Effects of Leading Edge Shape on Boundary Layer Parameters in Subsonic Airfoils (Postprint)

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Abstract

The influence of circular-arc, elliptical, and curvature-continuous leading edges without suction peaks on blade profile boundary layer development was investigated using numerical simulation methods. By extracting blade surface boundary layer parameters under different operating conditions and combining them with changes in the blade surface boundary layer velocity profiles, it was found that the essential influence of leading-edge suction peaks on blade profile performance lies in that the suction peaks affect the initial development state of the boundary layer. An excessively strong diffusion parameter Dspike of the leading-edge suction peak leads to deterioration of the boundary layer' s initial development state, causing premature transition and even separation bubbles, rapid thickening of the blade surface boundary layer, increased profile loss, and a reduced usable angle-of-attack range. The study also demonstrates that the designed curvature-continuous leading edge without suction peaks can always eliminate suction peaks near the leading edge on either the suction surface or pressure surface under any operating condition, thereby resulting in a superior initial state of the blade surface boundary layer compared to other leading-edge profiles, and consequently better aerodynamic performance.

Full Text

Preamble

Effects of Leading-Edge Geometry on Boundary Layer Parameters in Subsonic Airfoils

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Abstract

This study employs numerical simulation to investigate how circular, elliptical, and curvature-continuous spikeless leading edges affect the development of boundary layers on airfoil surfaces. By extracting blade surface boundary layer parameters under various operating conditions and analyzing changes in the boundary layer velocity profiles, it is revealed that the essential mechanism by which leading-edge suction peaks influence airfoil performance lies in their impact on the initial development state of the boundary layer. An excessively strong leading-edge suction peak diffusion parameter, Dspike, deteriorates the initial boundary layer state, causing premature transition and even separation bubbles. This leads to rapid thickening of the blade surface boundary layer, increased airfoil losses, and a reduced usable incidence range. The research also demonstrates that the designed curvature-continuous spikeless leading edge can eliminate suction peaks on either the suction or pressure surface near the leading edge under any operating condition, thereby ensuring a superior initial boundary layer state compared to other leading-edge geometries and consequently achieving better aerodynamic performance.

Keywords: Leading edge; Subsonic compressor blade; Boundary layer; Numerical analysis; Compressor

0 Introduction

Future aircraft propulsion systems demand compressors with significantly higher pressure ratios and efficiencies than current operational levels to improve engine thrust-to-weight ratio and reduce fuel consumption. Translating these requirements to blade design entails high loading, a broad usable incidence range, and a healthy blade surface boundary layer. From both domestic and international research trends, Controlled Diffusion Airfoils (CDA) represent a key technology for high-efficiency, high-load compressor design and have found extensive engineering application in the subsonic environment of high-pressure compressor rear stages. The core philosophy of CDA design involves controlling the isentropic Mach number distribution or blade surface static pressure rise coefficient according to specific patterns to promote healthier boundary layer development and enhance airfoil performance.

In aero-engines, compressor blade leading-edge thickness is often extremely small, reaching minimum values of approximately 0.2 mm, which corresponds to a relative thickness of only about 1% of the chord length. In engineering design, considerations are predominantly structural strength-based. Additionally, during the airfoil design process, the dramatic curvature changes at the leading and trailing edges typically necessitate a piecewise approach—generating the blade body first before adding the leading and trailing edges. This often results in large curvature discontinuities at the junctions between the leading edge and the blade body. Furthermore, insufficient manufacturing precision and natural erosion and wear during service can alter the leading-edge geometry, with these geometric changes inevitably impacting blade performance.

With in-depth research on airfoils, numerous scholars have recognized that leading-edge geometry exerts a non-negligible influence on blade aerodynamic performance, making it a focal research topic. Traditional airfoil leading edges employ a circular design for manufacturing convenience. However, extensive numerical simulations and experimental studies have revealed that curvature discontinuities at the circular leading-edge-to-blade-body junction generate separation bubbles near the leading edge, causing premature flow transition and increased losses. Lu et al. conducted numerical simulations on elliptical leading edges and found they could significantly suppress suction peaks and improve aerodynamic performance. Based on the concept of controlling curvature transition between the leading edge and blade body, the authors proposed a circular leading edge with a platform, discovering this design could improve blade surface flow and achieve results comparable to elliptical leading edges. Addressing leading-edge issues, Song and Yuan et al. subsequently proposed curvature-continuous leading-edge design methods, demonstrating that such designs better suppress leading-edge suction spike generation, promote healthier boundary layer development, and effectively improve blade performance. Wheeler and Miller experimentally investigated the effects of upstream rotor wakes on airfoil losses, showing that under upstream wake interference, elliptical leading edges significantly reduce profile losses compared to circular ones. Goodhand and Miller optimized elliptical leading edges to eliminate suction surface leading-edge spikes, achieving spikeless airfoil design at design conditions. They further proposed that suction peak height possesses a critical value evaluable through the leading-edge suction spike diffusion factor Dspike; when the suction peak exceeds this critical value, airfoil losses increase rapidly.

These findings indicate that leading-edge geometry substantially impacts airfoil performance, with different leading-edge shapes affecting performance differently under identical conditions. However, the mechanism by which leading-edge geometry influences blade surface boundary layer development remains incompletely understood. Consequently, this paper focuses on a CDA airfoil, implementing circular, elliptical, and spikeless leading-edge designs without altering the blade body. Numerical analysis methods capable of simulating boundary layer transition are employed to model the incidence-loss characteristics, with detailed extraction of blade surface boundary layer evolution processes and in-depth analysis of boundary layer parameters. By comparing boundary layer development across different leading-edge geometries, the mechanism of leading-edge geometry effects on airfoil aerodynamic performance is investigated. This research provides essential theoretical support for further experimental studies and airfoil optimization.

1.1 Airfoil Introduction

The airfoil parameters are summarized in Table 1 , and the leading-edge geometry is shown in Figure 1 [Figure 1: see original paper]. The controlled diffusion airfoil parameters include: chord length, solidity, inlet Mach number, stagger angle, inlet metal angle, camber angle, turbulent intensity, and Reynolds number (based on inlet velocity and chord) of 5.45$×10^{5}$.

1.2 Mesh

Given the focus on boundary layer flow details, high mesh quality is essential. The mesh is generated using ICEM, with "O"-type grids employed on the blade surface to ensure good orthogonality while achieving local refinement. This guarantees a near-wall mesh $y^+ < 1$, with more than 10 grid points within the boundary layer and a growth ratio of 1.1 toward the mainstream region. The computational domain extends approximately 1.5 chord lengths upstream of the blade leading edge and 2 chord lengths downstream of the trailing edge. The spanwise direction is assigned 20 grid points, resulting in a total model mesh count of 640,000.

1.3 Turbulence Model Selection and Boundary Conditions

ANSYS CFX software is used for airfoil computations. To accurately predict boundary layer transition and flow details, the Shear Stress Transport (SST) turbulence model is selected with the Gamma-Theta transition model. High-precision schemes are employed to ensure second-order accuracy. Boundary conditions are specified as follows: inlet boundary with total temperature, total pressure, and flow angle; outlet boundary with mass flow rate varying according to operating conditions; periodic boundary conditions in the pitchwise direction; symmetric boundary conditions in the spanwise direction; and a smooth no-slip solid surface on the blade. Convergence is determined when residuals fall below 1.0$×10^{-4}$, with mass flow, velocity, and pressure residual curves required to be flat.

1.4 Numerical Simulation Validation

Following determination of the mesh, numerical scheme, turbulence model, transition model, and boundary conditions, experimental data are used to validate the CFX results and confirm computational reliability. Figure 3 [Figure 3: see original paper] compares CFX results with cascade experimental data. Due to three-dimensional flow effects in cascade experiments, particularly endwall boundary layers, the axial velocity density ratio (AVDR) is approximately 1.02. Since CFX simulations do not account for AVDR effects, slight deviations exist between computational and experimental results. However, overall agreement is satisfactory, especially near the leading edge, where suction peak simulation shows good consistency with experiments. Although separation-bubble-type boundary layer transition was not observed in experiments (likely due to blade surface roughness and higher inlet turbulence), computational results reveal separation-bubble transition near 40% chord location, creating a pressure plateau. Given that this study primarily analyzes the qualitative effects of leading-edge geometry on blade surface boundary layers, the numerical simulation results provide valuable reference.

2 Effects of Leading-Edge Geometry on Airfoil Aerodynamic Performance

Figure 4 [Figure 4: see original paper] presents the computed total pressure loss characteristics for airfoils with different leading-edge geometries. The minimum losses for the three geometries are essentially identical, but the usable incidence range (defined as the incidence range where loss remains below twice the minimum loss) shows significant variation. The circular leading-edge airfoil exhibits a usable incidence range approximately 2.5° narrower than the elliptical and spikeless designs, representing about 14.3% of the total operating range. The spikeless leading-edge airfoil demonstrates the widest usable incidence range and the lowest losses, indicating that different leading-edge geometries create distinct boundary layer flow states, resulting in variations in loss and operating range.

Figures 5 [Figure 5: see original paper], 6 [Figure 6: see original paper], and 7 [Figure 7: see original paper] show flow deflection angle characteristics, D-factor characteristics, and deviation angle characteristics, respectively. The circular leading-edge airfoil exhibits significantly poorer pressure rise performance at high incidence angles compared to the other two designs, with the elliptical leading edge performing intermediately and the spikeless design showing optimal performance. The circular leading-edge airfoil also displays consistently higher deviation angles across the incidence range, with premature boundary layer separation at large positive and negative incidence angles causing loss to increase with incidence deviation. In contrast, the spikeless leading-edge airfoil shows the smallest deviation angles and the latest boundary layer separation at high incidence angles, maintaining excellent flow conditions. In summary, with all other parameters held constant and only leading-edge geometry varied, significant performance differences emerge due to altered boundary layer development states. To further investigate leading-edge geometry effects, detailed comparative analysis of blade surface boundary layer development follows.

3.1 Boundary Layer Parameter Extraction Method

To extract boundary layer parameters, an in-house boundary layer extraction program was developed. The computed flow field is first interpolated onto a grid strictly orthogonal to the airfoil surface, after which boundary layer parameters are extracted and analyzed on this orthogonal grid. The integration direction follows the surface normal direction, where $_{1}u$ represents local density and velocity, and $_{2}U$ represents mainstream density and velocity. Mainstream identification is based on extremum points of the velocity gradient along the normal direction; when no extremum exists, a minimum gradient threshold is used. Since boundary layer parameters are insensitive to gradient thresholds, the mainstream region' s maximum gradient can be appropriately increased as the criterion for distinguishing the boundary layer from the mainstream. Momentum thickness is calculated using appropriate integral formulations, and energy thickness is determined through standard boundary layer analysis procedures. The shape factor is then determined accordingly.

3.2 Analysis of Airfoil Boundary Layer Development Under Different Incidence Conditions

Figures 8 [Figure 8: see original paper], 9 [Figure 9: see original paper], and 10 [Figure 10: see original paper] illustrate the evolution of boundary layer integral parameters on the suction surface at different incidence angles. At a large negative incidence of 35° inlet flow angle, the stagnation points of all three airfoils are located on the suction surface side, with minimal differences in the leading-edge flow process around the suction surface. Consequently, boundary layer development is essentially identical, with transition occurring at 60% chord, rapid boundary layer thickening, increased friction losses, and essentially identical suction surface losses.

At the design condition of 47° inlet flow angle, stagnation points are located near the leading edge. The spikeless airfoil exhibits the smallest leading-edge curvature variation, with the elliptical design showing intermediate curvature change. Both experience similar flow processes on the suction surface side without excessive sudden compression or expansion, undergoing transition at 40% chord, rapid thickening, and increased friction losses. However, the circular leading-edge airfoil features curvature discontinuities, resulting in strong sudden compression and over-expansion processes—i.e., a strong leading-edge spike —and a separation bubble at the leading edge. This causes separation-bubble transition near the leading edge on the suction surface, rapid boundary layer thickening, and increased friction losses. Therefore, under this condition, the circular airfoil exhibits the highest suction surface losses, while elliptical and spikeless designs show minimal and essentially identical losses.

At a large positive incidence of 50° inlet flow angle, stagnation points are located on the pressure surface side, requiring flow to wrap around the leading edge to reach the suction surface. The three leading-edge geometries exhibit substantial differences in curvature variation, leading to varying degrees of compression and expansion in the leading-edge flow. The circular leading-edge airfoil maintains maximum curvature except at the leading-edge point but features curvature discontinuities at the leading-edge-to-blade-body junction, resulting in the strongest sudden compression and over-expansion, the most intense leading-edge spike, and a separation bubble on the suction surface. Transition occurs prematurely near the leading edge, causing excessively rapid boundary layer thickening and large trailing-edge separation, increasing suction surface losses.

The elliptical leading-edge airfoil has larger curvature at the leading-edge point but smaller curvature elsewhere, with minimal curvature discontinuity at the junction, producing weaker compression and expansion than the circular design. Although transition also occurs near the leading edge, boundary layer thickening is slower, resulting in smaller loss increases. The spikeless design, despite having maximum curvature at the leading-edge point, features minimum curvature elsewhere and curvature continuity between leading edge and blade body, significantly reducing the expansion process and optimizing the leading-edge flow. Transition remains at 30% chord—further downstream than the other designs—with the weakest trailing-edge separation and minimal losses.

Figures 11 [Figure 11: see original paper], 12 [Figure 12: see original paper], and 13 [Figure 13: see original paper] show boundary layer integral parameter variations on the pressure surface at different incidence angles. At 35° large negative incidence, stagnation points are on the suction surface, making the pressure surface flow process similar to the suction surface flow at large positive incidence described above. Although pressure surface transition occurs at the leading edge (separation-bubble transition), the circular airfoil experiences the strongest compression and expansion, followed by the elliptical design, with the spikeless design being the weakest. This results in the fastest and thickest pressure surface boundary layer growth for the circular design, intermediate for the elliptical, and minimal for the spikeless. Consequently, the circular airfoil exhibits the highest pressure surface losses, the elliptical design intermediate, and the spikeless design the lowest.

At 47° design incidence, stagnation points are near the leading edge. Since the compression and expansion processes are stronger on the suction surface, they are correspondingly weaker on the pressure surface, with no leading-edge pressure surface transition occurring for any design (shape factor around 2.5). The pressure surface remains in a fully laminar state, resulting in relatively small overall losses. However, the elliptical and circular designs exhibit local disturbances near the leading edge due to curvature discontinuities, creating a friction coefficient spike without triggering typical laminar boundary layer separation. These different disturbances affect the initial boundary layer development state, leading to different development characteristics from leading edge to trailing edge, though losses remain similar. At 50° large positive incidence, the pressure surface compression and expansion processes weaken further, maintaining laminar conditions similar to the 47° case.

Figure 14 [Figure 14: see original paper] shows boundary layer velocity profiles at different chordwise positions on the suction surface at 50° inlet flow angle. The circular design exhibits the most severe acceleration and expansion, with velocity profiles changing from the fullest to the most near-wall deficient between X=0.0005 and X=0.0024, clearly showing separation bubble initiation before X=0.0009 and termination after X=0.0024. The existence of this separation bubble is also evident in Figure 10 [Figure 10: see original paper] as a sudden Cf drop to zero in the near-leading-edge region. At X=0.0075, the circular design' s separation bubble has reattached, but the velocity profile fullness is significantly poorer than the other designs. According to the shape factor distribution in Figure 8 [Figure 8: see original paper], transition should have occurred after the separation bubble, resulting in rapid boundary layer thickening substantially greater than the other leading-edge designs.

The spikeless design' s suction surface boundary layer development shows no separation characteristics in the near-leading-edge region, consistent with Figure 10 [Figure 10: see original paper]. After X=0.0009, the spikeless design exhibits the fullest velocity profiles and the smallest boundary layer thickness among the three designs. At 50° incidence, the spikeless design' s friction coefficient approaches zero between 20-30% chord (Figure 10 [Figure 10: see original paper]), indicating near-separation conditions where transition occurs (Figure 8 [Figure 8: see original paper]) between X=0.036 and X=0.054. Figure 14 [Figure 14: see original paper] shows a typical laminar velocity profile at X=0.022 with smaller near-wall velocity gradients than the other designs, transitioning to a typical turbulent profile by X=0.06, indicating small-scale separation-bubble transition. The spikeless design' s later transition location results in a healthier subsequent turbulent boundary layer development.

The elliptical design' s boundary layer development shows a separation bubble scale comparable to but weaker than the circular design, with complete transition after the bubble, resulting in a laminar boundary layer. Overall, the elliptical design' s boundary layer thickness and fullness lie between the circular and spikeless designs.

In summary, different leading-edge geometries create varying leading-edge flow processes that directly affect the initial boundary layer development state, producing significant differences in blade surface flow conditions and energy loss (Figures 9 [Figure 9: see original paper] and 12 [Figure 12: see original paper]), i.e., different airfoil losses. This explains why different leading-edge shapes substantially impact blade performance—the stronger the local acceleration and deceleration in the leading-edge flow process, the greater the flow losses.

4.1 Effects of Different Leading-Edge Configurations on Leading-Edge Spike Development

To analyze key factors affecting local leading-edge flow disturbances, it is necessary to employ the Dspike parameter proposed by Goodhand and Miller for in-depth analysis. This parameter, adapted from the D-factor, characterizes the sudden compression process at the leading edge.

Figure 16 [Figure 16: see original paper] shows the variation of local static pressure coefficient with inlet flow angle for different leading-edge geometries. Combined with Figure 17 [Figure 17: see original paper] showing Dspike variation on the suction surface leading edge, it is evident that Dspike increases with inlet flow angle, intensifying the leading-edge suction peak. Different leading-edge geometries exhibit different Dspike growth rates: the circular design increases fastest, more readily producing suction surface separation bubbles and increasing losses. The spikeless design maintains low Dspike at negative incidence with slow growth, accelerating only after 45° incidence. The elliptical design shows intermediate Dspike variation. Figure 18 [Figure 18: see original paper] reveals that pressure surface leading-edge spikes are smaller than suction surface spikes, decreasing with increasing inlet flow angle and thus having reduced impact on performance. The spikeless design achieves spikeless flow on the suction surface from minimum-loss incidence (around 45°) to all negative incidence conditions, and on the pressure surface from minimum-loss incidence to positive incidence limits. The circular design exhibits strong spikes in all conditions except at maximum negative incidence on the suction surface and maximum positive incidence on the pressure surface. The elliptical design achieves spikeless flow on the suction surface near negative incidence limits and on the pressure surface at maximum positive incidence, but exhibits spikes in other conditions. Clearly, spike intensity directly correlates with boundary layer development state and airfoil loss. The spikeless design's superior performance is closely associated with essentially eliminating or significantly weakening spikes across a broad operating range, necessitating deeper analysis of the relationship between boundary layer parameters and spikes.

4.2 Relationship Between Leading-Edge Spike and Blade Surface Boundary Layer Parameters

Based on the above analysis, boundary layer momentum thickness and shape factor at 10% chord are extracted to examine relationships with Dspike. Figures 19 [Figure 19: see original paper] and 20 [Figure 20: see original paper] show that airfoil energy loss thickness increases with Dspike, with rapid loss increase after Dspike reaches a certain value due to separation bubble formation. For suction surface results (Figure 19 [Figure 19: see original paper]), the spikeless design produces greater energy loss thickness than other designs at the same Dspike, with the elliptical design also producing greater losses than the circular design. Dspike > 0.17 causes significant suction surface loss increase (critical values near 0.2 for spikeless and circular designs), deviating substantially from the Dspike $>$ 0.1 criterion proposed by Goodhand and Miller. Pressure surface results (Figure 20 [Figure 20: see original paper]) show similar trends across designs, with the circular design producing slightly smaller losses at the same Dspike. Pressure surface results indicate Dspike > 0.12 causes significant pressure surface loss increase (critical value near 0.14 for circular design), closer to the Goodhand and Miller criterion, though the circular design' s critical value approaches 0.18.

Figures 21 [Figure 21: see original paper] and 22 [Figure 22: see original paper] present correlations between shape factor and Dspike in the leading-edge region. On the suction surface, shape factor decreases rapidly after Dspike $>$ 0.17, indicating transition from laminar to turbulent state via separation-bubble formation. On the pressure surface, the critical Dspike for triggering transition is approximately 0.12 (near 0.14 for circular design). The rapid shape factor decrease followed by increase suggests that excessively strong Dspike (spikeless and elliptical designs > 0.13, circular design > 0.15) likely causes large-scale negative-incidence pressure surface separation, rapidly increasing losses. While in laminar state, the circular design exhibits smaller shape factors (smaller near-wall velocity gradients), resulting in smaller energy loss thickness, which should be related to local leading-edge curvature.

5 Conclusions

This study investigates the mechanisms by which different leading-edge geometries affect blade surface boundary layer development through numerical simulation of three airfoil designs, yielding the following main conclusions:

1. Different leading-edge geometries create different leading-edge flow processes (i.e., different leading-edge spikes). Excessively strong leading-edge spikes degrade airfoil performance and reduce usable incidence range. The circular leading-edge design exhibits the most dramatic flow process changes, larger losses, and a smaller usable incidence range, while the spikeless design weakens leading-edge spikes, reduces losses, and expands the usable incidence range.
2. The primary impact of leading-edge spikes on airfoil performance lies in their influence on the initial boundary layer development state. Overly strong spikes produce less full velocity profiles, making the boundary layer less resistant to adverse pressure gradients, causing premature transition and even separation bubbles. This leads to rapid boundary layer thickening, increased airfoil losses, and reduced usable incidence range.
3. Suction surface leading-edge Dspike increases with inlet flow angle, increasingly degrading suction surface performance, while the opposite trend occurs on the pressure surface.
4. Airfoil losses increase with leading-edge Dspike, with performance deteriorating sharply after reaching a critical value.
5. The designed spikeless leading edge can eliminate suction peaks on either the suction or pressure surface near the leading edge under any operating condition, ensuring a superior initial boundary layer state compared to other leading-edge geometries.

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Note: Figure translations are in progress. See original paper for figures.