Postprint of Astrometric Calibration Experiment for Saturn’ s Major Satellites

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Abstract

In the precise positioning of Saturn’ s major satellites observed with ground-based telescopes, CCD imaging technology is currently widely employed. To measure the positions of faint satellites in the field of view, 4 major satellites (Tethys to Titan) are commonly used for calibration. Given technological advancements, CCD fields of view are gradually increasing, making it worthwhile to investigate how to precisely calibrate images. An experimental study was conducted using 105 CCD images with different orientations observed on January 20, 2010, with the 1 m telescope at Yunnan Observatory. Calculations were performed using the JPL (Jet Propulsion Laboratory) ephemeris of Saturn’ s major satellites and the UCAC4 stellar catalog from the United States Naval Observatory for calibration relative to different reference objects. The results indicate that using 4 major satellites for calibration yields good measurement accuracy and internal precision when measuring objects relatively close to the satellites. When measuring satellites farther from the calibration stars, this calibration method exhibits significant uncertainty. Using 6 major satellites (Tethys to Iapetus) for calibration, the measurement results for all satellites show better external and internal consistency. Nevertheless, the calibration results using both four-parameter and six-parameter models are unsatisfactory, revealing that there are significant distortion effects in the CCD field of view. If stars in the field of view are used for calibration, the external consistency and internal precision of the satellites deteriorate significantly, indicating that the UCAC4 stellar reference catalog is also not an ideal reference catalog for calibration.

Full Text

Study on Methods for Analyzing Satellite Navigation Signal Stability

Abstract

The analysis of Global Navigation Satellite System (GNSS) space signals is a critical task during both system design and operation phases. This paper first presents a space signal stability analysis method based on observations from GNSS monitoring receivers. The feasibility of the method is then verified through digital simulation experiments. Finally, using measured pseudorange data from a BeiDou System (BDS) geostationary Earth orbit satellite as an example, signal stability is evaluated from three aspects: original pseudorange observations, pseudorange fitting residuals, and the standard deviation of pseudorange fitting residuals. The experimental results further validate the feasibility and effectiveness of the proposed method, which holds significance for ensuring the continuity and reliability of satellite navigation signals.

Keywords: Global Navigation Satellite System (GNSS); signal quality analysis; stability; pseudorange

1. Introduction

With the advancement of satellite navigation technology, major countries worldwide have successively developed their own global navigation satellite systems, including the U.S. Global Positioning System (GPS), Russia' s GLONASS, Europe' s Galileo, and China' s BeiDou System. The quality of GNSS space signals directly affects positioning, navigation, and timing (PNT) performance and serves as a crucial metric for evaluating system excellence. Moreover, signal quality reflects the in-orbit status of satellite payloads and various electrical performance parameters. Signal quality assessment technology provides essential support for signal design, on-orbit testing, on-orbit monitoring, fault diagnosis, and space signal integrity monitoring [1].

Given that GNSS space signal evaluation is vital during system design and operation, it has attracted significant attention globally and become a research hotspot in the satellite navigation field. Various GNSS systems have established dedicated space signal quality monitoring and analysis systems. For instance, GPS is monitored by the Stanford Research Institute using a large-aperture parabolic antenna for continuous real-time observation of GPS space signals [2]. Galileo' s signal characteristics are analyzed by the European Space Research and Technology Centre' s Navigation Laboratory, which employs an omnidirectional antenna, standard measurement instruments, and test receivers [3]. China' s BeiDou System is supported by the National Time Service Center of the Chinese Academy of Sciences, which has built a GNSS space signal analysis platform using standard measurement instruments, offline data evaluation, and monitoring receivers to assess signal quality for BeiDou and other GNSS signals

[1,4]. Offline data analysis evaluates short-term satellite status through baseband signal analysis [5] and long-term performance through monitoring receiver measurements [6-7].

This paper focuses on analyzing satellite navigation signal stability based on monitoring receiver observations.

2. Satellite Signal Composition

To understand the composition and characteristics of satellite navigation signals, we briefly describe the signals transmitted by GNSS satellites. Structurally, navigation satellite signals consist of three layers: ranging code, data code, and carrier [8]. Before transmission, the data code is modulated with the ranging code, and the resulting signal is then mixed with a sinusoidal carrier. At the receiver terminal, the GNSS receiver outputs two fundamental observables: pseudorange and carrier phase, based on the received satellite signals. Pseudorange derived from code measurements is called code-phase pseudorange, while that from carrier phase measurements is called carrier-phase pseudorange.

The observation equations for pseudorange (P) and carrier phase (L) can be expressed as:

$$P = \rho_s + c(\delta t_r - \delta t_s) + T + I + \varepsilon_P$$
$$L = \rho_s + c(\delta t_r - \delta t_s) + T - I + \lambda N_s + \varepsilon_L$$

where ρ_s represents the geometric distance from the satellite position at signal transmission time to the receiver position at signal reception time; c is the speed of light in vacuum; δt_r and δt_s are receiver and satellite clock errors, respectively; T and I are tropospheric and ionospheric refraction errors; λ is the signal wavelength; N_s is the integer ambiguity of carrier phase; and ε_P and ε_L represent pseudorange and carrier phase observation noise plus multipath effects, respectively.

3. Signal Stability Analysis Method Based on GNSS Receivers

GNSS space signal stability is generally assessed by evaluating the stability of pseudorange and carrier phase measurements from monitoring receivers. Signal stability is categorized as short-term (minutes, hours, or days) or long-term (months or years). Short-term stability is primarily affected by receiver momentary loss-of-lock and receiver clock jumps, causing sudden jumps in carrier phase and pseudorange observations that subsequently recover. Long-term stability mainly relates to satellite clock performance, such as frequency drift causing gradual changes in carrier phase and pseudorange measurements, reflecting overall signal drift [9].

This section introduces the GNSS signal stability analysis method. To evaluate the stability of carrier phase and pseudorange measurements, we analyze their curve fitting residuals. The process involves using a segment of carrier phase or pseudorange measurements for polynomial fitting to obtain residuals. Based on the least squares principle, polynomial fitting minimizes the sum of squared differences between fitted and original data. For given observations (x_i, y_i) $(i = 1, 2, \cdots, m)$, we seek an n-th order polynomial $f_n(x) = \sum_{j=0}^{n} a_j x^j$ that minimizes the fitting error $r_i = f_n(x_i) - y_i$ $(i = 1, 2, \cdots, m)$. The resulting $f_n(x)$ is called the least squares fitting polynomial, and r_i are the fitting residuals.

For carrier phase and pseudorange observations, slow-varying errors such as ionospheric and tropospheric refraction are largely eliminated through polynomial fitting. Thus, the residuals effectively reflect observation stability. The least squares fitting eliminates systematic errors while reducing random errors, yielding reliable results. The magnitude of residuals indicates the quality of polynomial fitting and is used to assess measurement stability.

The algorithm flowchart is shown in [Figure 2: see original paper]. The basic steps are: 1. Read raw GNSS monitoring receiver observations, including carrier phase and pseudorange values. 2. Plot carrier phase and pseudorange curves to identify jumps in the data. 3. Extract continuous observation segments of a certain duration for polynomial fitting to obtain residuals, ensuring minimal difference between original data and fitted curves. 4. Analyze observation stability based on the variation range of polynomial fitting residuals.

4. Feasibility Verification

4.1 Polynomial Fitting Simulation The curves of raw pseudorange and carrier phase observations over several days approximate sinusoidal or cosine functions. For generality, we simulate pseudorange observations using a cosine function with added Gaussian white noise. Cosine curves over one period ($0 \sim 2\pi$) are generated with added Gaussian white noise. Two segments are extracted: $0 \sim 0.6\pi$ and $0.7\pi \sim 1.3\pi$, both containing inflection points, for polynomial fitting.

Table 1 lists the standard deviations of polynomial fitting residuals under different Gaussian white noise levels. Even without added noise, the fitting residual standard deviation σ is very small, indicating good agreement between polynomial fitting results and original observations. Using the fitting residual standard deviation of noise-free data as reference, Gaussian white noise with standard deviations of 0.020, 0.040, and 0.050 is added to the cosine function. The resulting fitting residual standard deviations σ are very close or equal to the standard deviations of the added Gaussian white noise, demonstrating that polynomial fitting works well for noisy data. The remaining residuals generally represent random noise components mixed in the original data.

Table 1 Standard deviation of polynomial fitting residuals

Gaussian White Noise	$0 \sim 0.6\pi$ Segment	$0.7\pi \sim 1.3\pi$ Segment
$\sigma = 0.020$	0.001	0.001
$\sigma = 0.040$	0.002	0.002
$\sigma = 0.050$	0.024	0.044

To visually demonstrate polynomial fitting results, comparisons between noisy data curves and polynomial fitted curves are shown in [Figure 4: see original paper]. Although fitting at curve inflection points is suboptimal, the overall fitting residual is small, and the fitting residual standard deviation closely matches the added Gaussian white noise standard deviation. This indicates that curve inflection points have minimal impact on polynomial fitting effectiveness, and the method remains valid.

4.2 Observation Anomaly Simulation Digital simulation is used to analyze polynomial fitting performance under observation anomalies, including data gaps caused by receiver loss-of-lock and data jumps caused by satellite clock jumps. A one-period ($0 \sim 2\pi$) noisy cosine curve is simulated as carrier phase or pseudorange observations. Data missing points are set at $0.13\pi \sim 0.15\pi$, and data jump points at 0.76π. The data is divided into 10 segments at intervals of 0.2π to calculate polynomial fitting residual standard deviations for each segment under different noise conditions.

Table 2 shows the difference between fitting residual standard deviations and their mean values under two anomaly conditions. Only results for Gaussian white noise standard deviation of 0.050 are plotted. When observations are missing or anomalous, polynomial fitting residual standard deviations increase significantly, indicating that the polynomial fitting method fails when carrier phase or pseudorange observations contain anomalies. Therefore, normal observation data must be selected for signal assessment.

Table 2 Statistics of polynomial fitting in presence of observational anomaly

Gaussian White Noise	Standard Deviation Mean	Missing Data	Jump Data
$\sigma = 0.020$	0.048	0.176	0.205
$\sigma = 0.040$	0.065	0.161	0.188
$\sigma = 0.050$	0.074	0.154	0.184

5. Real Data Analysis

According to the signal stability assessment method, pseudorange measurements from a BDS geostationary Earth orbit satellite over 24 hours are analyzed. The monitoring receiver sampling interval is 30 seconds. [Figure 7: see original paper] shows the original pseudorange observation curves and polynomial fitting residual curves. The pseudorange observation curve is smooth without data gaps or jumps, and the polynomial fitting residual curve varies within a certain range with absolute values less than 0.5 m.

The 24-hour period is divided into eight 3-hour segments. The 06:00 ~ 09:00 segment shows poorer fitting residual performance compared to other periods. Table 3 presents the pseudorange fitting residual standard deviations and their deviations from the mean for each segment. The 06:00 ~ 09:00 segment has the largest standard deviation of 0.434 m, with a deviation of 0.183 m from the mean. The segment with the smallest deviation is 12:00 ~ 15:00, with a standard deviation of 0.218 m and deviation of -0.033 m.

Table 3 Statistics of pseudorange polynomial fitting residuals

Time Segment	Fitting Residual Standard Deviation (m)	Deviation from Mean (m)
00:00 ~ 03:00	0.193	-0.058
03:00 ~ 06:00	0.299	0.048
06:00 ~ 09:00	0.434	0.183
09:00 ~ 12:00	0.298	0.047
12:00 ~ 15:00	0.218	-0.033
15:00 ~ 18:00	0.203	-0.048
18:00 ~ 21:00	0.154	-0.097
21:00 ~ 24:00	0.212	-0.039

The larger residuals in the 06:00 ~ 09:00 segment occur because the pseudorange observations fall near an inflection point of the curve, where polynomial fitting is generally less effective than at other segments. However, this local fitting issue does not affect the overall fitting result, making the polynomial fitting results credible and usable. The mean standard deviation shows that the 06:00 ~ 09:00 segment did not impact the overall fitting result. Compared with the simulated anomaly results in Section 4, this segment' s fitting residual standard deviation is much smaller than when pseudorange anomalies occur, indicating that the larger residuals are due to segment division rather than signal stability fluctuations.

To further analyze the statistical properties of residuals, the 06:00 ~ 09:00 and 15:00 ~ 18:00 segments are compared. Theoretically, pseudorange fitting residuals should follow a zero-mean Gaussian distribution. However, satellite signal propagation through the troposphere and ionosphere affects the overall variance and mean. Table 4 provides the statistical parameters for these two segments.

Table 4 Statistical parameters of pseudorange polynomial fitting residuals

Time Segment	Sample Size	Standard Deviation (m)	Mean (m)
06:00 ~ 09:00	361	0.040	0.189
15:00 ~ 18:00	361	0.021	0.041

A 95% confidence interval for the difference between the two population means is estimated as [-0.358, 0.237]. Since this interval includes zero, there is no significant difference between the means of the two segments' pseudorange fitting residuals. The probability distributions are shown in [Figure 8: see original paper].

6. Conclusion

Based on the least squares fitting principle, this study evaluates the stability of GNSS space signals by analyzing carrier phase and pseudorange measurements from monitoring receivers. Digital simulation results demonstrate that while polynomial fitting is less effective at curve inflection points, it does not affect the overall fitting result. However, when carrier phase or pseudorange observations contain anomalies due to receiver loss-of-lock or satellite clock jumps, the polynomial fitting method becomes invalid, necessitating the selection of normal observation data for analysis.

The evaluation of real BDS geostationary satellite pseudorange data confirms that the satellite' s transmitted signals remain stable over 24 hours. Since polynomial fitting has certain limitations that may affect signal assessment results, improving the fitting method represents a future research priority. Additionally, developing methods for long-term stability assessment based on satellite signal trends is another important research direction.

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Note: Figure translations are in progress. See original paper for figures.