Postprint: Performance Study of Quantum Stirling Refrigeration Cycle

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Abstract

An irreversible quantum Stirling refrigeration cycle model is established with particles in infinitely many infinite potential wells as the working substance. The distribution of particles in the potential well over energy levels is determined by the Gibbs distribution function. This refrigeration cycle consists of two isoenergetic processes and two constant-potential-well-wall-width processes, and features heat leak between high- and low-temperature reservoirs as well as incomplete regeneration factors. Analytical expressions for the coefficient of performance, refrigeration rate, entropy production rate, and ecological objective function performance are derived; the relationship curve between refrigeration rate and ecological function is a loop-shaped curve passing through the origin. The influence of the heat leak coefficient and incomplete regeneration factor on the coefficient of performance, refrigeration rate, and ecological function is analyzed.

Full Text

Preamble

Study on the Performance of Quantum Stirling Refrigeration Cycle

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Abstract

An irreversible quantum Stirling refrigeration cycle model is established using particles in infinite potential wells as the working substance. The distribution of particles across energy levels in the potential well is determined by the Gibbs distribution function. The refrigeration cycle consists of two iso-energy processes and two iso-potential-well-width processes, featuring heat leakage between high- and low-temperature reservoirs and imperfect regeneration. Analytical expressions for the coefficient of performance (COP), cooling rate, entropy generation rate, and ecological objective function are derived. The relationship curve between cooling rate and ecological function is a loop-shaped curve that returns to the origin. The effects of heat leakage coefficient and imperfect regeneration factor on COP, cooling rate, and ecological function are analyzed.

Keywords: finite time thermodynamics; one-dimensional infinite potential well; quantum refrigeration cycle; ecological performance optimization

0 Introduction

At micro-nano scales, under low-temperature or high-density conditions, the quantum characteristics of working substances in thermodynamic cycles must be considered. Applying finite time thermodynamics theory and methods enables the establishment of various quantum cycle models to investigate their thermodynamic properties and performance parameters, and to optimize their operating regimes. Real heat engines operate irreversibly due to factors such as thermal resistance, heat leakage, friction, eddy currents, inertial effects, and non-equilibrium influences.

Several scholars have studied the effects of irreversible factors on quantum thermodynamic cycle performance. Jin et al. considered weak coupling between high- and low-temperature heat sources and introduced bypass heat leakage. Feldmann and Kosloff introduced an internal friction coefficient to describe quantum non-adiabatic phenomena. Wu et al. examined the effects of heat leakage, thermal resistance, and finite-rate heat transfer on the performance of irreversible quantum Stirling refrigerators. Jin Xiaochang investigated the optimal performance of quantum Stirling refrigerators from an exergoeconomic perspective.

Building upon the work documented in references [3,10,14,16,23,28,30], this paper establishes an irreversible Stirling refrigeration cycle model that incorporates heat leakage between high- and low-temperature reservoirs and imperfect regeneration, using countless particles confined in one-dimensional infinite potential wells as the working substance. We derive the coefficient of performance (COP), cooling rate, entropy generation rate, and ecological function, and analyze the relationships between these important performance parameters and the heat leakage rate and imperfect regeneration factor.

For simplicity, we consider a particle confined in a one-dimensional infinite potential well, taking into account only two energy levels. The working substance consists of countless such particles, each confined in its own one-dimensional infinite potential well. The occupation probabilities of particles at different energy levels are determined by the Gibbs distribution. The cycle comprises two iso-energy processes and two regenerative processes with constant potential well width, corresponding to the isothermal and isochoric processes in classical cycles, respectively.

1 Quantum Mechanical Description of the System

The system under study consists of a particle confined in a one-dimensional infinite potential well of width L. Its stationary wave function satisfies:

$$-\frac{\hbar^2}{2m}\frac{d^2\Psi}{dx^2} = E\Psi$$

Based on the boundary conditions that the wave function must satisfy, (0) = 0 and (L) = 0, the wave function can be obtained as:

$$\Psi(x) = \sum_{n=1}^{\infty} a_n \phi_n(x)$$

where ϕ (x) is the normalized eigenfunction:

$$\phi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$$

The coefficients a satisfy the normalization condition:

$$\sum_{n=1}^{\infty} |a_n|^2 = 1$$

where p and E (i = 1,2,L; n = 1,2) represent the probability density and the corresponding energy eigenvalue for each level, respectively, with i = L denoting an arbitrary state and n being the quantum number.

Substituting equations (4) and (5) into equation (9) yields:

$$p_n = |a_n|^2$$

where p is the probability density. The energy eigenvalues of the system are:

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}$$

where m and n are the particle mass and quantum number, respectively. The expectation value of the Hamiltonian is:

$$E = \sum_{n=1}^{\infty} p_n E_n = \sum_{n=1}^{\infty} p_n \frac{n^2 \pi^2 \hbar^2}{2mL^2}$$

In classical cycles, a piston moves reciprocally in a cylinder. Assuming the potential well wall moves like a piston in classical cycles, both the wave function and energy levels vary with L when the potential well width changes. The generalized force is defined as:

$$Y_n = -\frac{dE_n}{dL}$$

where y is the generalized coordinate corresponding to the generalized force Y . Therefore, the force acting on the potential well wall can be expressed as:

$$F = \sum_{n=1}^{\infty} p_n Y_n = -\sum_{n=1}^{\infty} p_n \frac{dE_n}{dL}$$

From equations (8) and (10), the force acting on the potential well wall during the iso-energy process 1-2 can be obtained as:

$$F_{12} = \frac{\pi^2 \hbar^2}{mL^3} \sum_{n=1}^{\infty} p_n n^2$$

During the iso-energy process, the total energy of the system remains constant. The heat absorbed from the low-temperature reservoir is entirely used for external work, therefore:

$$Q_{12} = W_{12} = \int_{L_1}^{L_2} F_{12} dL$$

Similarly, during process 3-4, the heat released to the high-temperature reservoir and the work done on the system equal:

$$Q_{34} = W_{34} = \int_{L_2}^{L_1} F_{34} dL$$

2 Quantum Stirling Refrigeration Cycle

[Figure 1: see original paper] shows the schematic diagram of the quantum Stirling refrigeration cycle. The system absorbs heat from a low-temperature reservoir at temperature T and releases heat to a high-temperature reservoir at temperature T . This cycle is an irreversible quantum Stirling refrigeration cycle. Process 1-2 is an iso-energy process during which the total energy of the system remains constant, leading to:

$$E_1 = E_2$$

Process 2-3 and process 4-1 are iso-potential-well-width processes during which the internal energy of the system changes. Assuming the rate of change of internal energy with time is constant, and since the internal energy is a single-valued function of temperature when the potential well width is fixed, the time required for these processes can be expressed as:

$$\tau_{23} = \frac{M(T_H - T_L)}{T_H}$$

$$\tau_{41} = \frac{M(T_H - T_L)}{T_L}$$

where M is a constant independent of the system energy, depending only on the regenerative characteristics of the regenerator material. The positive and negative signs correspond to processes 2-3 (i = 1) and 4-1 (i = 2), respectively. For simplicity, we assume $M_1 = M_2$.

In an ideal regeneration process, the heat exchanged in the regenerator would be:

$$Q_{23} = Q_{41}$$

As shown by equations (14) and (15), $Q_{23} = Q_{41}$, indicating that an ideal quantum Stirling refrigerator can achieve perfect regeneration. However, assuming heat leakage exists between the high- and low-temperature reservoirs, the heat leakage rate is determined by:

$$\dot{Q}_{leak} = \alpha(T_H - T_L)$$

where α is a constant. The heat leakage per cycle is then:

$$Q_{leak} = \dot{Q}_{leak}\tau = \alpha(T_H - T_L)\tau$$

In an ideal regeneration process, the system would complete regeneration at state 1. Due to internal irreversibilities, actual Stirling refrigerators experience regenerative losses, completing regeneration at state 1 instead of state 1. Introducing an imperfect regeneration factor (> 0), the regenerative loss can be expressed as:

$$Q_{loss} = \mu Q_{reg}$$

3 Cycle Period

Assuming the potential well wall moves with velocity v(t) and average velocity v̄, we obtain:

$$\bar{v} = \frac{L_2 - L_1}{\tau_{12}} = \frac{L_1 - L_2}{\tau_{34}}$$

The time required for iso-energy processes 1-2 and 3-4 is:

$$\tau_{12} = \frac{L_2 - L_1}{\bar{v}}$$

$$\tau_{34} = \frac{L_1 - L_2}{\bar{v}}$$

The total cycle period can be expressed as:

$$\tau = \tau_{12} + \tau_{23} + \tau_{34} + \tau_{41}$$

4 Performance Parameters and Analysis of Irreversible Stirling Refrigeration Cycle

The net heat absorbed from the low-temperature reservoir and the net heat released to the high-temperature reservoir can be expressed as:

$$Q_L = Q_{12} - Q_{leak} - Q_{loss}$$

$$Q_H = Q_{34} + Q_{leak} + Q_{loss}$$

The probability density of particles at a certain energy level is given by the Gibbs distribution:

$$p_n = \frac{1}{Z} \exp\left(-\frac{E_n}{kT}\right)$$

where Z is the partition function. From equations (12) and (13), the input work of the cycle is:

$$W = Q_H - Q_L$$

where x = L_2/L_1 represents the potential well width ratio. From equations (19) and (31), the coefficient of performance (COP) of this irreversible quantum Stirling refrigerator is:

$$\text{COP} = \frac{Q_L}{W}$$

Combining equations (19) and (30) yields the cooling rate:

$$R = \frac{Q_L}{\tau}$$

The entropy generation rate is:

$$\sigma = \frac{Q_H/T_H - Q_L/T_L}{\tau}$$

The exergy output rate is:

$$\dot{A} = Q_L\left(\frac{T_0}{T_L} - 1\right) - Q_H\left(\frac{T_0}{T_H} - 1\right)$$

The ecological objective function is defined as:

$$E = R - T_0\sigma$$

The dimensionless ecological objective function is:

$$E^* = \frac{E}{E_{max}}$$

From equations (27), (28), and (31), for given T_H, T_L, T_0, L_1, M = 15, and $\bar{v} = 1/(2L_1)$, the cooling rate, COP, and ecological objective function of the refrigerator are all functions of the potential well width ratio x.

[Figure 2: see original paper] shows the relationship between COP and x for different values of the imperfect regeneration factor . The results indicate that the COP decreases as the potential well width ratio x increases. As increases, the COP decreases.

[Figure 3: see original paper] illustrates the effect of α on COP versus x when = 0.01, with other parameters identical to those in Figure 2. The COP decreases as α increases.

[Figure 4: see original paper] presents the cooling rate as a function of x for different values when $\alpha = 0.1$, with other parameters identical to those in Figure 2. The relationship between cooling rate and x exhibits a parabolic-like shape. The extremum condition yields the optimal relationship between cooling rate and potential well width ratio x.

[Figure 5: see original paper] shows the cooling rate versus x for different α values when = 0.01. The cooling rate decreases as heat leakage increases.

[Figure 6: see original paper] depicts the optimization relationship between the dimensionless ecological objective function and cooling rate (E-*R). The E*-R optimization curve is loop-shaped, featuring a maximum ecological objective function point and a maximum cooling rate point. Each ecological objective function value (except at the maximum) corresponds to two cooling rate values, and the state point with the larger cooling rate should obviously be selected. Heat leakage does not change the curve type, but both the ecological objective function value and cooling rate decrease as heat leakage increases. The effect of heat leakage is smaller when the ecological objective function value is small, and larger when the ecological objective function value is large.

[Figure 7: see original paper] shows the effect of on the E-*R relationship curve. Comparing Figures 6 and 7 reveals that the influence of heat leakage on the E*-R relationship is greater than that of the imperfect regeneration factor, indicating that heat leakage is a more significant irreversibility than imperfect regeneration.

Conclusion

This paper establishes an irreversible quantum Stirling refrigeration cycle model using countless particles in infinite potential wells as the working substance. Based on solutions to the Schrödinger equation, we analyze the performance of this irreversible quantum Stirling refrigerator, deriving expressions for the coefficient of performance, cooling rate, entropy generation rate, and ecological objective function. We examine the effects of heat leakage coefficient and imperfect regeneration factor on COP, cooling rate, and ecological function. The relationship curve between cooling rate and ecological function is loop-shaped and returns to the origin. There exists a maximum ecological objective function value and a maximum cooling rate value, with the region between these two maxima representing the optimal operating range for the refrigerator. These results contribute to a deeper understanding of irreversible quantum Stirling refrigeration cycle performance using particles in infinite potential wells as the working substance.

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Note: Figure translations are in progress. See original paper for figures.