Postprint: Comparison of Maximum Entropy and MP-CLEAN Methods for Extended Source Image Reconstruction

Authors: Zhang Maolin, Zhou Jianfeng

Date: 2017-10-20T00:00:00+00:00

Abstract

In the field of radio synthesis imaging, deconvolution methods are typically required to complement sparsely sampled frequency domain data. Since extended sources possess richer frequency domain information, complementing this information is more difficult compared to point sources; therefore, image reconstruction of extended sources represents a major challenge in radio synthesis imaging. To investigate the characteristics of radio interferometric image reconstruction methods for extended sources, a comparison was conducted between Maximum Entropy and an accelerated CLEAN method (referred to in this paper as Multi-Point CLEAN, MP-CLEAN) for the image reconstruction of simulated interferometric array data of extended sources. Through this comparison, it was found that both methods can reconstruct images reasonably well for the same observational data; however, the MP-CLEAN method exhibits superior sidelobe removal and reconstruction performance compared to the Maximum Entropy method, and in the reconstruction of simulated data, the overall speed of MP-CLEAN is more than three times faster than that of Maximum Entropy. Finally, in the discussion section, by investigating the influence of parameter selection in both methods on the reconstruction results, it was discovered that the Maximum Entropy method exhibits weaker dependence on parameter selection compared to MP-CLEAN, indicating that the Maximum Entropy method possesses better robustness.

Full Text

A Comparison of Methods for Extended Source Image Reconstruction

Authors: Department of Engineering Physics and Center for Astrophysics, Tsinghua University, Beijing; Key Laboratory of Particle & Radiation Imaging

(Tsinghua University), Ministry of Education, Beijing

Abstract

In radio synthesis imaging, deconvolution methods are typically employed to fill in sparsely sampled points in the frequency domain. Extended sources possess more complex visibility information in the frequency domain, making their reconstruction considerably more challenging than that of point sources. Consequently, extended source image reconstruction represents a major difficulty in radio synthesis imaging. To investigate the characteristics of radio interferometric extended source reconstruction methods, this paper compares the Maximum Entropy Method (MEM) with an accelerated CLEAN method (termed Multi-Point CLEAN, or MP-CLEAN for short) for reconstructing images of extended sources from simulated interferometric array data. The comparison reveals that both methods can achieve satisfactory image reconstruction from identical observational data, but MP-CLEAN demonstrates superior sidelobe suppression and overall reconstruction quality compared to MEM. Furthermore, MP-CLEAN is at least three times faster than MEM in our simulations. Finally, through examining the parameter selection dependencies of both methods, we find that MEM exhibits weaker dependence on parameter choices, indicating better robustness.

Keywords: Image reconstruction; Radio interferometer array; Multi-Point CLEAN; Maximum Entropy Method; Extended source

1. Introduction

Radio astronomy is a branch of astronomy that observes celestial objects by detecting radio waves. The wavelength of radio waves is much longer than that of visible light, which historically imposed significant limitations on angular resolution in radio observations. For example, at a typical radio wavelength, the resolution of a radio telescope would be approximately one hundred thousand times worse than that of an optical telescope with equivalent aperture. This resolution limitation was a major obstacle in the development of radio astronomy until the advent of interferometric imaging technology, whose founders were awarded the Nobel Prize. Over subsequent decades, large radio interferometric arrays such as the Martin One-Mile Telescope, Green Bank Interferometer, Westerbork Synthesis Radio Telescope, and Very Large Array have greatly advanced radio astronomy. Modern radio interferometers now rank among the world' s highest-resolution telescopes—for instance, the Atacama Large Millimeter/submillimeter Array (ALMA) achieves resolution comparable to the Hubble Space Telescope.

An interferometer array obtains visibility data (equivalent to spatial frequency spectrum data) through interference principles. Since interference arises from the coherence of signals from two antennas, the obtained visibility data depends on both the number of antennas and their relative positions. However, practical constraints on construction technology and funding limit the number and size of antennas that can be built. As interferometric technology continues to evolve, increasingly larger arrays are being planned, which introduces new challenges in data processing. The arrangement of an interferometer array significantly affects imaging quality, as demonstrated in studies of domestic arrays.

Radio interferometric observations provide sparse sampling of the image' s frequency domain space. Direct inverse Fourier transformation of the acquired visibility data typically yields blurred images with strong sidelobes. To reconstruct clear images, deconvolution is necessary. Common deconvolution methods in radio astronomical image reconstruction include CLEAN and its derivatives such as Multi-Scale CLEAN, Multi-Resolution CLEAN, and the Maximum Entropy Method (MEM). While these methods perform well for simple point sources, they suffer from large computational requirements, slow convergence, and poor stability when reconstructing complex, large-area diffuse sources, often producing unsatisfactory results.

Representative methods suitable for extended source reconstruction fall into two categories: those based on the Maximum Entropy Method and those based on multiscale approaches derived from CLEAN. This paper compares MEM with MP-CLEAN, a simplified acceleration of the multiscale CLEAN method proposed by Cornwell. Unlike traditional CLEAN, which selects only the brightest point in each iteration, MP-CLEAN simultaneously selects multiple points exceeding a fraction of the maximum value (where $0 < \quad < 1$). This approach accelerates CLEAN while leveraging physical correlations between neighboring pixel values, making it suitable for complex extended source reconstruction. Both MEM and MP-CLEAN are considered fundamental methods for extended source reconstruction, and comparing these two primary approaches can reveal their respective characteristics and provide insights for further improvement of radio interferometric extended source imaging techniques.

2. Methodology

2.1 Model Establishment To simplify the model, this work assumes that the visibility data of a sky map is equivalent to its spatial frequency spectrum, using Fourier spectrum data directly as visibility data. For any given brightness distribution $I(x, y)$, its visibility can be expressed as:

$$V(u, v) = \iint I(x, y) \exp[-2\pi i(ux + vy)]\, dx\, dy$$

where $V(u, v)$ represents visibility data and $I(x, y)$ is the brightness distribution image. Thus, visibility is the Fourier transform of the image. Radio telescope observations effectively sample the frequency domain of the sky map. If complete visibility sampling were obtained, the true brightness distribution could be recovered directly through inverse Fourier transformation. However, actual data typically involve partial sampling.

The sampling function is given by:

$$S(u,v) = \sum_k \omega_k \delta(u - u_k, v - v_k)$$

where ω_k represents weighting factors (taken as constants in this paper), δ is the Dirac delta function, and (u_k, v_k) determines the position of the k-th sampling point. The sampled visibility data becomes:

$$V_k = V(u_k, v_k) = \iint I(x,y) \exp[-2\pi i(u_k x + v_k y)]\, dx\, dy$$

This process is clearly irreversible, as information is lost and the true complex image cannot be recovered without error. When observational noise is considered, the visibility data can be expressed as:

$$V_k^{\text{obs}} = \iint I(x,y) \exp[-2\pi i(u_k x + v_k y)]\, dx\, dy + n_k$$

where n_k denotes noise at the k-th sampling point in the UV plane. In the image domain, this relationship means we know the dirty map and point spread function, and we must employ deconvolution methods to determine the true brightness distribution $I(x,y)$.

2.2 Maximum Entropy Method Before introducing the Maximum Entropy Method, we first define entropy. Gull and Skilling proposed an entropy form:

$$H = -\sum_i b_i \ln\left(\frac{b_i}{m_i}\right)$$

where b represents the image to be reconstructed and m is the prior image model, with i indexing the pixels.

Combining this entropy definition with observational data, we construct an objective function using Lagrange multipliers:

$$J = H - \alpha\chi^2$$

where α is a constant and χ^2 represents the normalized sum of squared deviations between observed visibility data and reconstructed image visibility:

$$\chi^2 = \sum_k \omega_k \frac{|V_k - V_k'|^2}{\sigma_n^2}$$

Here, V_k is the observed visibility data, V_k' is the resampled visibility from the reconstructed image, ω_k is the weighting factor, and σ_n^2 is the variance of observational noise (taken as 1 in this paper). The solution is the image b that maximizes this objective function under observational constraints.

Various methods exist for solving this extremum problem. This paper employs Newton' s method, with iteration steps given by:

$$\Delta b = -(\nabla^2 J)^{-1} \nabla J$$

The implementation proceeds as follows: First, provide an initial reconstructed image b (in our simulations, the prior image m is a uniform image with all pixel values equal to the estimated total flux). Resample the Fourier domain of the reconstruction using the sampling function to obtain V_k'. Then calculate the iteration step size Δb using the observed data, resampled data, and sampling function parameters. Update the reconstructed image $b \leftarrow b + \Delta b$. Check termination conditions: if $\chi^2 \leq 1$ (normalized chi-square value less than or equal to 1) or maximum iterations reached, terminate. The final MEM result is obtained by convolving the reconstructed image with a Gaussian function corresponding to the resolution of the maximum baseline length.

2.3 MP-CLEAN Method The CLEAN method is one of the most commonly used deconvolution techniques in radio astronomical image reconstruction. Hogbom proposed a basic CLEAN algorithm in 1974. For description and comparison purposes, we refer to this as "CLEAN."

CLEAN requires an empirical assumption: the probability of different bright sources simultaneously affecting the same position is small, and the sidelobes of the point spread function are much lower than the main lobe. Even with sidelobe superposition, the peak values will not be excessively large. This enables selection of multiple real sources simultaneously. Therefore, in each iteration, we can select multiple points exceeding γ times the maximum value (where $0 < \gamma < 1$) rather than just a single point, thereby accelerating CLEAN. This approach is termed Multi-Point CLEAN (MP-CLEAN).

The parameter γ may vary in practice, but for simplicity, this paper uses a conservative constant value for γ within each reconstruction. The selection of γ is discussed in detail in Section 4.

The MP-CLEAN implementation proceeds as follows: First, provide an initial model M (typically all zeros). Calculate visibility data from the model using the sampling function. Compute the difference between model and observed visibility data, then perform inverse Fourier transformation and take the real part to obtain the residual map R. Find the maximum value MAX in R and identify all points greater than $MAX \times \gamma$. Create a map of the same size as the model where only these identified points retain their original positions and values, with all other pixels set to zero. Add this map to the original model.

Check termination conditions: iteration stops when the root-mean-square of the residual map's corresponding visibility data is less than or equal to the estimated noise level, or when $\chi^2 \leq 1$. Finally, convolve the result with a Gaussian function corresponding to the resolution of the maximum baseline length to obtain the MP-CLEAN reconstruction.

3. Numerical Simulations and Analysis

3.1 Simulation Data Generation Simulation data generation begins with interferometer array configuration and combination. The sampling function is determined by antenna positions and observational parameters. In this study, the sampling function consists of antennas arranged along a straight line, with positions referencing the Westerbork Synthesis Radio Telescope (WSRT). During observation, Earth' s rotation causes each baseline to sweep an arc in the sampling space. Longer observation times produce longer arcs. Given Earth' s rotational periodicity, this scanning process is equivalent to rotational sampling around a central diameter point. The scanned arcs are elliptical with an axis ratio of $1 : \sin\theta$, where θ is the declination of the observed source (taken as 45° in this paper). Since real signals produce complex visibility data, the sampling function must also sample the complex conjugate points.

An extended source image is selected as the true image (1200 × 1200 pixels, with each pixel representing 1 arcsec × 1 arcsec). Fourier transformation is applied to this image, and the sampling function is used to obtain visibility data. Adding noise to the sampled visibility yields simulated observational data. The interferometer' s maximum baseline length determines its resolution. With an observation wavelength λ of approximately 6 cm, the resolution is about 4.4 arcsec × 5.1 arcsec. For consistent resolution comparison, the "true image" used for evaluation is the original image convolved with a normalized two-dimensional Gaussian function corresponding to this resolution.

Since simulated visibility data are complex, the noise must also be complex. The amplitude follows a Gaussian distribution with standard deviation σ_n, while the phase follows a uniform distribution on $[0, 2\pi]$. The simulations assume accurate noise level estimation. The sampling function includes additional sampling at the origin (zero baseline), requiring total flux estimation, which is also assumed accurate in these simulations.

The simulation inputs are: the visibility data obtained by sampling the true image (bottom-right panel), estimated total flux, sampling function, and estimated noise level. [Figure 1: see original paper] shows the selected extended source (top-left), the UV coverage for a 12-hour observation (bottom-left), the logarithm of the visibility amplitude of the original image (top-right), and the logarithm of the sampled visibility amplitude with added noise (bottom-right).

3.2 Reconstruction Comparison Using the simulated data, the true image can be reconstructed via either MEM or MP-CLEAN. For the complex extended sources used in our simulations, no method can perfectly restore the original image because any deconvolution method essentially interpolates at unsampled frequency points. Reconstruction results can only be approximate estimates of the true image based on certain models.

In our simulations, we set $\alpha = 0.1$ for MEM and $\gamma = 0.6$ for MP-CLEAN. Parameter selection principles aim to make the converged χ^2 as close to 1 as possible while ensuring reconstruction stability. With these fixed parameters, both methods reconstruct the simulated visibility data. [Figure 2: see original paper] presents the reconstruction results for 8-hour and 12-hour observations.

The dirty map obtained by direct inverse Fourier transformation of the sampled data shows significant differences from the true image, primarily manifested by numerous sidelobes and stripe-like structures. Both MEM and MP-CLEAN suppress sidelobes to varying degrees. MP-CLEAN produces clearer results with better recovery of the extended source and its diffuse structures, while also recovering point sources within the sidelobes. MEM, however, fails to recover some sources and exhibits weak residual sidelobes.

Residual maps (difference between resampled reconstructed image visibility and observed visibility, inverse Fourier transformed) reveal that MEM residuals still faintly show shadows of the central extended source and many black point source remnants. In contrast, MP-CLEAN residuals show almost no such traces, indicating superior reconstruction performance. Overall, both methods produce good results that are vastly better than the dirty map.

For quantitative analysis, profile cuts are taken at $x = 601$ arcsec (right ascension direction). Profile difference maps are calculated as true image minus reconstructed image (distinct from residual maps). Due to random noise, statistical quantities vary slightly between reconstructions, but mean variations are less than 0.5 Jy and standard deviation variations are within 2 Jy—small enough not to affect comparative analysis.

shows the mean and standard deviation of profile differences for the 8-hour and 12-hour observations. Both methods recover total flux well. MP-CLEAN exhibits smaller standard deviations than MEM in both observations, indicating better consistency with the true image. Profile plots demonstrate that MP-CLEAN matches the true image even at faint edges, while MEM results still contain weak, difficult-to-remove sidelobes. Thus, MP-CLEAN' s sidelobe suppression is superior.

3.3 Reconstruction Speed Beyond reconstruction quality, speed is another crucial factor. [Figure 4: see original paper] shows χ^2 and maximum absolute residual versus iteration number and time. MP-CLEAN reaches $\chi^2 \approx 1$ in fewer than 15 seconds, while MEM requires about 50 seconds. MP-CLEAN needs roughly half the iterations of MEM, with each iteration taking approximately 0.2 seconds—about twice as fast as MEM' s 0.4 seconds per iteration. Consequently, MP-CLEAN' s total reconstruction time is less than one-third of MEM' s.

3.4 Reconstruction of Different Source Types To further validate these methods' characteristics, additional source images were tested using the same 12-hour observation setup and array configuration. Two irregular galaxy images were selected as Sources 1 and 2, along with one regular source to provide contrast. [Figure 5: see original paper] shows the reconstruction results.

Consistent with previous analysis, both methods recover flux well and reconstruct sources satisfactorily. MP-CLEAN again demonstrates better sidelobe suppression and smaller profile difference variance. For Source 1, MP-CLEAN completes iteration in under 15 seconds while MEM requires nearly 150 seconds. For Source 2, MP-CLEAN finishes in under 30 seconds versus over 150 seconds for MEM—five times faster. These results confirm that MP-CLEAN maintains its advantages across different extended sources.

4. Discussion of Parameter Selection

Reconstruction quality depends on parameter selection. This section examines how parameters affect results to better understand each method' s characteristics.

4.1 Maximum Entropy Parameter For MEM, parameter controls the relative weight between entropy and χ^2 constraints. [Figure 6: see original paper] shows χ^2 evolution curves for different values, while [Figure 7: see original paper] displays profile difference maps and provides quantitative statistics.

If is too large (e.g., $= 0.5$), χ^2 converges rapidly but to a value far from 1, causing significant overall deviation from the true image (large mean difference). As increases, the weight on observational data increases while entropy' s smoothing constraint decreases, leading to larger standard deviations. If is too small ($= 0.01$), χ^2 fails to decrease effectively.

The results show that $= 0.1$ provides balanced performance: χ^2 converges near 1, mean differences are small, and standard deviations are moderate. The method exhibits good robustness across values from 0.05 to 0.2, with mean values remaining stable. This stability demonstrates MEM's superior robustness to parameter selection.

4.2 MP-CLEAN Parameter Parameter in MP-CLEAN determines the threshold for selecting multiple points per iteration. [Figure 8: see original paper] shows χ^2 curves for various values, [Figure 9: see original paper] shows profile differences, and provides statistics.

Smaller values (e.g., $= 0.1$) result in more selected points per iteration, faster reconstruction, but increased risk of false sources and image oscillations. Larger values (e.g., $= 0.9$) make reconstruction slower and may prevent χ^2 from converging to 1. The optimal range appears to be $= 0.5$–0.7, where false sources are minimized while maintaining stability.

MP-CLEAN shows greater sensitivity to selection than MEM does to . If sampling is too sparse, the viable range for becomes narrow, potentially making good reconstruction difficult. This parameter sensitivity indicates that MP-CLEAN' s robustness is inferior to MEM' s, though it remains effective when its empirical assumptions hold.

5. Conclusion

This paper reconstructs simulated extended source visibility data using both MEM and MP-CLEAN, yielding the following conclusions:

1. **Reconstruction Quality:** Both methods perform well, but MP-CLEAN achieves better sidelobe suppression and smaller differences from the true image compared to MEM.
2. **Reconstruction Speed:** MP-CLEAN is significantly faster, requiring less than one-third the time of MEM in our simulations, due to both fewer iterations and faster per-iteration computation.
3. **Parameter Dependence:** MEM exhibits better robustness to parameter selection than MP-CLEAN. MP-CLEAN' s performance depends critically on appropriate selection, with optimal values typically in the 0.5–0.7 range.
4. **Applicability:** MP-CLEAN requires good visibility sampling and satisfies the empirical assumption that sidelobes from different sources rarely superimpose severely. When this assumption fails (e.g., due to symmetric bright source distributions causing severe sidelobe 叠加), MP-CLEAN becomes unsuitable, while MEM remains theoretically applicable. This represents another aspect of MEM' s superior robustness.

MP-CLEAN effectively exploits the potential of CLEAN at a single scale, providing a faster pathway for improving extended source reconstruction methods when visibility sampling is adequate. Future work should focus on developing more sophisticated multiscale approaches that combine the speed advantages of MP-CLEAN with the robust mathematical framework of MEM.

References

[1] Chen Xuelei, Tian Haijun, Xu Yang, et al. A preliminary study on GPU-based correlators for a radio interferometer array. *Astronomical Research & Technology—Publications of National Astronomical Observatories of China*, 209-217.

[2] Xu Yonghua, Wang Min, Hao Longfei, et al. Simulations of a low-frequency solar radio interferometry array. *Astronomical Research & Technology—Publications of National Astronomical Observatories of China*, 242-248.

[3] Högbom J A. Aperture synthesis with a non-regular distribution of interferometer baselines. *Astronomy and Astrophysics Supplement*, 417-426.

[4] Schwarz U J. Mathematical-statistical description of the iterative beam removing technique (CLEAN). *Astronomy and Astrophysics*, 345-356.

[5] Cornwell T J. Multiscale CLEAN deconvolution of radio synthesis images. *IEEE Journal of Selected Topics in Signal Processing*, 793-801.

[6] Wakker B P, Schwarz U J. The multi-resolution CLEAN and its application to the short-spacing problem in interferometry. *Astronomy and Astrophysics*, 312-322.

[7] Skilling J. Maximum entropy method in image processing. *IEEE Proceedings F*, 646-659.

[8] Starck J L. Deconvolution of astronomical images using the multiscale maximum entropy. *Astronomy and Astrophysics Supplement*, 575-586.

[9] Cornwell T J. A method of stabilizing the CLEAN algorithm. *Astronomy and Astrophysics*, 281-285.

[10] Gull S F, Pantin E. The maximum entropy method. *Astronomy and Astrophysics*, 377-378.

[11] Clark B G. An efficient implementation of the CLEAN algorithm. *Astronomy and Astrophysics*, 377-378.

[12] Skilling J, Gull S F. Bayesian maximum entropy image reconstruction. *Mathematical Statistics*, 341-367.

[13] Cornwell T J, Evans K F. A simple maximum entropy deconvolution algorithm. *Astronomy and Astrophysics*, 77-83.

Note: Figure translations are in progress. See original paper for figures.