

## Research on Weakened Shock Wave Turbine Cascades (Postprint)

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### Abstract

Shock wave issues severely impact the aerodynamic performance, cooling characteristics, and structural integrity of supersonic/transonic turbines; thus, shock-weakening design constitutes one of the active research areas in high-load turbine development. To address this problem, the present study employs two typical supersonic/transonic turbine cascades as case studies, utilizes the adjoint optimization method, applies mass flow and outlet flow angle constraints, conducts optimization at multiple operating points, and performs comparative investigations into off-design characteristics, flow features, and geometric parameters, aiming to explore the design techniques and underlying principles of shock-weakening turbine cascades.

### Full Text

#### Study on Optimal Transonic Turbine Cascade of Weakening Shock

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### Abstract

Shock wave phenomena severely impact the aerodynamic performance, cooling characteristics, and structural integrity of supersonic/transonic turbines. Consequently, weakened shock wave design represents a key research focus for highly-loaded turbines. Addressing this issue, this paper examines two typical supersonic/transonic turbine cascades as case studies. Using the adjoint method for optimization under constraints of mass flow rate and outlet flow angle, we perform optimizations at different operating points and conduct comparative studies on off-design characteristics, flow features, and geometric parameters to explore the design techniques and principles of weakened shock turbine cascades.

**Keywords:** Supersonic/Transonic turbine cascade; Turbine design with weakened shock wave; Adjoint optimization

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## 2. Weakened Shock Design and Its Design Philosophy

“Weakened shock design” is a shaping methodology that reduces or eliminates shock wave phenomena in supersonic/transonic turbines through appropriate selection of geometric parameters. In fact, this is neither a new concept nor a new technology—engineers worldwide have historically pursued “weakened shock” objectives proactively in supersonic/transonic turbine design. Designers and researchers typically achieve this by controlling a limited number of key parameters combined with CFD analysis. For instance, Russian designers control over 40 geometric parameters, particularly the 13 most independent key parameters [5], to obtain satisfactorily performing supersonic/transonic turbine cascades through experimental and CFD analysis. In Japan, supersonic blade design for the final stage of steam turbines only requires control of 5-6 parameters such as throat area, exit area, and installation angle to determine the shock structure [6], yielding blades with satisfactory performance. Nevertheless, current methods still cannot rapidly and precisely design shock structure and intensity, resulting in performance that, while good, remains far from optimal. Confronting this challenge—and with continuously increasing loads that make turbine exit velocities ever higher, shock intensities stronger, and impacts on aerodynamics, aeroelasticity, and heat transfer more severe—NASA and GE jointly launched the Highly-Loaded Turbine Research Program [4], which explicitly proposed “Reduced Shock Blading” technology to achieve the aggressive goal of increasing single-stage turbine load by 33%. This represents the first explicit documentation of “weakened shock design” in domestic and international literature, reportedly achieving effective shock intensity reduction primarily through measures such as rear straight sections, convergent-divergent channels, and thin trailing edges. However, from a practical design perspective, these are merely known key parameters for controlling shock structure and intensity, with their specific values entirely dependent on the designer’s or researcher’s judgment. Yet even slight variations in these parameters can lead to completely different shock structures and intensities, making “weakened shock” design still no easy task. From this perspective, we infer that despite knowing the key parameters for weakened shock design, its perfect implementation must rely on computer optimization techniques. We regard this as the primary design philosophy for “weakened shock design.”

Based on the above considerations, this paper employs the adjoint optimization method, whose computational cost is virtually independent of the number of design variables, to conduct analytical research on weakened shock supersonic/transonic turbine cascades and to explore and master the geometric parameter selection principles for weakened shock design.

### 3. Adjoint Optimization Method

The adjoint method developed in previous work [7-8] is applied in this paper for weakened shock design. The core of this method comprises the adjoint equations and their boundary conditions, as well as the sensitivity relationships between the objective function and design variables, whose derivations depend on the flow governing equations and objective function. This paper employs the thin-layer simplified N-S equations, using mass-averaged entropy increase between inlet and outlet as the objective function, with mass flow rate and pressure ratio as constraints to establish a three-dimensional viscous aerodynamic optimization design model for multi-stage turbomachinery. In application, the blade radius formed by stacking typical airfoil sections is taken as a large value to neglect centrifugal force terms, allowing the three-dimensional program to be used for two-dimensional cascade optimization.

To meet design requirements, the outlet flow angle must remain unchanged before and after optimization. Therefore, in addition to the mass flow constraint, the outlet flow angle is added as a new constraint condition to the objective function—mass-averaged entropy increase between inlet and outlet—forming the final objective function as follows:

where  $I$  denotes the objective function,  $s$ ,  $\bar{m}$ , and  $\beta$  represent the mass-averaged entropy increase between inlet and outlet, the arithmetic mean of mass flow rates at inlet and outlet, and the outlet flow angle, respectively; the subscript “0” indicates the corresponding value of the initial cascade;  $\sigma_1$  and  $\sigma_2$  are the penalty function weighting factors. (1)

In this optimization, the airfoil parameterization selects the circumferential coordinate of the suction surface, while the circumferential thickness remains unchanged during the optimization process.

### 4. Case Studies

The weakened shock research in this paper is conducted based on existing supersonic/transonic cascades with already satisfactory performance. The selected cascades are two typical Russian supersonic/transonic turbine cascades: Cascade No. 9 and Cascade No. 45 from reference [5], designated as Case 1 and Case 2 respectively. Their key geometric parameters are listed in , and their geometries are shown in [Figure 1: see original paper]. The two cases feature sequentially increasing exit/throat expansion ratios.

To distinguish the influence of different back pressures (i.e., different operating conditions) on optimization results, optimizations were performed at three operating points with dimensionless back pressures ( $P_b/P_0$ ) of 0.33, 0.40, and 0.47.

## 5.1 Comparison of Cascade Geometry and Key Parameters Before and After Optimization

[Figure 2: see original paper] presents a comparison of the original and optimized airfoils for both cascades under different back pressures. The figure shows significant geometry changes before and after optimization, particularly for Case 2. The changes primarily manifest as: rearward movement of maximum thickness; the rear portion of the chord is not straight, especially after the throat, but shows significantly increased camber; simultaneously, since circumferential thickness remains constant during optimization, the thickness represented by the inscribed circle diameter in the rear portion of the chord decreases substantially. The comparison between Case 1 and Case 2 indicates that the changes in the convergent-divergent channel cascade before and after optimization are much smaller than those in the convergent channel cascade.

To detail the geometric changes and shaping parameters of the cascades before and after optimization, a shaping program was used to back-calculate the key geometric parameters of the optimized cascades, with the significantly changed parameters summarized in .

The results show that it is difficult to obtain clear shaping rules for “weakened shock design” from specific numerical values alone. Simultaneously, these uncertain differences fundamentally determine that manual design optimization is extremely difficult, confirming that “weakened shock design” must rely on computer optimization.

## 5.2 Comparison of Aerodynamic Performance Before and After Optimization

summarizes the results at the optimization operating points. It can be seen that under different optimization conditions, the outlet flow angle constraint is basically achieved as intended, while cascade performance improves to varying degrees. For example, for Cascade No. 9, the total pressure recovery coefficient increases by 0.001, 0.005, and 0.007 at dimensionless back pressures of 0.33, 0.40, and 0.47, respectively. For Cascade No. 45, the total pressure recovery coefficient increases by 0.002, 0.01, and 0.023, respectively.

To verify the full operating range performance of the optimized cascades, viscous flow field simulations were conducted for both original cases and all six optimized results across all operating conditions, with results summarized in [Figure 3: see original paper]. The horizontal axis represents the exit isentropic velocity coefficient, the vertical axis represents the total pressure recovery coefficient, and large hollow icons indicate the optimization operating points. Clearly, within the exit isentropic velocity coefficient range of 0.5 to 1.3, the total pressure recovery coefficients of the cascades optimized under various back pressures are almost all higher than those of the original cascades, particularly in the 0.9 to 1.2 range where performance improvements are very significant. Analysis of the

results, combined with reference [9], attributes this entirely to improved shock structure near the trailing edge and consequently improved pressure distribution near the trailing edge and reduced trailing vortex region, which substantially decreases profile base drag. Reviewing the three original cascades, the exit/throat expansion ratios increase sequentially. Cascade No. 9, designed for lower exit velocity than Cascade No. 45, is a near-convergent transonic cascade, while Cascade No. 45 features a convergent-divergent channel. From the optimization results, for near-convergent transonic cascades where shock intensity is inherently weaker, the optimization benefits are not substantial. For convergent-divergent cascades specifically designed for higher exit velocities, where shock structures are complex and intense, conventional manual design cannot achieve fine organization for optimal performance. However, through optimization-based “weakened shock” design, performance can be improved regardless of the optimization operating point. This result stems from the automatic fine organization of shock waves and related flow through computer optimization. From the full operating range characteristic comparison of optimization results, selecting low back pressure conditions for optimization is more appropriate for high design exit Mach numbers, though this may cause performance degradation at low exit velocity conditions. Nevertheless, compared with the original cascades, this approach still yields efficiency gains. In the exit velocity coefficient range of 0.9-1.1, the “weakened shock” cascades obtained through optimization do not exhibit the typical “hump region” where loss first increases then decreases.

### 5.3 Flow Field Comparison and Analysis Before and After Optimization

The previous results show significant performance changes before and after optimization. How do the flow field details change? To investigate, numerical simulations were performed on the original and optimized cascades at the optimization back pressures. Taking Cascade No. 45 at a dimensionless back pressure of 0.47 as an example, [Figure 4: see original paper] shows the flow field, with the left image showing the original cascade and the right image showing the optimized cascade. The comparison reveals that optimization significantly weakens the trailing edge shock, consequently reducing the reflection of the pressure-side trailing edge shock on the suction surface, with the suction-side trailing edge shock also noticeably weakened or even eliminated. In the original cascade, the suction-side trailing edge oblique shock is relatively weak, while the pressure-side oblique shock angle increases, developing toward a near-normal shock. Influenced by the pressure ratio across the shock, the boundary layer thickens at the shock impingement point on the suction surface, even forming a separation bubble that leads to increased loss. After optimization, the suction-side trailing edge shock basically disappears, and the pressure-side trailing edge shock is no longer distinct. The overall flow development becomes uniform, and the suction surface boundary layer is no longer thickened or separated by shock effects, thereby reducing overall loss.

These results demonstrate that using optimization methods to weaken shock waves can effectively organize the flow field, reducing or eliminating shock waves and thereby improving supersonic/transonic turbine performance.

## Conclusions

Shock wave phenomena severely impact the aerodynamic performance, cooling characteristics, and structural integrity of supersonic/transonic turbines. Addressing this issue, this paper explored weakened shock turbine cascade design techniques using two typical supersonic/transonic turbine cascades and the adjoint optimization method. The main contributions and findings are:

- 1) The “weakened shock design” technology was analyzed, demonstrating that perfect “weakened shock design” must rely on computer optimization. The key geometric parameters of optimized cascades show that manual design cannot achieve optimal “weakened shock” performance.
- 2) Two typical Russian supersonic/transonic turbine cascades were selected and optimized under different back pressures using the adjoint method. Full operating range characteristics were verified through numerical simulation, indicating that selecting low back pressure for optimization design relative to the design operating point can achieve optimal full-range performance.
- 3) Weakened shock design achieved through optimization can reduce trailing edge base drag, thereby essentially eliminating the “hump region” where loss first increases then decreases at exit velocity coefficients of 0.9-1.1.

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