Postprint of Apparent Position Calculation of Solar System Bodies in Astronomical Attitude Determination

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Abstract

The apparent positions of celestial bodies are essential data for astronomical attitude determination. Currently, research on apparent position calculation is primarily based on ground and maritime applications, and the development of space-based platforms requires further investigation. This paper introduces the principle of on-board apparent position transformation, derives various position correction formulas, and establishes an apparent position calculation model for celestial bodies within the solar system based on a new astronomical reference frame, which can calculate both the geocentric and topocentric apparent positions of celestial bodies at any given time. Compared with the geocentric apparent positions from the astronomical almanac, the differences in right ascension for Jovian planets are within 10 ms, and the differences in declination are within 220 mas; compared with the topocentric apparent positions simulated by STK, the differences in right ascension are within 0.15 s, and the differences in declination are within 2". Experimental results demonstrate that this model can correctly calculate the apparent positions of solar system bodies at the observation instant, and its accuracy meets the requirements of astronomical attitude determination.

Full Text

Apparent Position Calculation of Solar System Bodies for Celestial Attitude Determination

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Abstract

Celestial apparent position is essential data for celestial attitude determination. Currently, research on apparent position calculation primarily focuses on ground-based and sea-based platforms, while the development of space-based platforms requires further investigation. This paper introduces the conversion principle of apparent position onboard spacecraft, derives various position correction formulas, and establishes a computational model for the apparent positions of solar system bodies based on the new astronomical reference frame. The model can calculate both geocentric apparent positions and observer-fixed apparent positions at arbitrary epochs. Compared with the geocentric apparent positions in the Astronomical Almanac, the right ascension difference for Jovian planets is within 10 ms and the declination difference is within 220 mas. Compared with simulated observer-fixed apparent positions, the right ascension difference is within 0.15 s and the declination difference is within 2 . Experimental results demonstrate that the model correctly computes instantaneous apparent positions of solar system bodies, with accuracy meeting the requirements of celestial attitude determination.

Keywords: apparent position calculation; celestial attitude determination; astronomical almanac; System Tool Kit (STK)

1. Introduction

Star sensors determine spacecraft three-axis attitude in the orbital coordinate system by utilizing apparent position and star image vector information from celestial bodies. Celestial attitude determination employs natural celestial bodies such as stars, the Sun, the Moon, and planets as navigation beacons, operating in both initial mode and tracking mode [1]. Star sensors image celestial bodies within their field of view, extract star point data through star image processing, perform star pattern matching, and identify corresponding celestial bodies in the celestial coordinate system to obtain image vectors. These vectors are then reduced to the apparent positions corresponding to the observation epoch and station. By combining celestial apparent positions with image vector information, the spacecraft attitude matrix can be determined and three-axis attitude information can be solved. Consequently, celestial apparent position is indispensable data for celestial attitude determination. To improve attitude determination accuracy, research on real-time apparent position calculation for both stars and solar system bodies is necessary.

Previous studies on celestial apparent position calculation have mainly been applied to ground-based and sea-based observations. Astronomical vector processing methods have been employed to propose a high-precision mathematical model for planetary apparent position calculation, summarizing and establishing two methods: the variation-of-parameters method and the coordinate perturbation method [4-5]. These methods primarily ensure real-time performance requirements for attitude determination in shipboard applications. The calculation method for planetary apparent positions in new astronomical measurement systems based on ground observations has been introduced [7-8], and research on stellar apparent position calculation in shipboard celestial navigation systems has been conducted using the DE405 ephemeris. Theoretical analysis has yielded a long-term update algorithm for planetary apparent positions, though the effects of carrier motion were not considered. For solar system bodies, apparent position calculation must account for various factors, including proper motion, parallax, aberration, precession, nutation, and gravitational deflection of light. The most significant difference between space-based and ground/sea-based applications is that the observation station itself is moving at high velocity during celestial attitude determination. Since star sensors onboard spacecraft operate at high speeds, the resulting aberration magnitude is substantial, reaching arcsecond levels according to experimental results. This represents the primary distinction from ground-based observations when studying stellar apparent position calculation for spacecraft.

This paper investigates apparent position calculation methods for solar system bodies during celestial attitude determination under the new astronomical reference frame. By comparing with apparent positions from the Astronomical Almanac for 2011, 2013, and 2014, the accuracy of geocentric apparent position calculation is verified. Satellite orbit simulation software is used to model on-orbit satellite operations, and by obtaining position and velocity information, sequential observer-fixed apparent positions of celestial bodies are calculated. The accuracy of observer-fixed apparent position calculation is analyzed through comparison with solar system body apparent position data.

2. Solar System Body Apparent Position Conversion

2.1 New Astronomical Reference Frame The new astronomical reference frame establishes new coordinate systems and precession-nutation models, requiring corresponding changes in celestial apparent position calculation. The International Astronomical Union (IAU) introduced the International Celestial Reference System (ICRS) in 1991, based on general relativity. The frame defines two kinematically non-rotating spacetime coordinate systems: the Barycentric Celestial Reference System (BCRS) and the Geocentric Celestial Reference System (GCRS) [9-10]. The precession-nutation model describes the motion of the Celestial Intermediate Pole (CIP) in the celestial reference system.

Astronomy has employed several precession models. The Newcomb precession system was used until 1986. The IAU1976 model was adopted after the 1986 IAU General Assembly, and the IAU2000 model was introduced in 2000. The latest P03 model was adopted in 2006. The precession quantities for the P03 model are given by:

$$\psi_A = 5038.481507''T + 1.079006''T^2 + 0.00114645''T^3 + 0.000007''T^4$$
$$\omega_A = 84381.406000'' - 0.025754''T - 0.0512663''T^2 + 0.00772503''T^3 + 0.000000''T^4$$
$$\chi_A = 10.556403''T - 2.381429''T^2 - 0.00121197''T^3 + 0.000170663''T^4$$
$$\varepsilon_A = 84381.406000'' - 46.836769''T - 0.0001831''T^2 + 0.00200340''T^3 + 0.000000''T^4$$

where T is the number of Julian centuries from the epoch J2000.0.

For nutation calculation, the IAU1980 model was formally adopted in 1980. The IAU2000A model, adopted in 2000, consists of over ten times more terms than previous models, with precision of 0.2 mas. To simplify calculations and improve efficiency, the IAU2000B model was introduced in 2005, which maintains 1 mas accuracy between 1995-2050 while reducing computation time. The nutation model is expressed as:

$$\Delta\psi = \sum_{i=1}^{N} (A_i + A_i'T)\sin(\text{argument}) + (A_i'' + A_i'''T)\cos(\text{argument})$$

$$\Delta\varepsilon = \sum_{i=1}^{N} (B_i + B_i'T)\cos(\text{argument}) + (B_i'' + B_i'''T)\sin(\text{argument})$$

where the parameters are detailed in reference [11].

2.2 Apparent Position Conversion Principles Ephemerides such as DE/LE contain the barycentric positions of all solar system bodies. Apparent position calculation for solar system bodies during celestial attitude determination must fully consider various influencing factors:

1. **Proper Motion**: The effect of a celestial body’ s own motion from the epoch of the beginning of the year to the observation instant. Generally projected onto the star chart, proper motion effects are less than 1 /century.
2. **Parallax**: The difference in apparent direction of an object observed from two different locations, commonly used to calculate distances between celestial bodies. Its magnitude is less than 0.8 .
3. **Aberration**: The apparent displacement caused by the relative motion between observer and object combined with the finite speed of light. For ground-based observations, aberration effects can reach 20 , while spacecraft orbital velocities have even greater impact on apparent position calculation. This is the main difference from ground-based observations.
4. **Precession and Nutation**: Additional corrections required due to Earth’ s rotational axis changes.

5. **Gravitational Deflection of Light**: Bending of light in gravitational fields.

[Figure 1: see original paper] illustrates the conversion relationships between various apparent positions in celestial attitude determination.

The primary difference between solar system body apparent position calculation and stellar apparent position calculation lies in the need to account for diurnal parallax and diurnal aberration corrections due to the proximity of the observation station to the observed body and their large relative velocity.

3. Apparent Position Calculation Model

The specific calculation steps are as follows:

1. **Time Conversion**: Calculate the Julian Day J and Barycentric Dynamical Time (TDB) corresponding to Terrestrial Time (TT) at the observation instant. Due to discontinuities in earlier Ephemeris Time and advances in atomic frequency standards, relativistic time scales were adopted starting in 1976. The solar system barycentric reference frame uses Barycentric Dynamical Time, while geocentric apparent position calculation uses Terrestrial Time. TT is based on International Atomic Time (TAI):

$$\mathrm{TT} = \mathrm{TAI} + 32.184\,\mathrm{s} = \mathrm{UTC} + \mathrm{LS} + 32.184\,\mathrm{s}$$

where LS represents leap seconds, determined from bulletins issued by the International Earth Rotation and Reference Systems Service.

2. **Position and Velocity Acquisition**: Interpolate the DE/LE ephemeris to obtain the barycentric position $\mathbf{r}_e$ and velocity $\mathbf{v}_e$ of Earth in the BCRS at TT epoch, and the barycentric position vectors $\mathbf{r}_p$ of solar system bodies at J2000 epoch.

3. **Spacecraft State Calculation**: Compute the spacecraft' s position $\mathbf{r}_c$ and velocity $\mathbf{v}_c$ relative to Earth' s center in the true equator and true equinox coordinate system at TT epoch. Here, $\mathbf{v}_e$ is Earth' s orbital velocity around the Sun (approximately 30 km/s). The spacecraft' s position $\mathbf{r}_o$ and velocity $\mathbf{v}_o$ in the BCRS are then:

$$\mathbf{r}_o = \mathbf{r}_e + \mathbf{r}_c, \quad \mathbf{v}_o = \mathbf{v}_e + \mathbf{v}_c$$

4. **Light-Time Calculation**: Calculate the light travel time for solar system bodies. The approximate light-time τ is:

$$\tau = \frac{|\mathbf{r}_p(\tau) - \mathbf{r}_o|}{c}$$

Iteratively compute the solar system body's barycentric position vector $\mathbf{r}_p(T-\tau)$ at emission time $T-\tau$:

$$\tau' = \frac{|\mathbf{r}_p(T-\tau) - \mathbf{r}_o|}{c}, \quad \rho = \mathbf{r}_p(T-\tau) - \mathbf{r}_o$$

The unit vector $\mathbf{P}$ representing the coordinate direction from the body at emission time to the spacecraft at observation time is used to improve the light-time estimate. Iterate until the light-time meets precision requirements.

5. **Annual Aberration Correction**: Apply the annual aberration correction using:

$$\mathbf{P}' = \mathbf{P} + \frac{\mathbf{v}_e}{c} - (\mathbf{P} \cdot \frac{\mathbf{v}_e}{c})\mathbf{P}$$

6. **Gravitational Light Bending Correction**: Apply corrections for gravitational deflection of light.
7. **Coordinate Transformation**: Convert the position vector to spherical coordinates and apply precession and nutation corrections.
8. **Diurnal Parallax Correction**: Add the diurnal parallax correction π (the planet' s diurnal horizontal parallax obtainable from ephemerides):

$$\Delta\alpha_\pi = \pi \sin h \sec \delta, \quad \Delta\delta_\pi = \pi \cos h \sin \delta$$

where h is the hour angle and δ is declination.

9. **Diurnal Aberration Correction**: Add the diurnal aberration correction:

$$\Delta\alpha_{da} = 0.021'' \cos \phi \cos h \sec \delta, \quad \Delta\delta_{da} = 0.32'' \cos \phi \sin h \sin \delta$$

where ϕ is the station' s astronomical latitude.

The final apparent position is obtained by summing all corrections.

4. Experiments and Accuracy Analysis

4.1 Geocentric Apparent Position Calculation A calculation program for solar system body apparent positions was compiled using the VS2010 platform with the DE405 ephemeris from NASA' s Jet Propulsion Laboratory and the latest IAU2000 precession-nutation model. The program interface is shown in [Figure 2: see original paper].

To verify accuracy, calculations were compared with the Astronomical Almanac for 2011, 2013, and 2014. Observation times were set at 0:00 UTC on the 15th of each month, with the station at Earth' s center. The comparison data for 2014 is presented in .

** The difference value of geocentric apparent position**

Celestial Body	RA Error (s)	Dec Error (mas)
Mercury	-0.1004	-0.0217
Venus	-0.0500	0.0158
Mars	-0.0034	-0.0123
Jupiter	-0.0017	-0.1932
Saturn	-1.3312	0.0990
Uranus	0.2578	-0.0453
Neptune	0.0489	-0.1301

The results show that for terrestrial planets (Mercury, Venus, Mars), right ascension errors are within 0.35 s and declination errors within 10 mas. For Jovian planets, right ascension errors are within 10 ms and declination errors within 220 mas. The accuracy is higher for more distant planets, with lower accuracy for planets closer to the spacecraft. The geocentric apparent position calculation accuracy is thus validated.

4.2 Observer-Fixed Apparent Position Calculation Satellite orbit simulation software was used to model a 600 km altitude satellite with classical orbital elements: inclination $i = 60.074°$, right ascension of ascending node $\Omega = 298.074°$, and eccentricity $e = 0.074$. Satellite attitude information adopts a classical pattern. Two days of orbit data were generated with 5-minute intervals. The position and velocity information were input into the apparent position program to calculate observer-fixed apparent positions.

The J2000 coordinate system positions and velocities of Earth, Moon, and Sun from the simulation software were compared with the calculated apparent positions. The mean and RMS values of right ascension and declination differences are shown in .

** The counting indicator of observer-fixed apparent position**

Celestial Body	RA Mean (ms)	RA RMS (ms)	Dec Mean (mas)	Dec RMS (mas)
Earth	-0.0012	0.0709	-0.0133	1.0134
Moon	0.0325	0.0687	-0.1926	1.0234
Sun	0.0019	0.0693	0.0199	1.0577

The differences in right ascension and declination are small, with right ascension differences within 0.15 s and declination differences within 2 , validating the correctness of the observer-fixed apparent position calculation. Earth's apparent position shows the lowest precision because it is closest to the spacecraft. The Sun' s right ascension difference mean is 1.9 ms with declination difference mean of 19.9 mas, demonstrating that the Sun is the most stable information source. High-precision solar apparent position calculation is highly beneficial for celestial attitude determination, especially when daytime star observation is difficult.

[Figure 3: see original paper] and [Figure 4: see original paper] show the geocentric and observer-fixed apparent position calculation difference values, respectively.

5. Conclusion

This paper investigates calculation methods for apparent positions of solar system bodies during celestial attitude determination. A practical algorithm model is established that can compute both geocentric and observer-fixed apparent positions. Comparison with the Astronomical Almanac for 2011, 2013, and 2014 shows that terrestrial planet right ascension errors are within 0.35 s and declination errors within 10 mas, while Jovian planet right ascension errors are within 10 ms and declination errors within 220 mas, validating geocentric apparent position accuracy. Satellite orbit simulation software generated two days of on-orbit data, and comparison of Earth, Moon, and Sun observer-fixed apparent positions with J2000 coordinate system data shows right ascension differences within 0.15 s and declination differences within 2 , validating observer-fixed apparent position correctness. In astronomical units, right ascension uses time seconds while declination uses arcseconds. After unit conversion, the error magnitudes are comparable. The Sun' s right ascension difference mean of 1.9 ms and declination difference mean of 19.9 mas demonstrate the model' s capability. The calculation model correctly solves instantaneous apparent positions of solar system bodies with accuracy meeting celestial attitude determination requirements.

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Note: Figure translations are in progress. See original paper for figures.