

Effect of Low-Energy Cutoff Behavior on Temperature-Anisotropy-Driven Cyclotron Maser Radiation (Postprint)

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Date: 2017-09-26T00:00:00+00:00

Abstract

Cyclotron maser radiation excited by high-energy electrons trapped in celestial magnetic fields is an important mechanism for astrophysical radio emission, widely applied to explain various non-thermal radio emission phenomena, particularly coherent radio burst phenomena on various timescales. In previous studies, the source of maser radiation excitation has been the anisotropic velocity distribution of high-energy electrons. However, observations show that high-energy electrons in the Sun and other celestial bodies often exhibit power-law spectral distributions with low-energy cutoff behavior. We calculated the growth rates of cyclotron maser instability driven by temperature anisotropy distributions and by temperature anisotropy distributions with low-energy cutoff behavior. The results demonstrate that the low-energy cutoff behavior of power-law spectrum electrons has an important impact on cyclotron maser radiation.

Full Text

Effects of Low-Energy Cutoff Behavior on Temperature-Anisotropy-Driven Cyclotron Maser Radiation

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Abstract

Electron cyclotron maser emission (ECME) excited by energetic electrons trapped in astrophysical magnetic fields represents an important mechanism for astrophysical radio emission and has been widely applied to explain various nonthermal radio phenomena, particularly short-timescale coherent radio

burst phenomena. Previous studies have assumed that the source of maser radiation is the anisotropic velocity distribution of energetic electrons. However, observations show that energetic electrons in the Sun and other celestial bodies often exhibit power-law spectral distributions with low-energy cutoff behavior. This work calculates the growth rates of cyclotron maser instability driven by temperature anisotropy distributions and by temperature anisotropy distributions with low-energy cutoff behavior. The results demonstrate that the low-energy cutoff behavior of power-law electrons has a significant impact on cyclotron maser radiation.

Keywords: Radio emission mechanism; Electron cyclotron maser emission; Temperature anisotropy; Low-energy cutoff behavior

1. Introduction

Electron cyclotron maser emission is a crucial radiation mechanism in which energetic electrons trapped in astrophysical magnetic fields directly amplify electromagnetic waves. The cyclotron maser instability can directly amplify electromagnetic waves near the electron cyclotron frequency and its harmonics. This mechanism has been extensively applied to interpret various coherent radio burst phenomena.

The theoretical foundation for ECME was established through several key developments. Twiss (1958) first proposed the possibility of negative absorption in radio astronomy. Schneider (1959) demonstrated stimulated emission of radiation by relativistic electrons in a magnetic field. Bekefi (1960s) applied these concepts to explain decameter radiation from Jupiter. Wachtel (1970) experimentally demonstrated electron cyclotron maser radiation using relativistic electrons in the laboratory. Melrose (1976) interpreted both Jupiter's decametric radiation and terrestrial kilometric radiation as direct amplified gyroemission. The breakthrough came in 1979 when Wu & Lee introduced weak relativistic effects into the resonance condition, which qualitatively changed the physical picture of resonance excitation and dramatically increased the amplification efficiency of the instability. Since then, electron cyclotron maser radiation has become a cornerstone mechanism for explaining various coherent radio burst phenomena in astrophysics.

Early work on ECME made two fundamental assumptions: (1) the electron plasma frequency is much smaller than the cyclotron frequency, and (2) energetic electrons have a loss-cone distribution. However, further calculations revealed that the growth rates driven by loss-cone distributions remain too low to explain observed radiation intensities. Consequently, researchers proposed various alternative electron velocity anisotropy distributions, including horseshoe distributions, electron hole distributions, ring-shell distributions, and ring-beam coupled distributions, depending on the specific magnetized plasma environment.

Temperature anisotropy represents a common distribution in magnetized plas-

mas. Classical double-adiabatic theory predicts that solar wind electrons moving along spiral magnetic field lines develop large temperature ratios T_{\perp}/T_{\parallel} . Observations confirm that solar wind electron temperature anisotropy is limited near 1 due to kinetic instabilities, with the temperature ratio increasing with distance from the Sun. Although in-situ measurements of charged particles and electromagnetic fields are limited to regions near Earth, we can infer that temperature anisotropy distributions with ratios of 1.5 to 4 likely exist in magnetized astrophysical plasmas such as the solar corona and accretion disks.

Previous studies of ECME have primarily focused on various strongly anisotropic distributions of energetic electrons as the driving source. However, observations demonstrate that energetic electrons in solar flares and other astrophysical sources frequently exhibit power-law spectral distributions with low-energy cutoff. Hard X-ray spectral analysis of flare emissions reveals that energetic electrons produced by flare bursts mostly display power-law distributions with low-energy cutoff, though the specific cutoff behavior is difficult to determine observationally.

Recent work has introduced a steepness index parameter δ to describe the general form of the low-energy cutoff behavior of power-law electrons through a continuous hyperbolic tangent function. Considering that energetic electrons develop velocity-space anisotropy in different magnetized plasma environments, the complete distribution function should represent an appropriate combination of power-law spectrum with low-energy cutoff and velocity anisotropy distribution. Cyclotron maser radiation is thus excited jointly by low-energy cutoff behavior and velocity anisotropy.

This paper investigates the effects of low-energy cutoff behavior on temperature-anisotropy-driven maser radiation by calculating the growth rates of cyclotron maser instability driven by temperature anisotropy distributions and by temperature anisotropy distributions with low-energy cutoff behavior. The results show that the presence of low-energy cutoff can effectively excite cyclotron maser instability even for weak temperature anisotropy.

2. Electron Distribution and Growth Rate Expressions

2.1 Distribution Functions

In spherical coordinates, we first consider a bi-Maxwellian velocity distribution:

$$F_b = \frac{1}{\pi^{3/2} \alpha_{\perp}^2 \alpha_{\parallel}} \exp \left[-\frac{u_{\perp}^2}{\alpha_{\perp}^2} - \frac{(u_{\parallel} - U)^2}{\alpha_{\parallel}^2} \right]$$

where $u = p/m$ is the momentum per unit mass, u_{\perp} and u_{\parallel} are the perpendicular and parallel components of u , α_{\perp} and α_{\parallel} are the thermal velocities in the perpendicular and parallel directions respectively, and U is the beam velocity in the parallel direction.

To incorporate the low-energy cutoff behavior of energetic electrons, we consider a power-law distribution with low-energy cutoff:

$$F_b = A \frac{\tanh[(u/u_c)^\delta]}{(u^2 + 1)^{\alpha/2}}$$

where A is the normalization coefficient, the hyperbolic tangent function $\tanh[(u/u_c)^\delta]$ describes the low-energy cutoff behavior, δ and α are the steepness index and energy spectral index respectively, u_c is the cutoff momentum, and $\gamma = \sqrt{1 + u^2/c^2}$ is the Lorentz factor.

Combining electron temperature anisotropy with the power-law distribution, the complete distribution function becomes:

$$F_b = A \frac{\tanh[(u/u_c)^\delta]}{(u^2 + 1)^{\alpha/2}} \exp \left[-\frac{(u_{\parallel} - U)^2}{\alpha_{\parallel}^2} \right]$$

2.2 Dispersion Relation and Growth Rate

Assuming the background thermal electron density is much higher than the energetic electron density, we can neglect energetic electrons when discussing the wave dispersion relation. For high-frequency electromagnetic waves, the dispersion relation can be approximated using cold plasma theory:

$$N_q^2 = \frac{\omega_q^2 + \tau_q \Omega^2}{\omega_q^2 - \tau_q \Omega^2}$$

where the subscript q represents the wave mode, ω_q is the radiation wave frequency, θ is the angle between the wave vector and magnetic field direction, Ω is the electron cyclotron frequency normalized by the plasma frequency, and $\tau_q = -s_q + \sqrt{s_q^2 + 4\omega_q^2 \Omega^2 / 2\omega_q \Omega}$ with $s_q = \omega_q^2 - \omega_p^2 - \Omega^2$. The indices $q = O, X$ represent the ordinary mode and extraordinary mode respectively.

The wave polarization vector \mathbf{a} can be expressed as:

$$\mathbf{a} = \frac{K_q \hat{\mathbf{k}} + T_q \hat{\mathbf{t}} + i \hat{\mathbf{p}}}{\sqrt{K_q^2 + T_q^2 + 1}}$$

where $\hat{\mathbf{k}} = (\sin \theta, 0, \cos \theta)$, $\hat{\mathbf{p}} = (0, 1, 0)$, and $\hat{\mathbf{t}} = \hat{\mathbf{p}} \times \hat{\mathbf{k}} = (\cos \theta, 0, -\sin \theta)$.

The growth rate expression for waves, assuming the wave growth rate ω_{qi} is much smaller than the wave frequency ω_q and neglecting background electron damping, is given by:

$$\omega_{qi} = - \frac{\text{Im}(\mathbf{a}^* \cdot \varepsilon \cdot \mathbf{a})}{\partial \text{Re}(\mathbf{a}^* \cdot \Lambda \cdot \mathbf{a}) / \partial \omega} \Big|_{\omega=\omega_q}$$

where ε and Λ are the dielectric tensor and dispersion tensor respectively, and \mathbf{a}^* is the complex conjugate of the polarization vector.

Using the cold plasma dispersion relation, this can be written as:

$$\omega_{qi} = \frac{\omega_q}{2} \frac{\text{Im}(\mathbf{a}^* \cdot \varepsilon \cdot \mathbf{a})}{\text{Re}(\mathbf{a}^* \cdot \Lambda_0 \cdot \mathbf{a})} \Big|_{\omega=\omega_q}$$

where $\Lambda_0 = \partial(\omega^2 \Lambda) / \partial \omega^2$.

Through calculation we obtain:

$$\text{Re}(\mathbf{a}^* \cdot \Lambda_0 \cdot \mathbf{a}) = \frac{1}{N_q^2} \left[\frac{\omega_{pe}^2 \Omega \tau_q}{\omega_q (\omega_q + \tau_q \Omega)} + \frac{s_q^2 + \omega_q^2 + \omega_{pe}^2}{\omega_q^2 - \omega_{pe}^2} \right]$$

The imaginary part of the dielectric tensor is:

$$\text{Im}(\mathbf{a}^* \cdot \varepsilon \cdot \mathbf{a}) = - \frac{\omega_{pe}^2 n_b}{\omega_q^2 n_0} \frac{\pi}{N_q \sin \theta} \int_0^\infty u_\perp^2 du_\perp \int_{-\infty}^\infty du_\parallel \sum_{n=-\infty}^\infty \delta(\gamma - N_q u_\parallel \cos \theta - n \Omega / \omega_q) \times \left[\frac{n \Omega}{\omega_q} K_q + \left(\frac{u_\parallel}{u_\perp} - N_q \cos \theta \right) \right]$$

where n_0 and n_b are the background plasma electron density and energetic electron density respectively, ω_{ce} is the electron cyclotron frequency, and J_n and J'_n are the Bessel functions of the first kind and their derivatives.

From equations (9)-(11), the wave frequency satisfies $\omega \approx \omega_q$, and the growth rate can be expressed as:

$$\frac{\omega_{qi}}{\omega_{ce}} = \Omega \frac{\omega_q (1 + T_q^2)}{N_q^2 (K_q^2 + T_q^2 + 1)} \frac{n_b}{n_0} \frac{\pi}{N_q \sin \theta} \int_0^\infty u_\perp^2 du_\perp \int_{-\infty}^\infty du_\parallel \sum_{n=-\infty}^\infty \delta(\gamma - N_q u_\parallel \cos \theta - n \Omega / \omega_q) \times \left[\frac{n \Omega}{\omega_q} K_q + \left(\frac{u_\parallel}{u_\perp} - N_q \cos \theta \right) \right]$$

3. Growth Rate Calculation Results

Under the weakly relativistic approximation and cold plasma assumption, substituting the energetic electron distribution function into the growth rate expression yields the results. All velocities are normalized by the speed of light.

[Figure 1: see original paper] shows the relationship between the peak growth rate of cyclotron maser radiation driven by temperature anisotropy and the

propagation angle for a bi-Maxwellian distribution. The different curves represent the ordinary mode fundamental wave and extraordinary mode harmonic wave. The energetic electron parameters and background plasma parameters are indicated in the figure. Frequencies are normalized by the plasma frequency, and growth rates are normalized by $\omega_{ce}n_b/n_0$.

The results indicate that temperature anisotropy can excite cyclotron maser instability, with stronger anisotropy producing higher growth rates. However, calculations show that only when the temperature anisotropy is sufficiently strong can cyclotron maser radiation be excited. For example, when $T_{\perp} = 0.5$ and $T_{\parallel} = 0.2$, the instability growth rate is essentially zero, meaning that only strong temperature anisotropy can drive instability growth.

[Figure 2: see original paper] illustrates the relationship between the growth rate and propagation angle when both low-energy cutoff behavior and temperature anisotropy drive the cyclotron maser instability. The different curves represent different steepness indices δ . The results demonstrate that even for weak temperature anisotropy that originally could not excite cyclotron maser radiation, the presence of low-energy cutoff can excite the instability, with growth rates increasing as the steepness index increases. For instance, when $T_{\perp} = 0.5$ and $T_{\parallel} = 0.2$, the low-energy cutoff can still effectively excite cyclotron maser radiation, significantly broadening the angular range of instability and shifting the propagation angle for maximum growth rate closer to the perpendicular direction.

[Figure 3: see original paper] further confirms that the combination of low-energy cutoff behavior and temperature anisotropy can drive cyclotron maser instability even under conditions where pure temperature anisotropy would be insufficient.

4. Conclusion

Electron cyclotron maser radiation represents an important mechanism for astrophysical radio emission and has been widely applied to explain various short-timescale, high-brightness-temperature coherent radio burst phenomena. Previous studies assumed that energetic electrons driving ECME have strongly anisotropic velocity distributions. However, observations show that energetic electrons often follow power-law spectral distributions.

This paper calculates the excitation of cyclotron maser instability by pure temperature anisotropy distributions and by temperature anisotropy distributions with low-energy cutoff behavior. While pure temperature anisotropy can excite cyclotron maser radiation, it requires relatively strong anisotropy. When energetic electrons exhibit low-energy cutoff behavior, even weak temperature anisotropy that would normally be insufficient to drive instability (such as $T_{\perp} = 0.5$, $T_{\parallel} = 0.2$) can effectively excite maser instability. Moreover, the angular range of instability increases substantially, and the propagation angle for maximum growth rate becomes more perpendicular to the magnetic field.

These results demonstrate that the low-energy cutoff behavior of energetic electrons plays a crucial role in cyclotron maser radiation and must be considered in comprehensive models of ECME in astrophysical environments.

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