Discrete Adjoint-Based Optimization Design Study of Single Expansion Ramp Nozzle Postprint

Authors: Song Hongchao, Li Xin, Ji Lucheng

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Abstract

Currently, the primary design methods for hypersonic nozzles are the method of characteristics and optimization design methods based on CFD analysis. When employing the method of characteristics for nozzle design, the flow must be simplified based on inviscid and irrotational assumptions, which cannot guarantee that the design outcome is optimal under realistic flow conditions; when utilizing optimization algorithms such as genetic algorithms or conventional gradient-based methods for optimization design, the increase in the number of design variables presents a significant computational challenge. To overcome the limitations of the aforementioned methods in the design process, leveraging the characteristic that the computational cost of the adjoint method is nearly independent of the number of design variables, this study develops a design methodology for single expansion ramp nozzles based on the discrete adjoint method, employing the control of the nozzle' s area distribution along the flow path as the parameterization approach to achieve refined adjoint optimization design on the basis of a prototype nozzle with excellent performance from preliminary design, with the optimized thrust coefficient improved by 0.8 percentage points compared to the prototype nozzle.

Full Text

Preamble

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Research on the Optimization of Unilateral Expansion Nozzle Based on the Discrete Adjoint Method

HongChao Song, Li Xin, Ji Lucheng
(Beijing Institute of Technology, Beijing, 100081, China)

Abstract

Currently, the primary design methods for hypersonic nozzles are the method of characteristics and optimization methods based on CFD analysis. The method of characteristics relies on simplifying assumptions of inviscid, irrotational flow, which cannot guarantee optimal performance under real flow conditions. Meanwhile, optimization algorithms such as genetic algorithms or conventional gradient-based methods face enormous computational challenges as the number of design variables increases. To overcome these limitations, this paper develops a design methodology for unilateral expansion nozzles based on the discrete adjoint method, leveraging its characteristic that computational cost is nearly independent of the number of design variables. By controlling the nozzle' s area distribution along the flow path as the parameterization approach, the study achieves refined adjoint-based optimization starting from a prototype nozzle with initially good performance. The optimized nozzle demonstrates a thrust coefficient improvement of 0.8 percentage points over the prototype.

Keywords: Discrete Adjoint Method; Unilateral Expansion Nozzle; Parameterization Method

Unilateral expansion nozzles are widely employed in scramjet engines for hypersonic vehicles. Current design methodologies for such hypersonic nozzles primarily include the method of characteristics and various optimization approaches.

The method of characteristics has been extensively used in supersonic flow profile design, enabling the creation of nozzles with smooth internal flow fields free of shock waves. A pioneering application of this method was Rao' s maximum-thrust nozzle design [1]. Early implementations of the method of characteristics proved effective for hypersonic internal flow component design. However, this method requires assumptions of inviscid, irrotational flow that deviate significantly from real flow conditions. Additionally, nozzles designed using this method typically feature excessively long ducts that require truncation, resulting in substantial performance losses.

With the rapid advancement of computational fluid dynamics and computer technology, optimization design methods based on CFD analysis have made significant progress in recent decades. Chen Bing [4] established a two-dimensional nozzle model using NS equation-based optimization design and solved it using the complex method. Subsequently, combining genetic algorithms with efficient, high-precision space-marching methods [5], he conducted aerodynamic optimization design studies on two-dimensional supersonic scramjet engine nozzles. Gan Wenbiao et al. [6] constructed an optimization methodology integrating experimental design methods, surrogate model techniques, and genetic algorithms, applying it to the integrated design of hypersonic vehicle afterbody/nozzle configurations.

The genetic algorithms and conventional gradient-based methods employed in the aforementioned hypersonic nozzle designs, while effective, suffer from computational costs that grow geometrically or exponentially with the number of design variables. This limitation makes them inadequate for meeting the demands of reduced design cycles, increased automation, and refined design in full three-dimensional engineering applications.

In contrast to conventional optimization methods, the adjoint method offers the distinct advantage that its computational cost is nearly independent of the number of design variables, thereby overcoming the computational limitations of traditional approaches. Through the efforts of Jameson, Giles, and others [7-11], aerodynamic shape optimization using adjoint methods for airfoils, wings, wing-body configurations, and even complete aircraft has reached maturity over the past two decades. This paper employs a discrete adjoint optimization method to design hypersonic unilateral expansion nozzles, enabling truly refined design capabilities to comprehensively address hypersonic unilateral expansion nozzle design challenges while providing technical references for subsequent integrated inlet and propulsion system design processes.

1.1 Discrete Adjoint Optimization Theory

For steady-flow optimization problems, the flow equation residual R can be considered a function of design variables D, grid X, and flow variables Q, expressed as R(Q, X, D) = 0. This residual equation essentially represents the constraint for the steady-flow optimization problem. The initial objective function J(Q, X, D) is augmented with the flow equation constraint through Lagrange multipliers to form the Lagrangian function L:

$$L(Q, X, D, \lambda) = J(Q, X, D) - \lambda^T R(Q, X, D)$$

where λ is the vector of Lagrange multipliers (adjoint variables). Differentiating with respect to D yields:

$$\frac{dL}{dD} = \frac{\partial J}{\partial D} - \lambda^T \frac{\partial R}{\partial D} + \left(\frac{\partial J}{\partial Q} - \lambda^T \frac{\partial R}{\partial Q}\right)\frac{dQ}{dD} + \left(\frac{\partial J}{\partial X} - \lambda^T \frac{\partial R}{\partial X}\right)\frac{dX}{dD}$$

The term involving dQ/dD is the primary contributor to the high computational cost of conventional gradient-based algorithms. By exploiting the arbitrariness of the Lagrange multiplier λ, we can eliminate this expensive term by setting the coefficient of dQ/dD to zero, which yields the discrete adjoint equation:

$$\left(\frac{\partial R}{\partial Q}\right)^T \lambda = \left(\frac{\partial J}{\partial Q}\right)^T$$

With the adjoint equation satisfied, the gradient simplifies to:

$$\frac{dL}{dD} = \frac{\partial J}{\partial D} - \lambda^T \frac{\partial R}{\partial D} + \left(\frac{\partial J}{\partial X} - \lambda^T \frac{\partial R}{\partial X} \right) \frac{dX}{dD}$$

1.2.1 Flow Control Equations

The three-dimensional dimensionless Reynolds-Averaged Navier-Stokes (RANS) equations can be expressed as:

$$\frac{\partial}{\partial t} \int_{\Omega} Q dV + \oint_{\partial \Omega} \mathbf{F} \cdot \mathbf{n} dS = 0$$

where n is the outward unit normal vector at the control volume boundary, Q represents the cell-averaged conserved variables, $\mathbf{F}_i$ denotes the inviscid flux, and $\mathbf{F}_v$ denotes the viscous flux. The laminar viscosity calculation employs the perfect gas assumption and follows Sutherland' s formula.

1.2.2 Turbulence Model

Extensive experimental evidence demonstrates that the SA turbulence model selected for this study exhibits strong robustness and is highly suitable for general internal flow aerospace engineering applications. The dimensionless SA turbulence model control equation is:

$$\frac{\partial \tilde{\nu}}{\partial t} + u_j \frac{\partial \tilde{\nu}}{\partial x_j} = C_{b1} \tilde{S} \tilde{\nu} - C_{w1} f_w \left(\frac{\tilde{\nu}}{d} \right)^2 + \frac{1}{\sigma} \left[\frac{\partial}{\partial x_j} \left((\nu + \tilde{\nu}) \frac{\partial \tilde{\nu}}{\partial x_j} \right) + C_{b2} \frac{\partial \tilde{\nu}}{\partial x_j} \frac{\partial \tilde{\nu}}{\partial x_j} \right]$$

where $\tilde{\nu}$ is the turbulent viscosity variable, $\tilde{S}$ is the modified vorticity magnitude, and d is the shortest distance from the current grid cell to the wall.

The nozzle inlet is supersonic. According to characteristic theory, all aerodynamic parameters must be specified at the inlet. The inlet boundary conditions are: Mach number 1.4, static pressure 145600 Pa, static temperature 1500 K, and specific heat ratio 1.4. The outlet employs a pressure boundary condition with assumptions of adiabatic and isentropic flow. If the outlet flow is supersonic, all aerodynamic parameters are computed using extrapolation. Wall conditions employ no-slip walls. The current program initializes the entire flow field using freestream parameters.

1.3 Optimization Method

The optimization program developed in this work comprises seven modules: objective function formulation, grid generation, flow field solution, adjoint field solution, sensitivity calculation, optimization algorithm, and grid deformation.

Detailed optimization procedures (including flow solution methods, adjoint solution methods, optimization algorithms, and grid deformation techniques) are described in Reference [12].

1.3.1 Objective Function To maximize the thrust coefficient of the hypersonic nozzle, the objective function takes the form:

$$J = -C_T$$

where C_T is the nozzle thrust coefficient and C_{T0} is the thrust coefficient of the initial nozzle. Gradient-based algorithms are local optimization methods that locate an extremum rather than the global optimum within the design space, and cannot guarantee global optimality of the result.

1.3.2 Initial and Boundary Conditions The unilateral expansion nozzle designed in this study operates at Mach 4.5 at an altitude of 18.5 km, where the standard atmospheric static pressure is 6994.8 Pa.

1.3.3 Geometric Parameterization Method This study selects 500 nodes on the nozzle surface as design variables. When perturbed, each design variable moves within a plane perpendicular to the streamline direction, thereby altering the nozzle' s area distribution along the flow path. Consequently, the parameterization object is the perturbation of the aerodynamic shape rather than the shape itself. By controlling the nozzle' s area distribution, this approach captures the critical design factors while reducing the number of optimization design variables. The nozzle parameterization is expressed as:

$$\mathbf{r}_n(v) = \mathbf{r}_n^0 + \Delta\mathbf{r}_n(v)$$

where v is the vector containing design variables, $\mathbf{r}_n(v)$ represents the coordinates of the nozzle profile at point n, $\mathbf{r}_n^0$ denotes the initial model profile coordinates, and $\Delta\mathbf{r}_n(v)$ is the perturbation coordinate with the specific expression:

$$\Delta\mathbf{r}_n(v) = \sum_{i=1}^{m} v_i \cdot f_i(s_n) \cdot \mathbf{n}_n$$

where m represents the number of design variables, p_i denotes the exponent for the i-th design variable, and $\mathbf{n}_n$ represents the unit normal vector at point n.

2 Prototype Nozzle Design

Gradient-based algorithms are local optimization methods whose ultimate goal is to identify an extremum of the objective function within the optimization domain rather than the global maximum, and cannot guarantee global optimality of the optimized result. To address this limitation, a viable approach is to pre-determine the local optimization domain where the optimal solution resides, thereby narrowing the optimization range for the adjoint method to ensure result validity. Consequently, the design of the prototype nozzle is critical. Serving as the starting point for adjoint optimization, the prototype nozzle acts as a "genetic inheritor," and its performance largely determines the effectiveness of subsequent adjoint optimization.

To enhance performance, the prototype nozzle design should employ optimization algorithms with global search capabilities. Therefore, the parameters controlling the original nozzle geometry must be minimized to reduce computational cost during prototype design. In this study' s prototype nozzle design process, the nozzle inlet cross-section is specified as circular and the outlet as rectangular, with the inlet and outlet shapes and nozzle length held constant throughout optimization. Based on these constraints, each streamwise cross-section is defined as a super-ellipse, with the area and exponent n distribution along the flow path serving as optimization parameters. By adjusting nine parameters—including the area distribution law, super-ellipse exponent n distribution law, and lower plate angle—the nozzle geometry can be uniquely determined, as illustrated in [Figure 1: see original paper].

[Figure 1: see original paper]

This modeling concept enhances three-dimensional surface control capability in nozzle design, with particular emphasis on the area distribution along the flow path and application of dihedral angle principles.

When employing the genetic algorithm for prototype nozzle design, the nozzle mesh contains approximately 600,000 cells. To ensure computational accuracy, the Y-plus value is maintained below 1, with the mesh shown in [Figure 2: see original paper].

[Figure 2: see original paper]

Building upon the nine control parameters, this study utilizes a genetic algorithm for evolutionary optimization to bring the initial nozzle geometry close to the global optimum. The genetic algorithm population size is 10. As shown in [Figure 3: see original paper], when the population evolves to the eighth generation, the thrust coefficient rapidly increases from below 91% to approximately 93%, and the optimization process stabilizes. The result from the eighth generation of the genetic algorithm optimization is adopted as the final prototype nozzle design, providing the initial geometry for subsequent adjoint optimization. [Figure 4: see original paper] presents the Mach number distribution in the nozzle symmetry plane and along the flow path for the prototype nozzle, revealing a relatively uniform Mach number distribution at the exit.

[Figure 3: see original paper]

[Figure 4: see original paper]

3 Adjoint Optimization Results Analysis

Using the prototype nozzle as the starting point, refined adjoint optimization is performed with 500 design variables on the nozzle surface—a level of refinement difficult to achieve with genetic algorithms or other conventional gradient-based methods. [Figure 5: see original paper] shows the objective function iteration history during the optimization process. The improvement in the objective function is relatively slow during the first three optimization iterations, which is related to the optimization algorithm, objective function formulation, and parameterization method. The most significant improvement occurs during the fourth optimization iteration, with thrust increasing by approximately 0.8 percentage points. Subsequently, the objective function essentially converges, indicating that the optimization has reached an extremum and the process terminates. This adjoint optimization required only seven optimization cycles, with computational cost equivalent to just 14 flow field solutions.

[Figure 5: see original paper]

The adjoint optimization process employs conventional geometric constraints, maintaining constant inlet and outlet geometries. [Figure 6: see original paper] compares the geometry before and after optimization, revealing substantial changes, with the circled area marking the location of maximum change. [FIGURE:6(b)] shows that the dihedral angle region exhibits particularly pronounced modifications.

[Figure 6: see original paper]

[Figure 7: see original paper] compares the Mach number and static pressure distributions in the flow symmetry plane before and after optimization. The flow field changes are evident: the prototype nozzle features gradual geometric variation from inlet to outlet, resulting in relatively smooth flow expansion. In contrast, the optimized nozzle exhibits lower expansion in the forward section and more abrupt expansion in the aft section, particularly noticeable on the lower plate. [Figure 8: see original paper] illustrates the Mach number distribution along the flow path, confirming that the optimized nozzle has reduced expansion in the forward section but greater expansion at the exit compared to the prototype nozzle. Near the corner regions, the optimized nozzle demonstrates more complete expansion, with significantly improved corner flow behavior.

[Figure 7: see original paper]

[Figure 8: see original paper]

This study first constructs the nozzle with a limited number of parameters and employs a genetic algorithm for global optimization to obtain a well-performing prototype nozzle. Subsequently, discrete adjoint methods are applied for refined optimization, achieving a final design with a thrust coefficient improvement of 0.8 percentage points over the prototype. The adjoint-based design system developed in this work requires no simplifying assumptions about the flow and is not constrained by the number of design variables, overcoming certain limitations of existing design methods and satisfying the development trend toward refined design. Furthermore, the parameterization model provides a key technical approach for integrated inlet and propulsion system optimization design.

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Note: Figure translations are in progress. See original paper for figures.