

## Impact of Anisoplanatic Effects on NVST High-Resolution Reconstruction Results (Postprint)

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### Abstract

Based on the high-resolution reconstruction algorithm for the 1 m New Vacuum Solar Telescope at Fuxian Lake, this study theoretically analyzes the effects of anisoplanatism on speckle interferometry and speckle masking. High-resolution data reconstruction is performed using measured photospheric data with different sub-block sizes. The impact of anisoplanatism is evaluated by comparing the power spectra and phases of reconstructed images across various sub-block sizes. The results indicate that when the reconstruction region exceeds the isoplanatic patch size, anisoplanatism limits the effectiveness of speckle interferometry and speckle masking, causing degradation in the accuracy of modulus and phase reconstruction due to algorithmic failure. Finally, recommendations for further algorithm optimization are provided based on the specific conditions of the image processing program.

### Full Text

## The Influence of Anisoplanatism on High-Resolution Reconstruction Results from NVST

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### Abstract

Based on the high-resolution reconstruction algorithm of the 1m New Vacuum Solar Telescope (NVST) at Fuxian Lake Solar Observatory, this paper theoretically analyzes the effects of anisoplanatism on speckle interferometry and speckle masking. Using observed photospheric data, high-resolution reconstructions were performed with different sub-block sizes. By comparing the power

spectra and phases of the reconstructed images under various sub-block sizes, the impact of anisoplanatism was evaluated. The results demonstrate that when the reconstruction region exceeds the isoplanatic patch size, anisoplanatism limits the effectiveness of both speckle interferometry and speckle masking, reducing the accuracy of modulus and phase reconstruction due to algorithmic failure. Finally, considering the specific implementation of the image processing pipeline, recommendations for further algorithm optimization are provided.

**Keywords:** Anisoplanatism; 1m New Vacuum Solar Telescope; High-resolution reconstruction

Atmospheric turbulence limits the imaging resolution of ground-based large telescopes to approximately 1 arcsecond. Real-time adaptive optics compensation and high-resolution image processing methods can overcome atmospheric turbulence effects to achieve diffraction-limited imaging from ground-based telescopes [1]. Several high-resolution image processing methods have been applied to solar telescopes, primarily including speckle reconstruction methods, phase diversity, and solar multi-frame blind deconvolution. For speckle reconstruction methods such as speckle interferometry, Knox-Thompson algorithm, and speckle masking, effectiveness is only maintained within the isoplanatic patch. The isoplanatic patch can be approximated as a linear space-invariant system where the point spread function (PSF) is considered identical. In astronomy, a typical isoplanatic patch does not exceed 5 arcseconds. For high-resolution image reconstruction of wide-field targets, images must be reconstructed in sub-blocks, with the principle that sub-block size should match the isoplanatic patch size [2]. When the reconstruction region exceeds the isoplanatic patch size, the linear space-invariant condition no longer holds, and atmospheric anisoplanatism causes speckle reconstruction algorithms to fail.

The 1m New Vacuum Solar Telescope (NVST) at Fuxian Lake is China's largest ground-based vacuum solar telescope, primarily used for high-resolution imaging observations of the solar photosphere and chromosphere, as well as solar spectroscopic observations [3-4]. The 1m solar telescope currently employs speckle interferometry and speckle masking for statistical image reconstruction of observed data [5], with a reconstruction sub-block size of 4.5 arcseconds determined empirically. The actual isoplanatic patch size depends on atmospheric conditions, making it difficult to determine precisely during observations. This leads to imprecise sub-block sizing during image reconstruction. Clearly, using a fixed empirical value for block-based reconstruction risks algorithmic failure and reduced reconstruction accuracy, potentially decreasing contrast between photospheric magnetic bright points and granules and reducing the resolution of magnetic bright points.

Numerous researchers have investigated atmospheric anisoplanatism. For instance, reference [6] used numerical simulations to analyze the effects of anisoplanatism on speckle image power spectra; reference [2] studied atmospheric anisoplanatism using binary star speckle images and speckle interferometry; reference [7] measured isoplanatic patch size using speckle holography; and ref-

erence [8] investigated the influence of anisoplanatism on the Knox-Thompson algorithm. This paper analyzes the impact of anisoplanatism under different reconstruction sub-block sizes for the 1m solar telescope based on actual observational data using speckle reconstruction methods.

## 1 Theoretical Analysis

Complete speckle reconstruction requires separate reconstruction of the target's Fourier modulus (speckle interferometry) and Fourier phase (speckle masking or Knox-Thompson algorithm). When reconstructing wide-field target images in sub-blocks, if the sub-block size exceeds the actual isoplanatic patch size, both the modulus and phase of the reconstruction results may contain errors due to algorithmic failure. The following theoretical derivation uses two point sources as an example to illustrate how atmospheric anisoplanatism affects modulus and phase reconstruction algorithms.

### 1.1 Isoplanatic Region

During observation, because light from different parts of the target travels through different atmospheric paths, the instantaneous point spread functions vary across the field of view. However, within a certain field of view range, each small region approximately satisfies the linear space-invariant assumption with an identical point spread function. This region is called the isoplanatic patch. As shown in [Figure 1: see original paper], light beams from targets O1 and O2 located in different directions experience different atmospheric perturbations. When the two targets are relatively close (e.g., 1-2 arcseconds), they can be considered to have experienced the same atmospheric perturbation and thus lie within the same isoplanatic patch. The dashed rectangle in the figure represents the identical atmospheric perturbation experienced by both targets; the closer the targets, the greater the correlation of atmospheric perturbations. According to reference [9], for Kolmogorov turbulence, the atmospheric isoplanatic angle can be expressed as:

$$\theta_0 = \left[ 2.91k^2 \sec(\xi) \int_0^\infty C_n^2(h)h^{5/3}dh \right]^{-3/5}$$

where  $k$  is the wavenumber,  $\xi$  is the zenith angle,  $h$  is altitude, and  $C_n^2$  is the atmospheric refractive index structure constant. Equation (1) shows that a strict measurement of isoplanatic patch size requires parameters including zenith angle and the refractive index structure constant from low to high atmospheric layers. In practice, atmospheric conditions continuously change, making real-time determination of isoplanatic patch size difficult.

## 1.2 Influence of Anisoplanatism on Speckle Interferometry

Speckle interferometry, proposed in reference [10], restores the target's Fourier amplitude through statistical averaging of speckle image energy spectra. Under the isoplanatic assumption, each short-exposure image frame  $i(x, y)$  can be considered as the convolution of the target  $o(x, y)$  and the system point spread function  $h(x, y)$ :

$$i(x, y) = o(x, y) \otimes h(x, y)$$

After performing energy spectrum statistics on speckle images and introducing the ergodicity assumption, the target power spectrum can be written as:

$$\langle |I(u, v)|^2 \rangle = |O(u, v)|^2 \cdot \langle |H(u, v)|^2 \rangle$$

where  $\langle \rangle$  denotes arithmetic averaging,  $I(u, v)$ ,  $O(u, v)$ , and  $H(u, v)$  are the Fourier transforms of the corresponding terms in equation (2). The denominator is called the speckle transfer function (STF).

Assuming two point sources separated by angular distance  $\theta$  with intensities  $m_1$  and  $m_2$ , expressed as  $m_1\delta(x, y) + m_2\delta(x - \theta, y)$ , when imaging through different atmospheric paths, the instantaneous point spread functions are  $h_1(x, y)$  and  $h_2(x, y)$ , respectively. The speckle image intensity is:

$$i(x, y) = [m_1\delta(x, y) \otimes h_1(x, y)] + [m_2\delta(x - \theta, y) \otimes h_2(x, y)]$$

Under ergodic conditions, the average power spectrum of the speckle image is:

$$\langle |I(u, v)|^2 \rangle = m_1^2 \langle |H_1(u, v)|^2 \rangle + m_2^2 \langle |H_2(u, v)|^2 \rangle + 2m_1m_2 \Re \{ \langle H_1(u, v) H_2^*(u, v) \rangle e^{-j2\pi u\theta} \}$$

When  $\theta$  is sufficiently small such that the two targets lie within the isoplanatic patch, i.e.,  $H_1(u, v) = H_2(u, v)$ , equation (5) simplifies to:

$$\langle |I(u, v)|^2 \rangle = (m_1^2 + m_2^2 + 2m_1m_2 \cos(2\pi u\theta)) \langle |H(u, v)|^2 \rangle$$

Deconvolving equation (6) yields the target intensities. However, when  $H_1(u, v) \neq H_2(u, v)$ , the intensities of the two targets cannot be accurately obtained using the speckle interferometry transfer function from either point source. Therefore, when the reconstruction region exceeds the isoplanatic patch size, atmospheric anisoplanatism causes speckle interferometry to fail, resulting in inaccurate Fourier modulus reconstruction.

### 1.3 Influence of Anisoplanatism on Speckle Masking Phase Reconstruction

Speckle masking, proposed in reference [11] and also known as triple correlation or bispectrum method, performs statistical averaging of triple correlations of target speckle images. The target phase can be reconstructed recursively from the statistical results. The bispectrum expression is:

$$I^{(3)}(u_1, u_2) = I(u_1)I(u_2)I^*(u_1 + u_2)$$

For simplicity, equation (7) is written as:

$$I^{(3)}(f_1, f_2) = I(f_1)I(f_2)I^*(f_1 + f_2)$$

Under isoplanatic and ergodic conditions, the speckle image bispectrum statistics are:

$$\langle I^{(3)}(f_1, f_2) \rangle = O^{(3)}(f_1, f_2) \cdot \langle H^{(3)}(f_1, f_2) \rangle$$

The phase-related part in equation (9) is:

$$\langle H^{(3)}(f_1, f_2) \rangle = \langle H(f_1)H(f_2)H^*(f_1 + f_2) \rangle$$

According to atmospheric turbulence theory, wave phase fluctuations follow a Gaussian distribution. Based on the relationship that holds for any real-valued Gaussian random variable  $z$  and any complex constant  $a$ :

$$\langle e^{az} \rangle = e^{a\langle z \rangle + \frac{1}{2}a^2\sigma_z^2}$$

The result from equation (12) is real, demonstrating that the transfer function-related term in equation (9) is real and contributes no phase to the bispectrum.

From equation (9), the relationship between bispectrum phase and target phase can be written as:

$$\Phi_B(f_1, f_2) = \Phi_O(f_1) + \Phi_O(f_2) - \Phi_O(f_1 + f_2)$$

where  $\Phi_B$  is the bispectrum phase and  $\Phi_O$  is the Fourier phase at the corresponding frequency. According to equation (13), the complete target phase can be reconstructed recursively from low to high frequencies.

For the two point sources in Section 1.2, the bispectrum statistics of their speckle images are:

$$\langle I^{(3)}(f_1, f_2) \rangle = m_1^3 \langle H_1(f_1)H_1(f_2)H_1^*(f_1+f_2) \rangle + m_2^3 \langle H_2(f_1)H_2(f_2)H_2^*(f_1+f_2) \rangle e^{-j2\pi(f_1+f_2)\theta} + \text{cross terms}$$

Using equation (11), it can be proven that all transfer function-related terms in equation (14) are real and do not contribute to bispectrum phase. Although transfer function terms do not affect bispectrum phase, when an extended target exceeds the isoplanatic patch size, the relationship between target bispectrum phase and target Fourier phase no longer strictly satisfies equation (13). Therefore, when the reconstruction region exceeds the isoplanatic patch size, anisoplanatism causes the target phase and bispectrum phase to no longer satisfy the phase recursion relationship, introducing errors in speckle masking phase reconstruction that accumulate with increasing frequency.

## 2 Data Reconstruction

### 2.1 Brief Introduction to Data Reconstruction Flow

The brief flow diagram of 1m solar telescope data processing is shown in [Figure 2: see original paper], primarily including data preprocessing, image blocking, amplitude and phase reconstruction, and image mosaicking. Data preprocessing includes flat-field and dark-field correction and image alignment. Image blocking should ensure the reconstruction region falls within the isoplanatic patch size. The 1m solar telescope uses speckle interferometry to restore the Fourier modulus of each sub-block and speckle masking to reconstruct the Fourier phase of each sub-block. After reconstructing the amplitude and phase of each sub-block, they are combined into complex variables, and high-resolution sub-images are obtained through inverse Fourier transform. The sub-images are then mosaicked into a full field-of-view image [1].

### 2.2 Data Reconstruction

The theoretical analysis indicates that when the reconstruction region exceeds the isoplanatic patch size, anisoplanatism causes algorithm failure, reducing the accuracy of Fourier modulus and phase reconstruction. The following analysis investigates the impact of anisoplanatism on algorithm reconstruction results under different reconstruction sub-block sizes through modulus and phase analysis of actual data processing.

**2.2.1 Measured Data** Actual photospheric data from the 1m solar telescope were used for block-based reconstruction, collected on October 3, 2014. The photospheric data were acquired using an Andor sCMOS camera at a wavelength of 7058 Å, with image size 2560×2160 pixels, 200 frames per set, and acquisition time of 30 seconds. Data were reconstructed with sub-block sizes of 5, 10, 15, and 20 arcseconds. [Figure 3: see original paper] shows one raw data frame.

**2.2.2 Reconstruction Results** Due to the large size of reconstructed images, only the region within the red box in [Figure 3: see original paper] is compared here. [Figure 4: see original paper] shows the reconstruction results for this region, with sub-block sizes of 5, 10, 15, and 20 arcseconds from top to bottom and left to right.

Comparison of different results reveals that the 5 arcsecond reconstruction achieves the highest resolution and best image quality. When the sub-block size is 10 arcseconds, isolated magnetic bright points with similar intergranular spacing in the 5 arcsecond result appear as elongated chain-like structures in the 10 arcsecond result, with reduced contrast between magnetic bright points and granulation. The 15 and 20 arcsecond reconstruction results show significantly reduced resolution and noticeably degraded image quality.

### 2.3 Analysis of Reconstruction Results

The reconstruction results in [Figure 4: see original paper] demonstrate clear degradation in image quality with increasing sub-block size. Further analysis of the power spectrum and phase of reconstructed images follows.

**2.3.1 Power Spectrum Analysis** The power spectrum of an image can be obtained from the squared modulus of its Fourier transform. Equation (15) gives the image power spectrum expression:

$$PS(u, v) = |FFT[f(x, y)]|^2$$

Equation (15) shows that the image power spectrum contains no Fourier phase information, reflecting only the modulus of the reconstructed image.

[Figure 5: see original paper] shows the power spectrum curves of reconstruction results. For clarity, only three curves are presented. The solid line, solid line with asterisks, and dashed line correspond to the average power spectrum curves of reconstructed images for sub-block sizes of 5, 15, and 20 arcseconds, respectively.

The power spectrum plot reveals that sub-block size has minimal impact on the low-frequency portion of the image power spectrum, with more significant effects in the mid-to-high frequency range. The 15 arcsecond power spectrum curve is lower than the 5 arcsecond case in both mid and high frequencies, with noticeable decline in the mid-to-high frequency region, indicating changes in image contrast and resolution. The 20 arcsecond power spectrum curve is lower than the 15 arcsecond curve near mid-frequencies, with other portions being very similar. Analysis of the 10 arcsecond reconstruction power spectrum shows that the 10 and 5 arcsecond curves are very close in the low-frequency region, but the 10 arcsecond power spectrum curve is generally lower than the 5 arcsecond curve from mid-frequency to mid-high frequency. As reconstruction sub-block

size increases, the differences between power spectrum curves decrease and tend to concentrate in the mid-frequency region.

**2.3.2 Phase Analysis** Both phase and modulus determine reconstruction quality. To isolate phase effects, a fixed modulus was combined with reconstructed phases from different sub-block sizes. The fixed modulus was obtained from the 5 arcsecond sub-block reconstruction, and phases corresponding to 10, 15, and 20 arcsecond sub-block sizes were combined to produce the partial images shown in [Figure 6: see original paper]. The region represented in [Figure 6: see original paper] corresponds to that in [Figure 4: see original paper], with phases from left to right corresponding to 10, 15, and 20 arcsecond reconstructions.

[Figure 6: see original paper] shows that image quality gradually degrades with increasing reconstruction sub-block size, all being lower than the 5 arcsecond case in [Figure 4: see original paper]. Since the modulus is identical, the degradation in image quality is caused by reduced phase reconstruction accuracy. Comparison of image quality indicates that the 5 arcsecond sub-block reconstruction achieves the highest phase accuracy, closest to the true target phase.

The phases reconstructed at 10, 15, and 20 arcseconds in [Figure 6: see original paper] were expressed in complex exponential form and subtracted from the complex exponential form of the 5 arcsecond phase for the corresponding region, with the modulus of the subtraction result calculated. Equation (16) gives the calculation formula:

$$Z = |e^{j\Phi_5} - e^{j\Phi_m}|$$

where  $|\cdot|$  denotes the modulus of a complex number,  $\Phi_5$  is the phase reconstructed at 5 arcseconds, and  $\Phi_m$  represents phases reconstructed at 10, 15, and 20 arcseconds.

The calculation yields the exponential phase subtraction images shown in [Figure 7: see original paper], with the modulus of subtraction results between phases reconstructed at 10, 15, and 20 arcseconds and the 5 arcsecond phase for the same region, from left to right. [Figure 7: see original paper] reveals differences in mid-to-high frequency phases between the 10, 15, and 20 arcsecond cases and the 5 arcsecond case, with increasing differences in the low-to-mid frequency region as sub-block size increases. Phase reconstruction accuracy decreases with increasing reconstruction sub-block size.

### 3 Discussion and Summary

When reconstruction sub-block sizes of 10, 15, and 20 arcseconds exceed the typical astronomical isoplanatic patch size, anisoplanatism causes speckle interferometry and speckle masking to fail. The resulting reduction in accuracy of Fourier modulus and phase reconstruction leads to decreased image resolution

and loss of detail. The analysis demonstrates that reconstruction results vary with sub-block size, with image quality degrading as sub-block size increases. This indicates that larger reconstruction sub-blocks, exceeding the isoplanatic patch size by greater margins, experience more severe anisoplanatic effects on speckle interferometry and speckle masking.

For optimal reconstruction resolution, blocking according to the isoplanatic patch size minimizes anisoplanatic effects, potentially to negligible levels. However, real-time determination of atmospheric isoplanatic patch size is difficult. Smaller sub-blocks do not necessarily yield better results; reconstruction at 2 arcseconds was computed for the above data but proved less effective than 5 arcseconds. This occurs because data alignment is required before speckle reconstruction to compensate for telescope vibration or tracking errors, and excessively small sub-blocks reduce alignment precision, also affecting image quality.

In data processing, sub-block size must comprehensively consider multiple factors such as atmospheric seeing and wind speed. Given the actual implementation of the 1m solar telescope high-resolution reconstruction algorithm, it is recommended that when reconstructing high-resolution images from a period of observational data, a small subset of images first be reconstructed using various sub-block sizes. By comparing results from different sub-block sizes, the most reasonable reconstruction sub-block size can be determined before proceeding with full dataset reconstruction.

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