Statistical Study of Sunspot Area as a Function of Sunspot Number (Postprint)

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Abstract

Data analysis was conducted on monthly sunspot numbers and sunspot areas from May 1874 to March 2015, statistically investigating variations in the ratio between sunspot number and sunspot area. The results indicate that sunspot number and sunspot area exhibit nonlinear variations, which becomes more pronounced during solar cycle maximum. The probability distributions of the two quantities are generally similar, both decreasing with increasing values; however, the sunspot area decreases more significantly, indicating a nonlinear relationship between them. The ratio of sunspot number to sunspot area is approximately 10. 2 during solar cycle minimum, about 21. 8 during solar cycle maximum, and approximately 16. 8 in the long-term average. The ratios determined for the rising and declining phases of each cycle show slight differences, and those determined for different solar cycles also exhibit small variations, with an overall decreasing trend in the ratio between the two.

Full Text

Preamble

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Statistical Study on Sunspot Area Varying with Sunspot Number

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Abstract

We analyzed monthly sunspot numbers and sunspot areas from May 1874 to March 2015 to conduct a statistical investigation of the variation in the ratio between sunspot number and sunspot area. The results demonstrate that the relationship between sunspot number and sunspot area is nonlinear, particularly pronounced during solar cycle maxima. The probability distributions of both quantities are generally similar, decreasing with increasing values, but the probability for sunspot area decreases more rapidly, implying a nonlinear relationship. The ratio of sunspot area to sunspot number is approximately 10.2 during solar cycle minima, about 21.8 during maxima, and 16.7 on a long-term basis. The ratios determined from the ascent and descent phases of each cycle show slight differences, and those from different solar cycles also exhibit small variations, displaying an overall decreasing trend across cycles.

Keywords: Solar activity; Sunspot number; Sunspot area; Correlation analysis

Introduction

The sunspot number represents the most classical and widely used parameter for characterizing solar activity, sometimes referred to as the Wolf number. It is defined by the formula $R = k(10g + s)$, where g denotes the number of sunspot groups on the solar disk, s is the number of individual spots, and k is a conversion factor related to the observing instrument, observer experience, and other factors. Since spots within a group are more easily observed and identified, they carry greater weight to ensure the reliability of the defined quantity[1-2]. Currently, daily values are compiled from observational data collected by astronomical observatories worldwide and are known as the international sunspot number, provided by the Solar Influences Data Analysis Center (SIDC).

Sunspot area is another frequently employed parameter to represent solar activity. It is defined as the projected area of sunspots on the solar disk, corrected to the center of the solar disk and expressed in millionths of the solar hemisphere area[2,4]. Early continuous sunspot area data were provided by the Royal Greenwich Observatory (GRO), and subsequently primarily by the United States Air Force (USAF) and the National Oceanic and Atmospheric Administration (NOAA). The relationship between these two quantities has been studied extensively.

The conversion factor between the 13-month smoothed sunspot numbers from GRO and the sunspot area data from USAF/NOAA is $16.7/11.2 = 1.39$. Some researchers have suggested that the ratio S/R may be related to the phase of solar activity. Although both quantities are used to represent solar activity, their relationship is complex. Cross Recurrence Plot (CRP) analysis has revealed that the phases are highly synchronized on timescales of years, but mixed at high frequencies. The ratio is more asynchronous at solar cycle maxima and minima than at other times.

This paper argues that the ratio between the two quantities represents the convergence degree of individual spots to spot groups and the average scale of spots, which is meaningful for studying the variation of this ratio. We define the ratio of sunspot area to sunspot number as a new parameter—sunspot unit area—which also reflects the relative relationship between sunspot number and sunspot area[8-9]. From a statistical perspective, this paper explores the relationship between the two quantities during different phases of solar cycles and investigates the temporal evolution of this parameter.

Data and Methods

We analyzed monthly sunspot area and sunspot number data from May 1874 to March 2015. The sunspot area data are composite data from GRO/USAF, while sunspot numbers are obtained from the Solar Influences Data Analysis Center. For visual clarity in figures, sunspot area values are scaled down by a factor of 16.7.

The monthly mean sunspot number and sunspot area from May 1874 to March 2015 are shown in [Figure 1: see original paper]. The difference between sunspot number and sunspot area during cycle maxima is significantly larger than during other periods, suggesting a potentially nonlinear relationship.

[Figure 2: see original paper] presents the probability distributions of sunspot number (top panel) and sunspot area (bottom panel). The distributions are similar, with probability inversely proportional to magnitude. However, the probability for sunspot area appears to decrease more rapidly with increasing area, indicating a nonlinear relationship between the two quantities.

Results

Nonlinear Relationship Between Sunspot Number and Sunspot Area

[Figure 3: see original paper] shows a scatter plot of the relationship between sunspot area and sunspot number, with linear and quadratic fits applied. Linear fitting yields the relationship $S = 16.792R - 82.4$, with a high correlation coefficient of 0.923, indicating statistical significance. Quadratic fitting produces

$S = 0.0094R^2 + 15.292R - 49.6$, with a correlation coefficient of 0.944. The two fitted curves are very close at low sunspot numbers but diverge increasingly as sunspot number increases, with the quadratic curve rising above the linear curve, demonstrating more pronounced nonlinearity. This nonlinearity is particularly evident during solar cycle maxima, when sunspot area grows more significantly with increasing sunspot number.

To determine whether the quadratic term is statistically significant, we randomly selected 1500 data pairs from the total 1690 sunspot area-number pairs and performed both linear and quadratic fits, repeating this process 5000 times. The distribution of correlation coefficients is shown in [Figure 4: see original paper]. In only 3.581% of cases did the linear fitting correlation coefficient equal that of quadratic fitting; in all other cases, the quadratic fit produced higher correlation coefficients.

[Figure 5: see original paper] shows the distribution of quadratic coefficients from these 5000 fits. The mean quadratic coefficient is $(9.339 \pm 3.5) \times 10^{-3}$, with a variance of 0.217. Only 3.58% of cases deviate from the mean by approximately one standard deviation, and the coefficient remains positive, confirming the statistical existence of the quadratic term.

Variation of Sunspot Area-Number Ratio with Sunspot Number

We first performed linear fits on data pairs with sunspot numbers less than or equal to a series of cutoff values, starting from zero each time and increasing the cutoff by 10. This yielded fitting parameters (intercept and slope) as functions of the cutoff sunspot number. The correlation coefficients for each fit are also presented.

As shown in [Figure 6: see original paper], when the cutoff sunspot number is small, both the correlation coefficient and the number of data points increase rapidly, leading to quickly improving confidence in the fit and strengthening linear relationship. Beyond a cutoff of about 100, the increases in both correlation coefficient and data point number slow down, as does the strengthening of the linear relationship.

The intercept values from these fits are small relative to the measured sunspot area values. For example, at a cutoff of 10.28, the intercept is -7.22, while the slope is 10.28. The ratio between sunspot area and number during cycle minima is approximately 10.2. As the cutoff increases to 69.55, the slope increases to 16.34, with the intercept at -82.4. The influence of the intercept on the slope-derived ratio remains small.

We repeated this analysis using quadratic fits, with results shown in [Figure 5: see original paper] (distribution of quadratic coefficients).

Variation of Sunspot Area-Number Ratio Across Solar Cycles

We also examined how the ratio varies when fitting different ranges of sunspot numbers, from minimum to maximum values. Starting with the full dataset, we performed linear fits on data pairs with sunspot numbers greater than or equal to progressively increasing starting values, always extending to the maximum sunspot number. This produced fitting parameters as functions of the starting sunspot number.

[Figure 8: see original paper] presents the intercept (top panel), slope (middle panel), and correlation coefficients (bottom panel) from these fits. The correlation coefficients are statistically significant throughout (minimum 0.654), indicating meaningful results for all calculations. The absolute intercept value shows a substantial increase when the starting sunspot number reaches 105. For a starting value of 9.61, the fit yields $S = 17.516R - 168.394$, giving a ratio of approximately 17.5. For a starting value of 21.811, the result is $S = 21.811R - 920.962$, yielding a ratio of about 21.8. The intercept' s influence on the slope-derived ratio remains small in all cases.

Our data comprehensively cover solar cycles 12 through 23. For each cycle, we performed linear fits between sunspot number and area for the ascent phase, descent phase, and entire cycle. The resulting parameters (intercept and slope) and correlation coefficients are shown in [Figure 10: see original paper].

The ratios determined from ascent and descent phases of each cycle show slight differences, and ratios from different cycles also vary. A linear fit to the slope values across cycles yields a correlation coefficient of 0.518, significant at the confidence level, indicating a decreasing trend in the ratio with increasing cycle number. However, correlation analysis reveals no significant relationship between these ratios and either the maximum sunspot number or cycle length.

Conclusion and Discussion

Using monthly sunspot area data from the Royal Greenwich Observatory and USAF, combined with international sunspot numbers from the Solar Influences Data Analysis Center, we conducted a statistical study of the variation in the ratio between these two parameters from May 1874 to March 2015.

The probability distributions of sunspot number and sunspot area are broadly similar but exhibit distinct differences. The distribution probability for sunspot area decreases more sharply with increasing area than does that for sunspot number. This difference in distribution leads to a nonlinear relationship between the two quantities, which is more pronounced during solar cycle maxima. During periods of high activity, the S/R ratio exceeds the value predicted by linear fitting, indicating larger average spot scales.

The ratio of sunspot area to sunspot number is approximately 10.2 during solar minima, about 21.8 during solar maxima, and 16.7 on a long-term basis. The ratio shows smaller fluctuations when sunspot numbers are low compared to during maxima. Slight differences exist between ratios determined from ascent and descent phases of individual cycles, and small variations occur between different solar cycles. The ratio values do not correlate with cycle maximum amplitude or length, but they exhibit a decreasing trend across solar cycles.

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A Statistical Study on Sunspot Area Varying with Sunspot Number

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Abstract: Statistical relation of sunspot area varying with sunspot number has been investigated through analyzing sunspot area and sunspot number with data acquired from May 1874 to March 2015. The obtained results are: (1) in general, distribution probabilities of both sunspot number and sunspot area are similar to each other; they decrease along with the increase of value; but the probability of sunspot area decreases more obviously, which implies that a nonlinear relation between the two exists; (2) the nonlinear relation is more clear during the solar maximum; (3) the ratio of sunspot area to sunspot number is about 10.2 during the solar minimum, 21.8 during the solar maximum, and 16.7

on long-term basis; (4) the ratio of every ascent phase is slightly different from that of the descent phase, and the determined ratios of different solar cycles are different from one another, displaying little relation to both the maximum amplitudes of cycles and cycles lengths; and (5) the ratios tend to decrease with increasing cycle number.

Keywords: Solar activity; Sunspot number; Sunspot area; Correlation analysis

Note: Figure translations are in progress. See original paper for figures.