A Method for Calculating Interference Fringe Angles Based on Fourier Transform and Image Binarization Threshold Traversal Postprint

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Date: 2017-09-26T00:00:00+00:00

Abstract

Eliminating the relative optical path difference between sub-apertures of a synthetic aperture telescope is a prerequisite for achieving high-resolution interferometric imaging, and the fringe detection method is an effective approach for detecting relative optical path difference. Due to the spatial arrangement of sub-apertures, interference fringes exhibit certain directionality. If the angle of the interference fringes cannot be accurately determined, sampling cannot be performed along the normal direction of the fringes, and consequently, the position of minimum optical path difference between sub-apertures cannot be obtained based on the maximum value of the contrast variation curve. A method for obtaining the angle of interference fringes based on Fourier transform and image binarization threshold traversal is proposed. First, the basic principle of the algorithm is introduced. Second, algorithm validation is performed using simulated data with a fringe angle of 43°, yielding an obtained angle of 43.0078°, with an error of 0.018% relative to the theoretical value, thereby confirming the feasibility of the method. Finally, the fringe contrast variation curves for two cases—camera without rotation and camera with rotation—are compared. It can be concluded that the approach of rotating the camera to align the fringes with the vertical axis of the camera sensor plane before sampling is more conducive to accurately obtaining the position of minimum relative optical path difference.

A Method for Calculating the Angle of Interference Fringes Based on Fourier Transform and Threshold Traversal of Image Binarization

Vol. 14 No. 3 Jul., 2017
ASTRONOMICAL RESEARCH AND TECHNOLOGY
1672-7673

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Abstract

Achieving high-resolution imaging of space targets has always been a fundamental goal for astronomers. For a given wavelength, the resolution of a telescope is fundamentally limited by its aperture size—larger apertures yield higher resolution. However, practical constraints in manufacturing and cost limit the feasible aperture size. Synthetic aperture imaging technology enables the construction of large-aperture systems by combining smaller, more easily fabricated sub-apertures, thereby achieving high resolution. A critical prerequisite for this approach is the elimination of relative optical path differences (OPD) between sub-apertures, and the fringe detection method represents an effective technique for measuring these differences.

The spatial arrangement of sub-apertures imparts a directional characteristic to interference fringes. Without precise knowledge of the fringe angle, sampling cannot be performed along the normal direction of the fringes, preventing determination of the minimum OPD position between sub-apertures from the maximum of the contrast variation curve. This paper proposes a novel method for determining interference fringe angles based on Fourier transform and threshold traversal of image binarization. The algorithm' s fundamental principle is introduced, and validation using simulated data with a fringe angle of 43° yields a calculated angle of 43.0078°, with an error of only 0.018% relative to the theoretical value, confirming the method' s feasibility. Finally, a comparison of contrast curves obtained with and without camera rotation demonstrates that rotating the camera to align fringes with the vertical axis of the sensor plane prior to sampling significantly improves the accuracy of determining the minimum relative OPD position.

Keywords: Fourier Transform; Threshold; Interference Fringe; Fringe Angle; Optical Path Difference

1. Introduction

High-resolution imaging of space targets remains a persistent objective in astronomical research. At a fixed wavelength, telescope resolution is fundamentally constrained by aperture diameter, with larger apertures providing higher resolution. However, manufacturing and cost considerations impose practical limits on aperture size. Synthetic aperture imaging technology addresses this limitation by combining multiple smaller, more easily fabricated sub-apertures to synthesize a large-aperture system capable of high-resolution imaging. The critical enabling requirement for this technology is the elimination of relative optical path differences between sub-apertures. The fringe detection method serves as an effective approach for measuring these relative OPDs.

Following acquisition of interference fringes between two sub-apertures using phase-shifting techniques, multiple frames of fringe data are continuously collected and sampled to determine the minimum OPD position. However, due to the non-collinear spatial arrangement of sub-apertures (rather than simple vertical or horizontal alignment), the resulting fringe patterns exhibit a specific angular orientation. Precise knowledge of this fringe angle is essential for sampling along the fringe normal direction, enabling reconstruction of the contrast variation curve and identification of the minimum relative OPD between sub-apertures—ultimately achieving high-resolution imaging with synthetic aperture telescopes.

2. Simulation Setup and Fringe Characteristics

2.1 Three-Aperture Interference Simulation

This study employs independent three-aperture interference fringes as the simulation object, with sub-apertures arranged in an equilateral triangle configuration. In an ideal system, beams transmitted to the entrance pupil of the beam-combining mirror would replicate proportionally those transmitted to the system entrance pupil. However, shear errors and magnification errors introduce pupil-mapping errors, causing the three beams on the combining mirror' s entrance pupil to deviate from a perfect equilateral triangle distribution [FIGURE:1(a)]. The actual relative positions are illustrated in [FIGURE:1(b)], where the angle between the baseline and horizontal direction is approximately -47° (with fringes oriented at approximately 43° relative to the horizontal).

When imaging a point target at infinity, a single-aperture telescope produces a Fraunhofer diffraction pattern at the focal plane. For sub-apertures imaging the same point target, the diffracted intensity distributions can be expressed as:

[Mathematical expressions for intensity distributions would appear here]

Double-aperture interference essentially superimposes a cosine modulation term dependent on the baseline onto the single-aperture diffraction pattern. The resulting interference fringe calculation formula is:

[Mathematical expression for fringe calculation]

The simulation parameters are: wavelength = 600 nm, focal length f = 1260 mm, bandwidth Δ = 100 nm, average baseline length B 16 mm, spot size D = 9 mm, pixel size = 7.4 m. The simulated fringe pattern is shown in [Figure 0: see original paper].

3. Algorithm Principles

3.1 Hough Transform Line Extraction Method

The Hough transform represents a classical line extraction algorithm that operates by accumulating evidence to determine which line contains the most feature points. The primary steps for fringe angle calculation involve: (1) edge extraction from the grayscale fringe image, and (2) line segment detection in the edge map to determine fringe slope and angle. However, since fringe edges are not perfectly straight lines, this approach yields significant deviations when applied to the simulated fringe pattern [Figure 2: see original paper]. The Hough transform requires threshold selection for both edge detection and line extraction, making the results highly sensitive to threshold values. While acceptable for fine, elongated fringes, performance degrades for wider fringe patterns.

3.2 Fourier Transform-Based Fringe Angle Calculation

For an optical imaging system, the Point Spread Function (PSF) describes imaging performance in the spatial domain, while the Optical Transfer Function (OTF) characterizes performance in the frequency domain. Fourier transform establishes the relationship between PSF and OTF. The modulus of the OTF is the Modulation Transfer Function (MTF), and the overall system MTF comprises contributions from sub-MTFs whose angular distribution in the frequency domain is determined by sub-aperture arrangement. Consequently, knowledge of the MTF' s spatial distribution in the frequency domain reveals the sub-aperture configuration, enabling determination of baseline direction and fringe angle.

The proposed algorithm proceeds as follows [Figure 4: see original paper]:

1. **Forward Fourier Transform**: Convert the fringe image's grayscale data to its frequency domain representation. Energy originally concentrated at the image center becomes distributed across the spectrum [FIGURE:5(a)].
2. **Diagonal Transform**: Shift the zero-frequency components from the corners to the spectrum center through a diagonal transformation (quadrant swapping) [FIGURE:5(b)]. This operation exchanges image blocks rather than individual pixel coordinates.
3. **Binarization**: Apply thresholding to the transformed spectrum to identify connected regions [FIGURE:5(c)].

4. **Centroid Calculation**: Determine the centroid of each connected region using crosshairs for marking [FIGURE:5(d)], with the image’ s top-left pixel as the coordinate origin.
5. **Slope and Angle Calculation**: Connect the centroids to calculate the line slope k. The fringe slope equals the average centroid connection slope, from which the fringe angle   is derived.

4. Threshold Traversal Method

Implementation reveals that threshold selection has a valid operational range—neither too high nor too low. To mitigate threshold dependency, a traversal approach is employed: for 8-bit unsigned data (threshold range 1–255), calculate fringe angles across all reasonable thresholds and average the results. For simulation data, the reasonable threshold range yields an average angle of 43.0078° with 0.018% error [Figure 6: see original paper].

5. Experimental Validation

5.1 Experimental Setup and Data Acquisition

Experimental validation employed a three-aperture interference setup with parameters closely matching the simulation. However, measurement errors and pupil-mapping errors introduced differences in spot sizes and baseline dimensions. The acquired experimental fringes are shown in [FIGURE:7(a)]. Applying the same algorithm yields a reasonable threshold range that overlaps with the simulation range between [50, 64] [Figure 6: see original paper]. The average angle calculated within this overlapping range is 44.7901°, with the deviation from simulation attributed to optical system aberrations and camera noise in the experimental setup versus the ideal, noise-free simulation.

5.2 Comparison of Sampling Methods

To determine the minimum OPD between sub-apertures, step-scanning acquires fringe data that must be sampled along the fringe normal direction. Fringe contrast, defined as (I_max - I_min)/(I_max + I_min) where I_max and I_min are the maximum and minimum intensities, serves as the key evaluation metric.

Two scenarios were compared:

Non-Rotated Camera: Using the calculated fringe angle of 44.7901°, sampling along the normal direction (-45.2099°) produces the contrast curve shown in [FIGURE:7(b)]. The curve exhibits indistinct trends, preventing identification of the minimum OPD position from the fitted curve.

Rotated Camera: The camera was rotated counterclockwise by 44.7901° to align fringes vertically (90°). Sampling along the normal direction (0°) yields the contrast curve in [FIGURE:8(b)], where the transition from blurred to clear fringes is clearly visible, enabling precise location of the minimum OPD position from the fitted peak.

The improved performance arises because fringe images consist of discrete pixel arrays (2048×2048 pixels). When fringes are angled, contrast calculations sample approximate integer values from pixels near the ideal line, reducing accuracy. Rotating fringes to vertical maximizes pixel alignment along a single column, improving contrast calculation precision.

6. Conclusion

The proposed Fourier transform and threshold traversal method effectively calculates fringe angles, with simulation results demonstrating 0.018% accuracy. Experimental validation confirms the method' s practical utility, though threshold ranges differ slightly between simulation and experiment due to real-world aberrations and noise. Rotating the camera to align fringes with the sensor' s vertical axis significantly enhances the ability to locate minimum OPD positions accurately, providing valuable guidance for camera positioning in synthetic aperture interferometry systems.

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Note: Figure translations are in progress. See original paper for figures.