Calibration of Mueller Matrix Measurement for Weakly Polarizing Devices and Its Applications

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Abstract

The high-resolution magnetograph of the 1 m New Vacuum Solar Telescope (NVST) employs synchronous reconstruction technology that utilizes beam splitters. Although beam splitters are generally polarization-optimized in design to minimize polarization effects, they inevitably influence polarization measurements of the solar magnetic field. Therefore, characterizing the polarization properties of beam splitters constitutes an essential step for solar magnetic field polarization measurements with the 1 m solar telescope. This study applies the air Mueller matrix calibration method to calibrate Mueller matrix measurement results, measures two beam splitter samples used for magnetic field polarization measurements, compares the Mueller matrices of different beam splitter configurations and their effects on polarization measurements, and investigates the variation of the Mueller matrix of flat glass with incident angle. By comparing deviations between measured results and theoretical values, the measurement system achieves a Mueller matrix measurement accuracy of 5$×10^{-3}$ after calibration using the air Mueller matrix.

Full Text

Calibration of Mueller Matrix Measurement for Weakly Polarizing Devices and Its Applications

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Abstract: The high-resolution magnetic imager of the 1 m New Vacuum Solar Telescope (NVST) adopts a synchronous reconstruction technique, in which a beam splitter is used. In design, the polarization of the beam splitter is generally optimized so as to minimize polarization effects as much as possible; nevertheless, it inevitably affects polarimetric measurements of the solar magnetic field. Therefore, measurement of the polarization characteristics of the beam splitter is an essential step for carrying out solar magnetic-field polarimetric measurements with the 1 m solar telescope. Using the air Mueller-matrix calibration method, the Mueller-matrix measurement results were calibrated. Two beam-splitter samples for magnetic-field polarimetric measurements were measured, the Mueller matrices of different beam-splitter schemes and their influence on polarimetric measurements were compared, and the Mueller matrix of a plate glass as a function of incident angle was measured. By comparing the deviation between the measurement results and theoretical values, after calibration with the air Mueller matrix the measurement system achieved a Mueller-matrix measurement accuracy of 5×10^{-3}.

Keywords: polarization optics; polarization characteristics of beam splitters; Mueller-matrix measurement of weakly polarizing devices; air Mueller-matrix

Fig. 1 Optical path diagram of NVST (a) and magnetic field measuring equipment (b)

Figure 1: Fig. 1 Optical path diagram of NVST (a) and magnetic field measuring equipment (b)

calibration;

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0 Introduction1

The solar magnetic field is the main factor governing solar activity[1], and high-resolution imaging of the solar magnetic field is the basic basis for analyzing the relationship between solar activity and magnetic-field variations. At present, many research institutions at home and abroad are devoted to studies of high-resolution observations of the solar magnetic field. Among them, the Swedish Solar Telescope (SST)[2], the New Solar Telescope (NST)[3] at Big Bear Solar Observatory in the United States, and Germany' s GREGOR Solar Telescope[4] have all achieved outstanding results in high-resolution imaging or spectroscopic observations of the solar magnetic field. In China, the solar magnetic-field telescope[5] at the Huairou Observing Station of the National Astronomical Observatories, as well as the Solar Stokes Spectral Telescope[6-7] at Yunnan Astronomical Observatory, have made important contributions to polarimetric measurements of the solar magnetic field. With its advantages in aperture and site, the 1 m solar telescope of Yunnan Astronomical Observatory is expected to obtain information on small-scale structures of magnetic fields in solar activity[8].

Fig. 1 Optical path diagram of NVST (a) and magnetic field measuring equipment (b)

At present, the solar magnetic field is mainly obtained by observing the polarization splitting of magnetically sensitive spectral lines[9]. However, for a solar telescope to carry out magnetic-field observations, polarization calibration of the telescope and related instruments is indispensable[10–12]. Figure 1(a) is the optical–

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shown in the optical-path schematic. Ref. [10] has already established the relevant model and carried out polarization calibration of the telescope. Fig. 1(b) is a schematic of the magnetic-field measurement terminal of the 1 m solar telescope; this terminal is installed at the rear end of the telescope optical-path system in Fig. 1(a), and the field lens in Fig. 1(b) is located at the telescope focus F_3. Since Ref. [10] has already performed the corresponding polarization calibration for the telescope system before F_3, it is now only necessary to measure the polarization properties of the optical elements from F_3 to the polarization analyzer, especially the polarization effect of the beam splitter.

Because the polarization effect of the beam splitter is weak (generally a few percent), the measurement of its polarization properties (represented in this paper by the Mueller matrix) is more readily affected by the accuracy of the measurement system. In this paper, an air Mueller-matrix calibration method is used to calibrate the measurement results of a dual-rotating-retarder Mueller-matrix measurement system[^13-^14], thereby improving the measurement accuracy of the system for Mueller matrices.

1 Dual-Rotating-Retarder Measurement System and Air Mueller-Matrix Calibration Method

1.1 Establishment of the dual-rotating-retarder Mueller-matrix measurement system

The principle of the dual-rotating-retarder Mueller-matrix measurement system is shown in Fig. 2. In the figure, PL_1 and WP_1 constitute the polarization generator, which is used to generate polarized light of known state; WP_2 and PL_2 constitute the polarization analyzer, which analyzes the state of the polarized light after it has passed through the sample. The transmission-axis directions of the two polarizers PL_1 and PL_2 in the figure are the same and remain fixed, while the retarders WP_1 and WP_2 rotate during the measurement. From the light source to the detector, the Mueller-matrix measurement system consists, in sequence, of a laser-diode light source, a Glan–Thompson prism PL_1, a quarter-wave plate WP_1, the sample to be measured, a quarter-wave plate WP_2, a polarizer PL_2, a filter, and an optical power meter. Here, WP_1 and WP_2 are respectively mounted on two electrically controlled precision rotation stages to adjust the wave-plate angles $(\vartheta_1, \vartheta_2)$. Through software control, continuous control of multiple sets of $(\vartheta_1, \vartheta_2)$ and synchronous acquisition of light intensity can be realized. The measurement system selects ϑ_1 and ϑ_2 to

Fig. 2 Schematic diagram of dual-rotating-retarder Mueller matrix polarimeter

Figure 2: Fig. 2 Schematic diagram of dual-rotating-retarder Mueller matrix polarimeter

be adjusted respectively to the eight positions $(0, 22.5, 45, \cdots, 157.5)^{\circ}$, and one measurement yields 64 light-intensity values. The adjustment positions of ϑ_1 and ϑ_2 are obtained by diagonalizing the modulation-frequency and response matrices of Q and U.

Fig. 2 Schematic diagram of dual-rotating-retarder Mueller matrix polarimeter

For the system, the initial polarization direction of the polarizer PL_1 is selected as the Q direction. The transmission axis of PL_2 and the fast axes of WP_1 and WP_2 are then aligned in sequence to the Q direction, and the retardances of WP_1 and WP_2 are fitted[^15-^16], giving δ_1 and δ_2 as 87.6° and 88.4°, respectively.

When establishing the measurement system in Fig. 2, a laser-diode light source is selected. This light source has good stability, high collimation, and a small spot radius. The light emitted by the source can pass directly through the polarization generator, the sample to be measured, the polarization analyzer, and the filter, and then enter the optical power meter directly, without using any lens. This both shortens the optical path and avoids the influence of the polarization properties of lenses on the measurement results.

1.2 Matrix calculation process for the Mueller matrix

The relationship between the light intensity measured by the system and the parameters of the various optical elements is as follows:

$$I(\theta_1, \delta_1, \theta_2, \delta_2) = [1, 0, 0, 0]*M_{\mathrm{pl2}}*M_{\mathrm{wp2}}(\theta_2, \delta_2)*M_s*M_{\mathrm{wp1}}(\theta_1, \delta_1)*M_{\mathrm{pl1}}*[I_0, 0, 0, 0]^T \quad (1)$$

Define $S_i = M_{\mathrm{wp1}}(\theta_{1i}, \delta_1) * M_{\mathrm{pl1}} * [I_0, 0, 0, 0]^T$, where S_i denotes the calibrated-light Stokes vector corresponding to ϑ_{1i}. Define $P_j = [1, 0, 0, 0] * M_{\mathrm{pl2}} * M_{\mathrm{wp2}}(\theta_{2j}, \delta_2)$, where P_j denotes the modulation matrix of the polarization analyzer for the light intensity corresponding to ϑ_{2j}.

From the measurement process described in Section 1.1, the measured $8 * 8$ light-intensity matrix is

$$I_{8,8} = P_{8,4} * M_s * S_{4,8}, \quad (2)$$

where $S_{4,8} = (S_1, \cdots, S_i, \cdots, S_8)$, $P_{8,4} = (P_1, \cdots, P_j, \cdots, P_8)^T$.

The least-squares solution of the equation is obtained through matrix operations, namely

$$M_s = (P_{8,4}^T * P_{8,4})^{-1} * P_{8,4}^T * I_{8,8} * S_{4,8}^T * (S_{4,8} * S_{4,8}^T)^{-1} \tag{3}$$

1.3 Air Mueller-Matrix Calibration Method

Since air is isotropic, when no sample is present, the Mueller matrix calculated from the measurement results according to Eq. (3) should be the identity matrix. In actual measurements, however, it deviates from the identity matrix. This is mainly caused by errors in the measured parameters of the polarizer and wave plate, as well as by nonideal factors of the polarizer and wave plate themselves.

This paper does not analyze in detail the causes of the above problems; instead, it adopts an air Mueller-matrix calibration method to process and optimize the measurement errors. The following analyzes the process of using the Mueller matrix of air, M_c, to calibrate samples such as beam splitters, for which the polarization effect is relatively weak.

The measured result of the air Mueller matrix, M_c, can be regarded as the sum of the identity matrix E and a 4×4 matrix m_0, i.e., $M_c = E + m_0$, where m_0 is a small quantity.

Since $m_0 \neq 0$ is caused by deviations of the polarization generator and polarization analyzer from their ideal conditions, the measurement of air can be obtained by expanding Eq. (2) as

$$I_{8,8}^0 = P_{8,4}^0 * (E + \Delta p) * E * (E + \Delta s) * S_{4,8}^0, \tag{4}$$

where $(E + \Delta s)$ and $(E + \Delta p)$ represent, respectively, the degrees to which the polarization generator and polarization analyzer deviate from the ideal condition, and Δp and Δs are small quantities. The measurement result for no sample is $M_c = (E + \Delta p) * (E + \Delta s)$.

When the Mueller matrix of the sample under test is close to the identity matrix E, i.e., when the sample is a weakly polarizing element, the actual Mueller matrix of the sample is $M_s^0 = (E + \Delta x)$, where Δx is a small quantity. The relationship between the result M_s calculated directly from Eq. (3) for the sample under test and M_s^0 is as follows:

$$\begin{aligned} M_s &= (E + \Delta p) * M_s^0 * (E + \Delta s) \\ &= (E + \Delta p) * (E + \Delta x) * (E + \Delta s) \\ &= [(E + \Delta x) * (E + \Delta p) - (\Delta x * \Delta p - \Delta p * \Delta x)] * (E + \Delta s) \\ &= M_s^0 * M_c - (\Delta x * \Delta p - \Delta p * \Delta x) * (E + \Delta s) \\ &\approx M_s^0 * M_c \end{aligned} \tag{5}$$

$$M_s^0 \approx M_s * M_c^{-1}, \tag{6}$$

where M_s^0 is the result obtained after calibration using the air Mueller matrix M_c. The accuracy of this approximation is related to the degree to which Δp and Δx approach infinitesimal quantities.

After calibration using the air Mueller matrix, the errors of the experimental measurement system mainly originate from the approximation error in the formula and from optical-intensity instability.

2 Measurement of the Beam-Splitter Mueller Matrix

First, the adjusted measurement system was used to measure the air Mueller matrix, and the measured result M_c was used for calibration in subsequent measurements. Next, Mueller-matrix measurements were performed on a standard K9 glass plate with an incidence angle from 0° to 50°, and the differences between the measured results and theoretical values were compared. Then the measurement system was used to measure the beam-splitter sample under test. Finally, the errors of the measurement system were analyzed.

2.1 Air Mueller Matrix with No Sample

According to the introduction in Section 1.2, the measured light-intensity matrix $I_{8,8}$, the retardances of the wave plates 87.6° and 88.4°, and the azimuth angles $(0, 22.5, \dots, 157.5)$ are substituted into Eq. (3) to obtain the Mueller matrix of air:

$$M_c = \begin{bmatrix} 1 & -0.0034 & 0.0175 & 0.0009 \\ -0.0018 & 1.0043 & -0.0039 & -0.0029 \\ -0.0196 & 0.0009 & 1.0050 & -0.0005 \\ 0.0007 & 0.0035 & 0.0007 & 1.0008 \end{bmatrix}$$

The measured air Mueller matrix is not a matrix with actual physical meaning, but rather a representation of the imperfection of the measurement system. This matrix reflects uncalibrated measurement errors of about 2%, which greatly limits the accuracy of sample measurements.

In subsequent measurements, the measured result of the air matrix, M_c, is used to calibrate the measured results of the sample Mueller matrix.

to improve the measurement accuracy.

2.2 Results of measuring the Mueller matrix of plate glass as a function of incident angle

The Mueller matrix of K9 plate glass was measured as the incident angle varied from 0° to 50°, and the difference between the measured results and those

Fig. 3 Measured Mueller matrix of K9 glass varies with the incident angle

Figure 3: Fig. 3 Measured Mueller matrix of K9 glass varies with the incident angle

Figure 4: Comparison of measured and theoretical values of Mueller matrix elements M_{12}, M_{21}, M_{33}, and M_{44}.

Figure 4: Figure 4: Comparison of measured and theoretical values of Mueller matrix elements M_{12}, M_{21}, M_{33}, and M_{44}.

obtained from the Fresnel formulas was compared. In this way, the accuracy of the Mueller-matrix measurement system can be obtained from the actual measurement angles.

Let the ratio of the transmitted intensities of the P and S components of the polarized light through the plate glass be t. The Mueller matrix of the transmitted light through K9 glass is

$$\mathrm{M}(n,\alpha)=\begin{bmatrix} 1 & \frac{1-t}{1+t} & 0 & 0 \\ \frac{1-t}{1+t} & 1 & 0 & 0 \\ 0 & 0 & \frac{2\sqrt{t}}{1+t} & 0 \\ 0 & 0 & 0 & \frac{2\sqrt{t}}{1+t} \end{bmatrix}, \tag{7}$$

where the ratio t of the transmitted intensities of the P and S components is a function of the refractive index n and the incident angle α.

Figure 3 shows the Mueller matrix of K9 plate glass when the incident angle α varies from 0° to 50°. The 4×4 plots correspond to the respective matrix elements of the 4×4 Mueller matrix in Eq. (7). It can be seen from Fig. 3 that the deviations between the measured results of the 0 and 1 matrix elements in Eq. (7) and the theoretical values are basically within 5×10^{-3}.

图 3 K9 平板玻璃 Mueller 矩阵随入射角变化

Fig. 3 Measured Mueller matrix of K9 glass vary with the incident angle

Figure 4 compares the measured values and theoretical values of the matrix elements M_{12}, M_{21}, M_{33}, and M_{44}. It can be seen that the maximum errors of the non-0 and non-1 matrix elements in Eq. (7) are also within 5×10^{-3}.

Comparing the Mueller-matrix measurement results for the standard sample K_9 glass with the theoretical values shows that, after calibration using the Mueller matrix of air, the error is less than 5×10^{-3}.

2.3 Measurement of the Mueller Matrix of Beamsplitter Samples

Mueller-matrix measurements were carried out on two beamsplitters used for magnetic-field measurements with the 1 m solar telescope. On the one hand, the results of polarization measurements with the 1 m solar telescope can be processed using the accurately measured beamsplitter Mueller matrix, thereby eliminating the offset and crosstalk caused by the polarization effect of the beamsplitter. On the other hand, by comparing the polarization characteristics of beamsplitters under different designs, a scheme with relatively small offset and crosstalk can be selected so as to avoid excessive degradation of the signal-to-noise ratios of Q, U, and V caused by offset and crosstalk.

Since all beamsplitter samples are coated, their Mueller matrices have the parameters P- and S-transmittance ratio t, and P- and S-phase retardance difference δ. The Mueller matrix of the transmitted light of a beamsplitter is

$$\mathrm{M}(t,\delta) = \begin{bmatrix} 1 & \frac{1-t}{1+t} & 0 & 0 \\ \frac{1-t}{1+t} & 1 & 0 & 0 \\ 0 & 0 & \frac{2\sqrt{t}}{1+t}\cos\delta & \frac{2\sqrt{t}}{1+t}\sin\delta \\ 0 & 0 & -\frac{2\sqrt{t}}{1+t}\sin\delta & \frac{2\sqrt{t}}{1+t}\cos\delta \end{bmatrix}, \tag{8}$$

where the S direction is parallel to the Q direction of the Stokes parameters.

When there is an included angle θ between the S direction of the beamsplitter and the Q direction defined by the measurement system, the Mueller matrix of the beamsplitter is

$$\mathrm{M}(t,\delta,\theta) = m_r(-\theta) * \mathrm{M}(t,\delta) * m_r(\theta)$$

$$= \begin{bmatrix} 1 & B\cos 2\theta & B\sin 2\theta & 0 \\ B\cos 2\theta & \cos^2 2\theta + C\sin^2 2\theta & \frac{1-C}{2}\sin 4\theta & -D\sin 2\theta \\ B\sin 2\theta & \frac{1-C}{2}\sin 4\theta & \sin^2 2\theta + C\cos^2 2\theta & D\cos 2\theta \\ 0 & D\sin 2\theta & -D\cos 2\theta & C \end{bmatrix}, \tag{9}$$

where

$$B = \frac{1-t}{1+t}; \qquad C = \frac{2\sqrt{t}}{1+t}\cos\delta; \qquad D = \frac{2\sqrt{t}}{1+t}\sin\delta; \qquad m_r(\theta) = \frac{1}{2}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\theta & \sin 2\theta & 0 \\ 0 & -\sin 2\theta & \cos 2\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

is the coordinate rotation matrix.

Table 1 gives the Mueller-matrix measurement results for beamsplitter A, a 1:9 beamsplitter of the NVST.

Mueller matrix of beamsplitter A at an incidence angle of 45°

1	−0.0729	0.0012	0.0017
−0.0720	1.0038	0.0033	−0.0027
0.0037	−0.0034	0.9898	−0.1272
−0.0020	0.0021	0.1278	0.9886

$$t = 1.156; \quad \delta = -7.35^\circ; \quad \theta = -0.64^\circ$$

After further polarization-optimization design, the Mueller matrix of beamsplitter B is shown in Table 2.

Mueller matrix of beamsplitter B at an incidence angle of 45°

1	0.0015	−0.0058	−0.0009
0.0018	0.9993	0.0081	0.0151
−0.0035	−0.0102	0.9904	0.1325
0.0012	−0.0114	−0.1329	0.9915

$$t = 1.004; \quad \delta = -7.67^\circ; \quad \theta = 2.87^\circ$$

Because the solar polarization signals Q, U, and V are relatively weak compared with the intensity I, when performing polarimetric measurements of the Sun, it is first necessary to avoid as much as possible the instrumental crosstalk effects ($I \rightarrow Q, U, V$), and next to avoid the crosstalk among Q, U, and V caused by the instrument. Comparing the measured Mueller matrices of beamsplitters A and B, it can be seen that the beamsplitter after polarization optimization suppresses the crosstalk effect very well, reducing the crosstalk from I to Q and U from the original approximately 7% to below 0.5%.

2.4 Systematic and random errors of the measurement system

The error treatment method in this paper is as described in Section 1.3; here the error after calibration with the air matrix M_c is analyzed. Using Eq. (5), the approximate error is obtained as follows:

$$\begin{aligned} \Delta M_s &= (\Delta x * \Delta p - \Delta p * \Delta x) * (E + \Delta s) * M_c^{-1} \\ &= (\Delta x * \Delta p - \Delta p * \Delta x) * (E + \Delta p)^{-1}, \end{aligned} \tag{10}$$

Fig. 5 The stability of light intensity over time (a) and random fluctuation of light intensity (b)

Figure 5: Fig. 5 The stability of light intensity over time (a) and random fluctuation of light intensity (b)

where Δx is the degree of polarization of the measured sample; the weaker the polarization characteristic of the sample, the closer Δx is to 0. Δp is the deviation of the polarization analyzer from the ideal case; the smaller the deviation, the closer Δp is to 0.

From the measurement results of M_c in Section 2.1, when $\Delta s = 0$, the maximum value of Δp is

$$\Delta p_{\max} = \begin{bmatrix} 0 & -0.0034 & 0.0175 & 0.0009 \\ -0.0018 & 0.0043 & -0.0039 & -0.0029 \\ -0.0196 & 0.0009 & 0.0050 & -0.0005 \\ 0.0007 & 0.0035 & 0.0007 & 0.0008 \end{bmatrix},$$

Combining the maximum values of Δx for the measured samples in Sections 2.2 and 2.3 gives

$$\Delta M_s < (\Delta x_{\max} * \Delta p_{\max} - \Delta p_{\max} * \Delta x_{\max}) * (E + \Delta p_{\max})^{-1}$$

The maximum approximate error of the above expression is

$$\begin{bmatrix} -0.0003 & -0.0004 & 0.0007 & 0.0003 \\ 0.0004 & 0.0002 & -0.0020 & -0.0001 \\ 0.0003 & -0.0022 & 0.0001 & 0 \\ 0.0004 & 0 & 0 & 0 \end{bmatrix}$$

Thus, for the measured samples in Sections 2.2 and 2.3, the approximate error ΔM_s obtained using Eq. (5) is $< 2 \times 10^{-3}$.

The random error in the Mueller matrix measured by the measurement system can be obtained from the random error of the light intensity. Figure 5 shows the results of stability measurements of the light intensity before the experiment. The root-mean-square value of the light intensity is 5×10^{-4}, and after the light intensity becomes stable, the intensity varies with time by less than 3×10^{-4}/min. Within one measurement period (2 min), the random error of the light intensity is

$$\frac{\delta I}{I} = 6 \times 10^{-4}.$$

Figure 5 Stability of light intensity over time (a) and random fluctuation of light intensity (b)

From Eq. (3), the measured (i, j)-th element of the Mueller matrix can be expressed as

$$M(i,j) = k_{11}(i,j)I_{11} + k_{12}(i,j)I_{12} + \cdots + k_{88}(i,j)I_{88}, \tag{11}$$

where $i, j = 1, 2, 3, 4$; $k_{11}(i,j), k_{12}(i,j), \cdots, k_{88}(i,j)$ are calculated from $P_{8,4}$ and $S_{4,8}$; and $I_{11}, I_{12}, \cdots, I_{88}$ are the elements of the light-intensity matrix $I_{8,8}$.

According to the formula for propagation of random errors, one obtains

$$\delta M(i,j)^2 = \left(k_{11}(i,j)\delta I_{11}\right)^2 + \left(k_{12}(i,j)\delta I_{12}\right)^2 + \cdots + \left(k_{88}(i,j)\delta I_{88}\right)^2, \tag{12}$$

where $\delta I_{11}, \delta I_{12}, \cdots, \delta I_{88}$ are the random errors of the light-intensity matrix $I_{8,8}$.

From Eq. (12), $I_{8,8}$, $\delta I/I$, $P_{8,4}$, and $S_{4,8}$, the random error of the light intensity propagated to the Mueller matrix, δM, can be calculated as

$$\delta M = \begin{bmatrix} 0.0005 & 0.0003 & 0.0003 & 0.0006 \\ 0.0003 & 0.0002 & 0.0002 & 0.0004 \\ 0.0003 & 0.0002 & 0.0002 & 0.0004 \\ 0.0006 & 0.0004 & 0.0004 & 0.0008 \end{bmatrix},$$

and the standard deviation of 20 sets of measurements of the air Mueller matrix M_c in the experiment is

$$M_{\text{std}} = \begin{bmatrix} 0 & 0.0002 & 0.0002 & 0.0002 \\ 0.0001 & 0.0002 & 0.0003 & 0.0007 \\ 0.0003 & 0.0005 & 0.0006 & 0.0004 \\ 0.0002 & 0.0003 & 0.0003 & 0.0001 \end{bmatrix}.$$

From the results of the error-propagation calculation and the experimental measurements, the influence of random errors on the Mueller-matrix measurement results is less than 10^{-3}.

After the Mueller-matrix system is calibrated using the air matrix M_c, the systematic error is $\Delta M < 2 \times 10^{-3}$, and the random error is $\delta M < 10^{-3}$. In Section 2.2, for the measurement of the K9 flat glass, the deviation between the measured values and the theoretical values is within 5×10^{-3}. Taking these factors together, the measurement error of this measurement system for the samples in this paper is less than 5×10^{-3}.

Because the air Mueller-matrix calibration method uses the approximation in Eq. (5), it is therefore applicable only to Mueller-matrix measurements of weakly polarizing components. For samples whose diattenuation and retardance effects are within 10%, a measurement accuracy within 5×10^{-3} can be achieved.

3 Conclusion

Starting from the measurement of the Mueller matrix of the beam splitter of a 1 m solar telescope, this paper uses the air Mueller-matrix calibration method to calibrate the Mueller-matrix measurement results, improving the Mueller-matrix measurement accuracy from 2% to 5×10^{-3}. The difference between the Mueller-matrix measurement results and the theoretical values for the standard sample K9 flat glass was compared, and the measurement system was used to accurately measure the beam splitter in the polarization measurement of the 1 m solar telescope. The beam splitters that had undergone polarization optimization were compared with those that had not been ···

differences and influences among beam splitters optimized for polarization; this is indispensable for the polarization measurements of the 1 m solar telescope and for optimization of the beam-splitter scheme.

After applying the air Mueller-matrix calibration method, the measurement accuracy of the Mueller-matrix measurement system can initially meet the requirements for measuring the polarization characteristics of weakly polarizing devices. If higher-precision measurement and calibration are required, more standard polarizers and wave plates must be used in the measurement system, and the stability of the light source must be improved.

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Calibration of Mueller Matrix Measurement of Weak Polarization Element and Its Application

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Abstract: Synchronous reconstruction technology has been used in the high-resolution magnetograph of the New Vacuum Solar Telescope, which uses a beam splitter. The beam splitter is designed to reduce the polarization effect, but it still inevitably affects the polarization measurement of the solar magnetic field; therefore, measurement of the polarization characteristics of the beam splitter is necessary for the NVST polarimeter. In this paper, the Mueller matrix of air is used to calibrate the measured Mueller-matrix results of samples, and two beam-splitter samples are measured by this method. The Mueller matrices of different beam splitters and their influences on polarization measurement are compared. The Mueller matrix of flat glass is measured as it varies with incident angle, and the deviation between the measured result and the theoretical value is compared. The accuracy of the Mueller-matrix measurement system is 5×10^{-3} after calibration using the Mueller matrix of air.

Key words: polarized optics; polarization characteristics of beam splitter; Mueller matrix measurement of weak polarization element; calibration using Mueller matrix of air;