by Zhongke Microelectronics Co., Ltd., GPS satellite broadcast ephemeris and single-frequency C/A code pseudorange measurement data in a custom binary format were output via serial port at 1 Hz. Real-time positioning experiments and verification were conducted based on this module and its output data.

To more clearly analyze and compare filter and positioning method performance under normal constellation conditions, long-term static measurements were performed at a known coordinate point with certain occlusion. Height coordinate values for all observation epochs were directly solved, and a 100-second velocity smoothing height filter was applied. The results are shown in [Figure 2: see original paper].

[Figure 2: see original paper]

The height after velocity smoothing is smoother and more consistent with the true height value. Based on this, enhanced positioning method solutions were performed according to the flow in [Figure 1: see original paper] and compared with original positioning results. As shown in the errors in [Figure 3: see original paper], the enhanced algorithm reduces original two-dimensional positioning error by about 30%, demonstrating significant effect due to high correlation between height coordinate components and receiver clock bias. The receiver clock bias sequence in [Figure 3: see original paper] is also smoother and more consistent with actual patterns, showing that more accurate height indeed yields more accurate receiver clock bias values, thereby helping improve receiver timing precision.

[Figure 3: see original paper]

#### 5. Conclusion

Affected and constrained by constellation geometry, height positioning accuracy has long been a major shortcoming of GNSS. Previous proposals using electronic map matching, barometric altimetry, and reference station differential correction have sought to obtain more accurate height values to improve GNSS height and three-dimensional positioning accuracy. However, these methods typically require additional hardware at the user or reference end, increasing cost and system complexity. Since electronic maps and barometric altimetry can generally only improve user height positioning accuracy to the meter level, they are only applicable to general civilian fields using single-frequency coarse code. When differential or carrier phase-based high-precision positioning is employed and GNSS height positioning accuracy is already better than one meter, these methods become difficult to improve further.

Similar to carrier phase smoothing of pseudorange, high-precision relative measurements in satellite positioning can always correct and improve absolute measurement accuracy. Based on this concept, this paper proposes combining height

coordinate components with velocity components, using high-precision velocity solutions to smooth and correct height positioning values. The resulting combined filter is simple and standardized, requiring no additional equipment. Through a short smoothing process, high-precision height coordinates can be obtained, demonstrating strong engineering practicality. Since velocity components can also have relatively higher precision when height component precision improves, the method is applicable to various precision requirements.

Field experiments and analysis prove that this method can significantly improve GNSS height and three-dimensional positioning accuracy. The height value sequence after filter processing has high precision and good curve smoothness, making it suitable for fitting and extrapolation. Due to high correlation between satellite positioning height coordinate components and receiver clock bias, when height precision improves, receiver clock bias solution precision also improves accordingly, enabling higher timing precision. This method can enhance and assist GNSS in multiple aspects, demonstrating good practical effects and application value.

## References

- [1] Balachandran W. Altitude aiding for GPS systems using elevation map. Proceedings of the Land/Vehicular Technology Conference, 1989: 451-462.
- [2] French R L. Map matching origins approaches and applications. Proceedings of the Land/Vehicular Technology Conference, 1986: 91-116.
- [3] Zavoli W B, Honey S K. Map matching augmented dead-reckoning. Proceedings of the Land/Vehicular Technology Conference, 1986: 359-362.
- [4] Ai Guoxiang, Shi Huli, Wu Haitao, et al. The principle of the positioning system based on communication satellites. Science in China Series G: Physics, Mechanics and Astronomy, 2009: 472-488.
- [5] Wu Haitao, et al. The principle of the positioning system based on communication satellites and the Chinese Area Positioning System (CAPS). Science in China Series G: Physics, Mechanics and Astronomy, 2009: 1615-1633.
- [6] Ai Guoxiang, Sheng Peixuan, Du Jinlin. A positioning system based on communication satellites and the Chinese Area Positioning System (CAPS). Chinese Journal of Astronomy and Astrophysics, 2008: 611-630.
- [7] Du Xiaohui, Pei Jun, Zhang Lirong, et al. Barometric altimetry system applied in mobile phone base stations as virtual constellation. Science in China Series G: Physics, Mechanics and Astronomy, 2012: 434-444.
- [8] Zhang Lirong, Shi Huli, Deng Zhongliang, et al. Differential barometric altimetry method based on ground mobile communication base stations. Journal of Beijing University of Aeronautics and Astronautics, 2011: 83-88.

- [9] Based on reference station barometric relative measurement to obtain precise height: China, 201110082773.5. 2011-05-20.
- [10] Misra P, Pratt M, Burke B, et al. Adaptive modeling of receiver clock for meter-level vertical positioning. Proceedings of the 8th International Technical Meeting of Satellite Division of The Institute of Navigation, 1995: 1127-1135.
- [11] Misra P. GPS principles and receiver design. Beijing: Publishing House of Electronics Industry, 2011.
- [12] Hatch R. The synergism of GPS code and carrier measurements. Proceedings of the Third International Geodetic Symposium on Satellite Doppler Positioning, 1982.
- [13] Eng P. Global Positioning System-signals measurements and performance.
- [14] Wuhan University Surveying Adjustment Discipline Group. Error theory and surveying adjustment foundation. Wuhan: Wuhan University Press.
- [15] Kong Xiangyuan, Guo Jiming, Liu Zongquan, et al. Geodesy foundation. Wuhan: Wuhan University Press.
- [16] Qiao Yangwen, Xin Jiuzhi, Wang Xiaohui, et al. A study on some problems of GPS height transformation. Bulletin of Surveying and Mapping, 2003: 17-19.
- [17] Shi Huli, Sun Xiyan, Li Zhigang. Transponder satellite navigation principle. Beijing: Science Press.
- [18] Solutions of navigation observation equations for CAPS. Science in China Series G: Physics, Mechanics and Astronomy.

*Note: Figure translations are in progress. See original paper for figures.*

*Source: ChinaXiv –Machine translation. Verify with original.*