Nonlinear Filtering Applications in SINS: Postprint

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Abstract

For the problem of attitude and angular velocity estimation of a vehicle, estimation algorithms based on Kalman filter and particle filter are respectively proposed. To address the singularity problem caused by Euler angles, quaternions are employed to describe the attitude angles of the vehicle. By collecting data through the mpu9250 module (gyroscope, accelerometer, and magnetometer) and utilizing both experimental data and simulation operations, the feasibility of the quaternion Kalman filter and particle filter algorithms is verified. The computational results from static experiments and dynamic simulations indicate that when measuring vehicle attitude using MEMS devices, the mean errors obtained by the particle filter and Kalman filter algorithms are comparable, but the standard deviation of the particle filter is relatively smaller.

Full Text

Application of Nonlinear Filtering in SINS

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Abstract

This paper presents two estimation algorithms for carrier attitude and angular velocity estimation: one based on Kalman filtering and the other on particle filtering. To address the singularity problem associated with Euler angles, quaternions are employed to describe the carrier’ s attitude. Data acquisition from an MPU9250 module (integrating a gyroscope, accelerometer, and magnetometer) was conducted, and both experimental data and simulation results verify the feasibility of the quaternion-based Kalman filter and particle filter algorithms.

Computational results from static experiments and dynamic simulations demonstrate that when measuring carrier attitude using MEMS devices, both particle filtering and Kalman filtering yield comparable error means, but the particle filter exhibits a relatively smaller standard deviation.

Keywords: Quaternion; Kalman filter; Particle filter; Attitude estimation

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1. Introduction

Attitude estimation for micro aerial vehicles and robots often employs filters to fuse multi-sensor data from Micro-Electro-Mechanical Systems (MEMS) Inertial Measurement Units (IMU). Common MEMS devices include three-axis gyroscopes, three-axis accelerometers, and three-axis magnetometers. This paper utilizes quaternions for attitude calculation. Reference [1] established quaternion Kalman filter equations, while reference [2] analyzed the performance of the quaternion Kalman filter proposed in [1]. Numerous researchers have also investigated particle filtering for attitude estimation [3]. References [4] and [5] designed quaternion particle filter algorithms for attitude determination through quaternion analysis. This paper collects and processes data from MEMS devices, designs nonlinear quaternion Kalman filtering and particle filtering algorithms, and further analyzes their attitude calculation results.

2. Quaternion Fundamentals

The output from micro-mechanical inertial measurement devices on a carrier can be converted into the carrier' s attitude, representing the angular position of the carrier' s body coordinate system (x_b, y_b, z_b) relative to the navigation coordinate system (x_n, y_n, z_n), expressed in matrix form as:

$$] = A_{z-y-x} * [], \quad (1)$$

where the Euler angles are the three attitude angles of the carrier: pitch, roll, and yaw. According to Euler' s rotation theorem, the body coordinate system can be made to coincide with the navigation coordinate system through three successive rotations, each about one axis of the navigation coordinate system. The rotation angles are the Euler angles, and the coordinate relationship after each rotation can be represented by a rotation matrix, i.e., the direction cosine matrix. Equation (1) employs a z-y-x rotation sequence:

$$A_{z-y-x}(\psi,\theta,\phi) = A_z(\phi)A_y(\theta)A_x(\psi) = \begin{bmatrix} \cos\theta * \cos\varphi & \cos\theta * \sin\varphi & \\ \sin\emptyset * \sin\theta * \cos\varphi - \cos\emptyset * \sin\varphi & \sin\emptyset * \sin\theta * \sin\varphi + \cos\emptyset * \cos\varphi & \mathrm{s} \\ \cos\emptyset * \sin\theta * \cos\varphi + \sin\emptyset * \sin\varphi & \cos\emptyset * \sin\theta * \sin\varphi - \sin\emptyset * \cos\varphi & \mathrm{c} \end{bmatrix}$$

where ψ, ϕ, and θ represent yaw, roll, and pitch angles, respectively. To avoid the singularity problem that may occur when using Euler angles to represent attitude, quaternions are widely applied in attitude representation. The carrier attitude quaternion is designed as:

$$q = q_0 + q_1 i + q_2 j + q_3 k, \quad (3)$$

The coordinate transformation between the navigation coordinate system and the body coordinate system can be represented by the direction cosine matrix, whose quaternion form is:

$$A(q)_n^b = \begin{bmatrix} 2 + q_1^2 - q_3^2 - q_2^2 & 2(q_1 q_2 - q_0 q_3) & 2(q_1 q_3 + q_0 q_2) \\ 2(q_1 q_2 + q_0 q_3) & 2 - q_1^2 - q_3^2 + q_2^2 & 2(q_2 q_3 - q_0 q_1) \\ 2(q_1 q_3 - q_0 q_2) & 2(q_2 q_3 + q_0 q_1) & 2 + q_3^2 - q_2^2 - q_1 \end{bmatrix}, \quad (4)$$

$$A(q)_n^b = \begin{bmatrix} T_{11} & T_{12} & T_{13} \\ T_{21} & T_{22} & T_{23} \\ T_{31} & T_{32} & T_{33} \end{bmatrix}. \quad (5)$$

During the rotation transformation from the navigation coordinate system to the body coordinate system, the coordinate system remains orthogonal, so $A(q)_n^b = (A(q)_n^b)^{-1} = (A(q)_b^n)^T$. Thus, the carrier' s attitude angles are:

$$\theta = -\arcsin T_{13}, \quad \emptyset = \arctan, \quad \varphi = \arctan. \quad (6)$$

The quaternion differential equation is as follows:

$$q = q \cdot \omega, \quad (7)$$

where q is the quaternion describing the carrier' s rotation and ω is the angular velocity of the carrier relative to the navigation reference coordinate system, also expressed in quaternion form:

$$\omega = 0 + \omega_x \cdot i + \omega_y \cdot j + \omega_z \cdot k, \quad (8)$$

$$\Omega = \begin{bmatrix} 0 & -\omega_x & \omega_y & -\omega_z \\ \omega_z & \omega_y & -\omega_y & -\omega_z \\ \omega_z & -\omega_y & & \end{bmatrix}, \quad (9)$$

$$\cdot \emptyset_w \cdot q. \quad (10)$$

By measuring the three-axis angular velocities from the gyroscope, the quaternion values can be updated in real time, thereby updating the attitude angles to obtain attitude information.

3. Quaternion Kalman Filter (EKF)

The most common algorithm combining quaternions with nonlinear filtering is the quaternion-based Kalman filter. This paper employs the quaternion Kalman filter from reference [1], with the filtering process summarized as follows:

Filter Initialization: Select initial values for Q and R by measuring the three-axis gyroscope, accelerometer, and magnetometer in static conditions and calculating the variance of each axis as the filter data error. The initial pitch and roll angles are measured from the initial accelerometer, and the azimuth angle is measured from the magnetometer to serve as initial attitudes, which are then converted to the initial quaternion $q_{0/0}$. The system initial noise is selected as $P_{0/0} = I_4$.

State Equation Propagation:

$$\Omega_b = H(\omega^b_{kq}), \quad (11)$$

$$\Phi_{k+1/k} = \exp(\Omega_b * \Delta t), \quad (12)$$

$$q_{k+1/k} = \Phi_{k+1/k} * q_{k/k}, \quad (13)$$

$$M_{k/k} = q_{k/k} q^T_{k/k} + P^q_{k/k}, \quad (14)$$

$$P_{k+1/k} = \Phi_{k+1/k} P^q_{k/k} \Phi_{k+1/k} [tr(M_{k/k}) I_4 - M_{k/k}] Q_k. \quad (15)$$

Measurement Equation Update: First, calculate the coefficient matrix $H(q_{k+1/k})$ in the observation equation using the predicted $q_{k+1/k}$ from equation (4):

$$M_{k+1/k} = q_{k+1/k} q_{k+1/k} + P_{k+1/k}, \quad (16)$$

$$B_{k+1} = \bar{H}(b_{k+1}), \quad (17)$$

$$P_{k+1/k} R_{k+1} [tr(M_{k+1/k}) I_4 - M_{k+1/k} - B_{k+1} M_{k+1/k} B_{k+1}], \quad (18)$$

$$S_{k+1/k} = H(q_{k+1/k}) * P_{k+1/k} * H(q_{k+1/k}) + P_{k+1/k}, \quad (19)$$

$$K_{k+1} = P_{k+1/k} * H(q_{k+1/k}) * S_{k+1/k}, \quad (20)$$

$$q_{k+1/k+1} = q_{k+1/k} + K_{k+1}[b_{k+1} - H(q_{k+1/k}) * n_{k+1}], \quad (21)$$

$$P_{k+1/k+1} = (I_4 - K_{k+1}H(q_{k+1/k}))P_{k+1/k}. \quad (22)$$

4. Particle Filter

Particle filtering approximates state variables by seeking a set of random samples propagated in the state space, replacing integration operations with sample means to obtain the system' s minimum variance process. These samples are called particles. Mathematically, for a stationary random process, assuming the system' s posterior probability at time $k-1$ is $p(x_{k-1}|z_{k-1})$, n random samples are selected according to certain principles. After measurement updates at time k and undergoing state and temporal processes, the posterior probability density of the n particles can approximate $p(x_k|z_k)$. As the number of particles increases, the particle probability density function gradually approaches the state's probability density function, enabling the particle filter to achieve optimal Bayesian estimation [3].

Assuming a nonlinear dynamic discrete system:

$$x_{k+1} = f_k(x_k, w_k),$$

$$z_k = h_k(x_k, v_k),$$

where $x_k \in R^n$ is the n-dimensional state vector at time k, $z_k \in R^m$ is the m-dimensional observation vector, and w_k and v_k are process noise and measurement noise, respectively.

The particle filter algorithm proceeds as follows:

(1) Initialization: At $k = 0$, generate N particle points $x_0^{(i)}$, $i = 1 \cdots n$, following the initial probability density $p(x_0)$.

(2) Update sample particle states through importance sampling:

$$x_k^{(i)} \sim q(x_k|x_{0:k-1}, z_{1:k}), \quad (24)$$

$$w_k^{(i)} = w_{k-1}^{(i)} \frac{p(z_k|x_k^{(i)})p(x_k^{(i)}|x_{0:k-1}^{(i)})}{q(x_k^{(i)}|x_{0:k-1}^{(i)}, z_{1:k})}, \quad i = 1 \cdots N. \quad (25)$$

(3) Calculate updated particle weights:

$$w_k^{(i)} = w_{k-1}^{(i)} p(z_k|x_k)p(x_k|x_{0:k-1}|x_{k-1}, z_{1:k}). \quad (27)$$

(4) Normalize particle weights:

$$w_k^{(i)} = \frac{w_k^{(i)}}{\sum w_k^{(i)}}. \quad (28)$$

(5) Resampling: Based on the normalized weights $w_k^{(i)}$, replicate or discard samples to obtain N samples $x_{0:k}^{(i)}$ approximately distributed according to $p(x_{0:k}^{(i)}|z_{1:k})$, and set $w_k^{(i)} = 1/N$, $i = 1 \cdots N$.

(6) Output results: The algorithm outputs the particle set $\{x_{0:k}^{(i)}, i = 1 \cdots N\}$, which can represent the posterior probability and the expectation of function $g(x_{0:k})$:

$$p(x_{0:k}|z_{1:k}) = \sum \delta_{x_{0:k}^{(i)}}(dx_{0:k}), \quad (29)$$

$$E(g(x_{0:k})) = \sum g_k(x_{0:k}^{(i)}). \quad (30)$$

**(7) Set $k = k + 1$ and repeat the above steps.

5. Experiments

The MPU9250 sensor module was selected for experiments, which integrates a three-axis gyroscope, three-axis accelerometer, and three-axis magnetometer, capable of directly outputting nine-axis sensor data through its onboard low-pass filters and A/D conversion modules. The MPU6050 was fixed on a turntable for measurement, installed away from environmental magnetic field interference. A microcontroller read the MPU9250 measurement data and transmitted it via I2C serial interface to a computer for MATLAB processing to verify algorithm feasibility and accuracy.

(1) Static Data Collection: The sensor module MPU9250 was measured in a static state at a sampling rate of 100 Hz for one minute to analyze the nine-axis output data under static conditions. The three-axis gyroscope static data is shown in [Figure 1: see original paper]. Figure 1 displays one minute of collected three-axis gyroscope data, with x-axis, y-axis, and z-axis from top to bottom. MATLAB analysis yields x-axis mean and variance of 0.0205°/s and 1.238e-4 $(°/s)^2$; y-axis mean and variance of -0.0048°/s and 1.57e-4 $(°/s)^2$; and z-axis mean and variance of 0.0154°/s and 2.3699e-4 $(°/s)^2$. The same method was applied to process raw data from the three-axis accelerometer and magnetometer, with results shown in .

Table 1 Mean and variance of accelerometer and gyroscope output data

Three-axis accelerometer (g)	Three-axis magnetometer (μT)

The measurements show that the accelerometer' s x-axis and y-axis means are non-zero at level position, introducing mean errors that must be accounted for in attitude calculation to improve measurement accuracy.

(2) Static Filter Performance: Using the variances from the static data as references for the quaternion Kalman filter and particle filter programs, the attitude angle calculation errors in static state are shown in [Figure 2: see original paper] and [Figure 3: see original paper]. The mean and variance of the three-axis attitude angles for both algorithms in static state are presented in .

Table 2 Mean and variance of the two algorithms for the quiescent state

Algorithm	Mean	Variance
Quaternion Kalman		

Static data analysis reveals that both algorithms provide similar mean improvement, but the particle filter shows improved variance compared to the quaternion Kalman filter, indicating more stable static measurements.

(3) Dynamic Simulation: Without turntable equipment, a stable attitude simulation trajectory was added to the static data to evaluate both algorithms under dynamic conditions. By adding rotation rates of 2, 3, and 5°/s and amplitude of 5 to the three-axis attitude angles, the simulation results shown in [Figure 4: see original paper] were obtained.

Table 3 Mean and variance of the two algorithms for simulation states

Algorithm	Mean	Variance
Quaternion Kalman		

Dynamic simulation analysis demonstrates that both algorithms maintain similar mean improvement as in static data, but the particle filter again shows variance improvement over the quaternion Kalman filter, indicating relative stability under dynamic conditions.

6. Conclusion

This paper introduced quaternion Kalman filtering and particle filtering, verified their feasibility through data collection from MPU9250 and simulation experiments, and demonstrated that particle filtering improves the standard deviation compared to quaternion Kalman filtering when using error mean and standard deviation as evaluation criteria.

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Note: Figure translations are in progress. See original paper for figures.