

## Mechanism Analysis of Time-Averaged Asymmetric Pressure Drop Induced by Jet Pump Postprint

**Authors:** Tang Ke, sealed leaf, Jin Tao, Wu Yunxiang

**Date:** 2017-11-07T00:00:00+00:00

### Abstract

Jet pumps capable of inducing asymmetric pressure drops have been incorporated into the loop structure of thermoacoustic engines to suppress Gedeon flow and thereby enhance the thermal efficiency of such engines. To reconcile the discrepancy between Swift's theoretical derivation regarding the time-averaged asymmetric pressure drop provided by jet pumps and the experimental results reported by Idelchik et al., this study employs computational fluid dynamics software to investigate the effect of the jet pump's thickness-to-diameter ratio  $1/df$  on the time-averaged asymmetric pressure drop. The results demonstrate that variation of the thickness-to-diameter ratio  $1/df$  can induce changes in the direction of the time-averaged asymmetric pressure drop. Consequently, in jet pump design, in addition to accounting for the influence of differing cross-sectional areas at the two ends of the flow channel, the thickness-to-diameter ratio must also be judiciously selected.

### Full Text

#### Analysis of Time-Averaged Asymmetric Pressure Drop Induced by Jet Pump

**TANG Ke<sup>1,2</sup>, FENG Ye<sup>1</sup>, JIN Tao<sup>1,2</sup>, WU Yunxiang<sup>1</sup>** <sup>1</sup>Institute of Refrigeration and Cryogenics, Zhejiang University, Hangzhou 310027, China  
<sup>2</sup>State Key Laboratory of Clean Energy Utilization, Zhejiang University, Hangzhou 310027, China

**Abstract:** Jet pumps capable of inducing asymmetric pressure drops have been introduced into the loop configuration of thermoacoustic engines to suppress Gedeon streaming and thereby improve thermal efficiency. Motivated by the contradiction between Swift's theoretical derivation of the time-averaged pressure drop provided by jet pumps and the experimental results reported

by Idelchik et al., this study employs computational fluid dynamics to investigate the effect of the jet pump's thickness-to-diameter ratio ( $l/de$ ) on the time-averaged asymmetric pressure drop. The results indicate that varying the thickness-to-diameter ratio  $l/de$  can alter the direction of the time-averaged asymmetric pressure drop induced by the jet pump. Consequently, in jet pump design, attention must be paid not only to the asymmetry of the flow areas at both ends of the channel but also to the rational selection of the thickness-to-diameter ratio  $l/de$ .

**Keywords:** jet pump; Gedeon streaming; oscillating flow; time-averaged pressure drop

## 0 Introduction

Thermoacoustic engines have attracted considerable attention due to their advantages of having no mechanical moving parts, simple structure, reliable operation, and long service life. Traveling-wave thermoacoustic engines are particularly promising because their thermoacoustic conversion cycle is intrinsically reversible, giving them a theoretical efficiency higher than that of standing-wave thermoacoustic engines. The traveling-wave thermoacoustic Stirling engine with a feedback loop developed by Swift et al. represents a typical example, achieving a thermal efficiency of 0.3 and sparking a surge of research interest in traveling-wave thermoacoustic engines.

However, the introduction of a feedback loop creates a closed circuit in the thermoacoustic engine, which may lead to the occurrence of Gedeon streaming. Gedeon streaming refers to a time-averaged flow that circulates along the loop, also known as acoustic DC flow. This streaming can transfer heat directly from the hot heat exchanger to the cold heat exchanger without participating in the thermoacoustic conversion process, representing a significant heat loss mechanism that substantially affects the thermal efficiency of thermoacoustic Stirling engines.

Swift et al. employed a jet pump to suppress Gedeon streaming. By utilizing a tapered slot structure with unequal flow areas at its two ends, the oscillating fluid in the thermoacoustic engine experiences a time-averaged asymmetric pressure drop when passing through the jet pump in opposite half-cycles, thereby suppressing Gedeon streaming in the loop. In Swift's derived formulas, the direction of the time-averaged asymmetric pressure drop is from the large cross-section to the small cross-section of the jet pump, indicating that the pressure drop for flow from the large to the small cross-section is greater than that for the reverse direction.

Idelchik et al. investigated the pressure drop when fluid flows in opposite directions through an asymmetric orifice plate under steady flow conditions. This orifice plate has a similar structure to a jet pump, differing in that Idelchik's experimental study used circular holes rather than rectangular slots, with a thickness-to-hole-diameter ratio of  $l/d = 0-0.015$ . The experimental results

showed that the pressure drop for flow from the small to the large cross-section is greater than that for the reverse flow direction.

Swift's analysis of the jet pump mechanism considered only the effects of different flow areas and edge shapes at the two ends of the narrow slot channel on the time-averaged pressure drop, while neglecting the influence of the jet pump thickness. This omission likely explains the contradiction between his theoretical conclusions and the experimental results reported by Idelchik et al.

Based on this analysis, the present study references Swift's jet pump model and investigates narrow-slot jet pumps using numerical simulation methods. We examine the effect of the thickness-to-diameter ratio  $l/d_e$  on the asymmetric pressure drop under different flow rates to achieve appropriate second-order time-averaged flow resistance while maintaining minimal first-order oscillating flow resistance, thereby realizing efficient and low-loss suppression of Gedeon streaming.

## 1 Validation of Computational Method

Following the approach of Swift et al., oscillating flow is treated as two opposite steady flows along the axial direction of the channel. We investigate the effect of the jet pump's thickness-to-diameter ratio  $l/d_e$  on the time-averaged asymmetric pressure drop under steady flow conditions. The three-dimensional steady incompressible Reynolds-averaged Navier-Stokes equations are employed. The Realizable  $k$ - $\epsilon$  (RKE) model is selected for the simulation because it can describe various flow types including free flows with jets and mixing layers, duct flows, and boundary layer flows. Given the large pressure gradients present at the jet pump inlet and outlet, the PRESTO scheme is used for pressure interpolation.

The wall is set as an adiabatic boundary condition, and the working fluid is helium. The continuity, momentum, and energy equations are discretized using a second-order upwind scheme. Convergence is considered achieved when the flow rate difference between the inlet and outlet is less than 0.1%. To accurately capture vortex information around the jet pump, the mesh is refined in the jet pump region and adjacent straight pipe sections.

To validate the computational method, we calculate the series flow resistance coefficient  $k$  for an asymmetric orifice plate and compare the results with the experimental data reported by Idelchik et al. The coefficient  $k$  characterizes the pressure loss when fluid flows through an asymmetric orifice plate and can be calculated using Equation (1):

$$\Delta p = k \cdot \frac{\rho v^2}{2}$$

where  $\Delta p$  represents the total pressure loss of fluid flowing through the asymmetric orifice plate. This loss comprises four components: local resistance loss due to cross-sectional change at the small opening, friction loss in the tapered

section, diffusion loss in the tapered section, and local resistance loss due to cross-sectional change at the large opening.  $\rho$  is the fluid density, and  $v$  is the flow velocity in the pipe.

compares the calculated series flow resistance coefficient  $k$  for flow from the large to the small cross-section of the asymmetric orifice plate with experimental results for area ratios  $A/a_s$  of 0.02, 0.04, and 0.06 at a flow velocity of 2 m/s. The comparison shows that the error between computational and experimental results is within 7%, demonstrating that the computational method can reasonably reflect the experimental data.

## 2 Selection of Computational Parameters

Swift et al. proposed that the time-averaged asymmetric pressure drop is caused solely by differences in flow area and edge shape at the two ends of the narrow slot channel. To eliminate these two factors and isolate the effect of thickness-to-diameter ratio  $l/d_e$  on the time-averaged asymmetric pressure drop, this study maintains constant flow areas at both ends and sharp edges (no fillets). Based on this approach, we propose appropriate  $l/d_e$  values that can provide suitable time-averaged asymmetric pressure drop while maintaining minimal pressure loss.

The physical model employs a pipe with an inner diameter of 60 mm. The jet pump structure references Swift's previous model, focusing on narrow-slot jet pumps with a small cross-sectional area  $a_s = 1.8 \text{ cm}^2$  and a large cross-sectional area  $a_b = 3.0 \text{ cm}^2$ , as shown in [Figure 1: see original paper]. The computational conditions use a working pressure of 3 MPa and temperature of 300 K. The study examines the effect of varying  $l/d_e$  on the time-averaged asymmetric pressure drop at flow velocities of 2 m/s, 3 m/s, and 4 m/s. The thickness-to-diameter ratio  $l/d_e$  is varied across the following values: 0.2, 0.5, 1.0, 1.5, 2.0, 3.0, 4.5, 5.5, 6.5, 7.0, 7.5, 9.0, and 10.0, where  $d_e$  is the equivalent diameter of the jet pump's small cross-section.

## 3 Computational Results

To facilitate analysis, three parameters are introduced to characterize jet pump performance: the time-averaged asymmetric resistance coefficient  $\Delta k$ , the time-averaged total resistance coefficient  $k_t$ , and the effectiveness coefficient  $\eta$ .  $\Delta k$  characterizes the time-averaged asymmetric pressure drop generated when fluid flows through the jet pump,  $k_t$  represents the time-averaged total pressure loss, and  $\eta$  indicates the time-averaged asymmetric pressure drop produced per unit of time-averaged total pressure loss. These coefficients are defined as:

$$\Delta k = \frac{k^+ - k^-}{2}$$

$$k_t = \frac{k^+ + k^-}{2}$$
$$\varepsilon = \frac{\Delta k}{k_t}$$

where  $T$  is the period, and the subscripts  $+$  and  $-$  represent the two opposite flow directions ( $+$  denotes flow from small to large cross-section,  $-$  denotes flow from large to small cross-section).

[Figure 2: see original paper] shows the variation of the series flow resistance coefficient  $k_+$  for flow from the small to large cross-section with  $l/de$ , while [Figure 3: see original paper] illustrates the variation of  $k_-$  for the opposite flow direction. The resistance coefficient  $k_+$  decreases gradually with increasing  $l/de$ , indicating reduced pressure loss. When  $l/de$  ranges from 5.5 to 10.0, the trend becomes gradual, and with further increase in  $l/de$ , the pressure drop shows a slight upward trend. The variation trend of  $k_-$  is similar to that of  $k_+$ , but the change in  $k_-$  is slightly smaller. When  $l/de$  exceeds 3.0, a relatively flat region appears where increasing  $l/de$  has minimal effect on  $k_-$ .

[Figure 4: see original paper] and [Figure 5: see original paper] display the velocity distributions for flow velocities of 2 m/s at  $l/de$  values of 0.2, 2.0, and 5.5. [Figure 4: see original paper] shows flow from small to large cross-section, while [Figure 5: see original paper] shows the reverse direction. For flow from small to large cross-section, fluid constriction occurs due to the small opening constraint, causing fluid particles to accelerate toward the orifice, increasing velocity and decreasing static pressure. After entering the small cross-section, the flow continues to contract under inertial forces until radial inertial forces balance with expansion forces, at which point the flow reaches its minimum cross-sectional area, maximum velocity, and minimum static pressure. Subsequently, the flow expands, velocity decreases, and the flow fills the pipe.

At small  $l/de$  values, the flow contraction caused by sudden cross-section reduction has a significant influence length, and mainstream detachment from the wall near the abrupt edge must be considered when calculating outlet expansion pressure loss. When  $l/de = 5.5$ , the actual flow area approaches the outlet wall area, reducing the effect of mainstream detachment. The fluctuation in  $k_+$  is then primarily caused by variations in internal friction and diffusion losses with thickness. For flow from large to small cross-section, the tapered contraction creates a positive pressure difference aligned with the flow direction, causing acceleration without mainstream detachment.

The simulation results show that changes in pipe flow velocity have minimal effect on the series flow resistance coefficient. Since oscillating flow is treated as two opposite steady flows with constant velocity magnitude in each half-cycle, the definitions in Equations (2) and (3) are applied accordingly.

[Figure 6: see original paper] presents the variation of the time-averaged asymmetric resistance coefficient  $\Delta k$  with  $l/de$ . At small  $l/de$  values, the pressure drop for flow from small to large cross-section exceeds that for the reverse flow, consistent with Idelchik's experimental results. As  $l/de$  increases, the pressure drop for small-to-large cross-section flow decreases rapidly while the reverse flow pressure drop decreases more slowly, causing  $\Delta k$  to decrease. When  $l/de$  increases further, the small-to-large cross-section pressure drop becomes smaller than the reverse flow pressure drop, making  $\Delta k$  negative. Swift's theoretical model predicts the direction of the time-averaged asymmetric pressure drop correctly when  $\Delta k$  is negative but incorrectly when  $\Delta k$  is positive, primarily because Swift neglected the mutual influence between local resistances at the two abrupt cross-sections.

[Figure 7: see original paper] shows that the time-averaged total resistance coefficient  $k_t$  decreases with increasing  $l/de$ , then levels off, and increases slightly with further  $l/de$  increase. [Figure 8: see original paper] illustrates the effectiveness coefficient  $\eta$ , which should be maximized for efficient DC flow suppression with minimal loss. At small  $l/de$ ,  $\eta$  first decreases then increases with  $l/de$ , exhibiting a minimum at a certain  $l/de$  value.

The results indicate that both small and large  $l/de$  values can provide substantial time-averaged asymmetric pressure drop, but small  $l/de$  values incur large total pressure losses. Therefore, larger  $l/de$  values are preferable in jet pump design to achieve appropriate asymmetric pressure drop with minimal loss. For the simulated conditions (3 MPa pressure,  $a_s = 1.8 \text{ cm}^2$ ,  $a_b = 3.0 \text{ cm}^2$ ), an  $l/de$  range of 7.0-10.0 yields minimal time-averaged total resistance loss while providing significant time-averaged asymmetric pressure drop, representing a suitable design range.

## 4 Conclusion

This study addresses the contradiction between Swift's theoretical derivation of the time-averaged asymmetric pressure drop provided by jet pumps and the experimental results of Idelchik et al. by investigating the effect of thickness-to-diameter ratio  $l/de$  on the time-averaged asymmetric pressure drop under various flow velocities.

The results demonstrate that at small  $l/de$  values, flow from the jet pump's small to large cross-section experiences mainstream detachment from the wall, causing its pressure drop to significantly exceed theoretical predictions and become larger than the reverse flow pressure drop. The time-averaged asymmetric pressure drop direction is then from small to large cross-section, contradicting Swift's theoretical model. As  $l/de$  increases, the small-to-large cross-section pressure drop decreases rapidly and eventually becomes smaller than the reverse flow pressure drop, making the time-averaged asymmetric pressure drop direction from large to small cross-section, consistent with Swift's theoretical conclusions. The variation of  $l/de$  thus changes the direction of the time-averaged

asymmetric pressure drop. Therefore, jet pump design must comprehensively consider both the area asymmetry at the two ends and the thickness-to-diameter ratio  $l/de$ ; otherwise, the design may fail to suppress DC flow or even aggravate it.

## References

- [1] Ceperley P H. A Pistonless Stirling Engine—the Traveling Wave Heat Engine[J]. Journal of the Acoustical Society of America, 1979, 66(5): 1508-1513
- [2] Backhaus S, Swift G W. A Thermoacoustic Stirling Heat Engine[J]. Nature, 1999, 399: 335-338
- [3] Backhaus S, Swift G W. A Thermoacoustic-Stirling Heat Engine: Detailed Study[J]. Journal of the Acoustical Society of America, 2000, 107(6): 3148-3166
- [4] Yu G, Luo E, Dai W, et al. An Energy-Focused Thermoacoustic-Stirling Heat Engine Reaching a High Pressure Ratio Above 1.40[J]. Cryogenics, 2007, 47(2): 90-93
- [5] Jin T, Mao C, Tang K, et al. Characteristics Study on the Oscillation Onset And Damping of a Traveling-Wave Thermoacoustic Prime Mover[J]. Journal of Zhejiang University (Science A), 2008, 9(7): 944-949
- [6] Tijani M E H, Spoelstra S. A High Performance Thermoacoustic Engine[J]. Journal of Applied Physics, 2011, 110(9): 093519
- [7] de Blok K. Novel 4-Stage Traveling Wave Thermoacoustic Power Generator[C]// In ASME 2010 3rd Joint US-European Fluids Engineering Summer Meeting collocated with 8th International Conference on Nanochannels, Microchannels, and Minichannels, American Society of Mechanical Engineers, 2012: 73-79
- [8] Gedeon D. DC Gas Flows in Stirling and Pulse Tube Cryocoolers. Cryocoolers 9[M]. New York: Plenum Press, 1997: 385-392
- [9] Idelchik I E. Handbook of Hydraulic Resistance, 3rd ed[M]. New York: Begell House, 1996: 170-172
- [10] Fluent 6.1 User' s Guide Fluent Inc., Lebanon, NH, 2001

*Note: Figure translations are in progress. See original paper for figures.*

*Source: ChinaXiv –Machine translation. Verify with original.*