Extraction and Synthesis of Frequency Information from Degraded Images in Golay3-type Optical Sparse Aperture Systems: Postprint

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Abstract

Utilizing small satellites constituting a constellation, each carrying separate sub-telescopes and a beam-combining imaging telescope to form a Fizeau-type optical synthetic aperture interferometric system for high-resolution imaging of extended targets represents a current research focus. Such optical interferometric imaging systems exhibit high sparsity, resulting in incomplete UV coverage, i.e., discontinuous spatial frequency sampling, which manifests as zero values in the system' s optical transfer function. To overcome the effects of incomplete UV coverage and achieve imaging performance equivalent to a large-aperture telescope, it is necessary to vary the spatial arrangement of the sub-apertures, acquire images under different baseline conditions, extract and synthesize spatial frequency information, and finally employ inverse filtering methods to improve image quality. Based on analyzing the relationship between the single sub-aperture transfer function and the system transfer function, this approach optimizes sub-aperture configurations, utilizes different frequency-domain filters to extract and synthesize frequency regions with high signal-to-noise ratio from images obtained at various baselines, then transforms to the spatial domain for inverse filtering processing to obtain improved synthetic images. Simulation results demonstrate that this method can effectively enhance synthetic image quality when the obtained interferograms exhibit low signal-to-noise ratios.

Full Text

Frequency Information Extraction and Synthesis of Degraded Images from Golay3-Type Optical Sparse Aperture Systems

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Abstract

A Fizeau-type sparse-aperture imaging system composed of a constellation of small satellites, each carrying a separate sub-telescope and a combined imaging telescope, represents a current research focus for achieving high-resolution imaging of extended targets. Due to the high sparsity of such optical interferometric systems, the spatial frequency coverage is incomplete, resulting in zero values in the optical transfer function. To overcome this incomplete coverage and obtain imaging performance equivalent to a large-aperture telescope, the spatial arrangement of sub-apertures must be varied to acquire images under different baseline conditions, enabling extraction and synthesis of spatial frequency information, followed by inverse filtering to improve image quality. Based on analyzing the relationship between single-sub-aperture transfer functions and the system transfer function, we optimize sub-aperture configurations and employ different frequency-domain filters to extract high signal-to-noise ratio (SNR) frequency regions from images obtained at various baselines. After synthesis and inverse filtering in the spatial domain, improved composite images are obtained. Simulation results demonstrate that when interferograms have low SNR, this method can effectively enhance the quality of synthesized images.

Keywords: sparse aperture; image reconstruction; frequency-domain filtering; image synthesis

1. Introduction

Traditional telescope imaging resolution is limited by aperture diffraction effects. Constrained by manufacturing capabilities and cost, optical system apertures cannot be increased indefinitely [1]. To address this limitation, researchers have proposed optical interferometry methods. Fizeau-type interferometric techniques, with their capability for wide-field imaging and short-exposure imaging of extended sources, have attracted increasing attention as a current research hotspot. The US Air Force Weapons Laboratory initiated the UltraLITE project to establish an Earth observation system using deployable telescope arrays [2]. Combined with recent advances in small satellite constellation technology, researchers have proposed Fizeau-type interferometric imaging systems based on small satellites. The basic approach involves each small satellite carrying a small-aperture telescope, with sub-satellites and a master satellite arranged in specific spatial configurations. Light beams from the sub-telescopes interfere coherently at the focal plane of the combined imaging telescope carried by the central master satellite.

Such interferometric imaging systems inevitably become highly sparse optical synthetic aperture systems due to collision avoidance requirements for small satellites and constraints on payload mass and volume. This sparsity causes missing samples in the spatial frequency domain, resulting in incomplete coverage. To better sample the spatial frequency information of extended targets and overcome incomplete coverage, the relative positions between sub-satellites and the master satellite can be changed to alter the system's sampling frequency coverage. By acquiring interferograms at different baseline configurations—thereby obtaining previously missing spatial frequency sampling information—and subsequently performing image synthesis and inverse filtering, a final image containing more complete target information can be obtained. This approach effectively achieves resolution equivalent to a much larger optical system.

Conventional synthesis methods align and directly superimpose images obtained at different baselines before deconvolution [3]. However, due to factors such as small sub-telescope apertures and limited integration time, the acquired images typically suffer from low SNR. When interferograms have poor SNR, direct superposition followed by deconvolution yields unsatisfactory results. This paper proposes extracting high-SNR frequency-domain information from images obtained at different baselines using appropriate frequency-domain filters, then synthesizing this information to reduce noise in the composite image and achieve better reconstruction quality.

2. Imaging Model and Noise Analysis

The imaging process can be viewed as the target' s frequency function $F(f_x, f_y)$ being multiplied by the optical transfer function $OTF(f_x, f_y)$ to produce the output image spectrum. Under incoherent illumination, a diffraction-limited system modifies only the intensity of each frequency component without altering their phase. The optical transfer function numerically equals the modulation transfer function $MTF(f_x, f_y)$.

A sparse aperture imaging system can be idealized as comprising N circular sub-apertures of diameter d. The aperture function $P(x, y)$ can be expressed as the convolution of a single sub-aperture pupil function with a δ function array:

$$P(x,y) = \mathrm{circ}\left(\frac{\sqrt{x^2+y^2}}{d/2}\right) \otimes \sum_{n=1}^{N} \delta(x - x_n, y - y_n)$$

where circ is the circular aperture function and (x_n, y_n) are the center coordinates of the n-th sub-aperture.

The Golay3 configuration, the simplest two-dimensional array structure, consists of three circular sub-apertures with centers at the vertices of an equilateral triangle [4]. From this, the normalized modulation transfer function for a Golay3 sparse aperture can be derived as:

$$MTF = MTF_d + \frac{1}{3} MTF_d * \sum_{i,j} \delta(\xi - (x_i - x_j), \eta - (y_i - y_j))$$

where MTF_d represents the modulation transfer function of a single sub-aperture, and $(x_i - x_j, y_i - y_j)$ denotes the relative positions of sub-apertures. This shows that the sparse aperture system' s MTF consists of single-sub-aperture MTFs distributed at different positions. If the MTF distribution is known, the sub-aperture arrangement can be determined. When the system MTF exhibits zero values or severe attenuation in certain spatial frequency regions, corresponding spatial positions can be identified to optimize sub-aperture configurations. Degraded images from different arrangements (different baselines) are then synthesized.

The primary noise sources in sparse aperture systems are photon noise and detector noise, with the overall noise distribution approximating Gaussian distribution—zero-mean white noise with uniform power spectral density across the frequency domain [5]. If the noise power of a single degraded image is n_i, direct superposition yields $\sum_i n_i$ noise, significantly higher than the proposed method.

3. Frequency Extraction and Synthesis Method

By calculating the relationship between sub-aperture configuration and system MTF, the SNR characteristics in different frequency regions of degraded images can be readily determined. Figure 1 shows a schematic MTF for one sub-aperture configuration, where the central peak indicates maximum signal transmission (normalized to 1), while surrounding secondary peaks transmit only 1/3 of the maximum signal intensity. The corresponding interferogram SNR maxima are $10 \log(3S/N)$, $10 \log(S/N)$, and 0 dB, where S and N represent effective signal and noise power, respectively.

We define corresponding frequency-domain filters H_i for various baseline configurations. Each filter extracts the high-SNR frequency portion from its respective image, with weights n_i significantly smaller than those from direct superposition. Since the filters' passbands do not completely overlap, the final synthesized image' s noise level $\sum_i H_i \cdot n_i$ is substantially reduced, yielding markedly improved SNR.

Starting from the tangential arrangement of sub-apertures and moving them outward twice at increments of one sub-aperture diameter produces three configurations, with the maximum enclosing circle diameter of 5.4 m. Analysis reveals that spatial frequency information near the cutoff frequency boundaries between configurations 4 and 5 shows significant attenuation. Therefore, additional sub-aperture configurations are introduced to compensate and enhance the MTF near these boundaries, resulting in five total configurations with an equivalent aperture diameter of 6.15 m.

Figure 2 shows partial cross-sections of the optimized MTFs for all configurations in the maximum cutoff frequency direction. Based on each configuration' s MTF distribution, we define corresponding frequency-domain filters H_i ($i = 1$ to 5). Equations (4) through (8) represent low-pass and band-pass filters with different passband ranges.

The frequency spectrum synthesis method extracts high-SNR frequency information from five images using these filters. The synthesis process is illustrated in Figure 3, where direct spectrum superposition follows:

$$G_{\text{direct}} = \sum_i G_i$$

while the proposed extraction method follows:

$$G_{\text{synthesis}} = \sum_i G_i \cdot H_i$$

4. Deconvolution and Image Reconstruction

Synthesized images from different baselines require deconvolution for proper restoration. Inverse filtering is the most direct deconvolution method:

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)}$$

where $G(u,v)$ is the degraded image' s Fourier spectrum and $H(u,v)$ is the point spread function' s Fourier spectrum.

Wiener filtering improves upon inverse filtering by incorporating statistical properties of both the degradation function and noise:

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)} \cdot \frac{|H(u,v)|^2}{|H(u,v)|^2 + \gamma \frac{S_n(u,v)}{S_f(u,v)}}\right] G(u,v)$$

where S_n/S_f is the noise-to-signal power spectrum ratio, typically approximated by a constant K, and γ is a weighting coefficient. After frequency extraction and synthesis, the composite image' s noise-to-signal ratio exhibits noticeable variations across regions. When noise statistics are unknown, the system becomes a pure deconvolution filter. We employ an improved Wiener filter using different K values for different regions based on the synthesized image' s noise-to-signal ratio variations.

Figure 5 shows reconstructed images: (a) equivalent full-aperture restoration, (b) direct superposition restoration, and (c) frequency extraction method restoration. The frequency extraction method produces superior results, particularly as image SNR decreases.

5. Evaluation Metrics

In addition to subjective assessment, we employ objective metrics including Mean Squared Error (MSE) and Structural Similarity Index (SSIM). MSE calculates gray-level differences at each pixel based on error theory, while SSIM quantifies brightness, contrast, and structure separately through weighted multiplication, better aligning with human visual perception [7].

$$MSE = \frac{1}{MN}\sum_{i=1}^{M}\sum_{j=1}^{N} e_{ij}^2$$

$$MSSIM = \frac{1}{M}\sum_{k=1}^{M} SSIM(x_k, y_k)$$

Lower root-mean-square error and higher mean structural similarity indicate better image quality.

Table 1 presents objective evaluation results for degraded images at different SNR levels. At 35 dB SNR, the proposed method shows only slight improvement, with visually indistinguishable differences. However, at 25 dB SNR, objective metrics clearly favor the frequency extraction method, and subjective evaluation reveals significantly clearer image details.

6. Conclusion

When original images have limited SNR, using the frequency extraction algorithm to synthesize interferograms from different sub-aperture configurations into a single degraded image containing more complete target information, followed by improved Wiener filtering, yields a clearer final reconstructed image. By preserving high-SNR frequency components from each interferogram, the synthesized image quality shows marked improvement over direct superposition, as confirmed by both visual assessment and objective metrics.

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