Study on Fractal Transport Properties of Reservoir Pores Based on Multiscale Dimensional Unification Principle (Postprint)

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Abstract

The pore space distribution in oil reservoirs exhibits fractal characteristics. Current research methodologies for applying fractal description methods to investigate fluid transport properties within reservoir porous media have certain limitations, primarily the failure to consider principles of dimensional consistency and scale effects. Based on the principle of dimensional consistency, this study utilizes fractal description methods to derive permeability and seepage flow rate for transport processes in reservoir porous media, obtaining unit scale conversion coefficients for transport characteristic parameters, thereby enabling analysis of macroscopic transport processes based on descriptions of microscopic fractal characteristics. The fractal transport description equation established herein facilitates conversion between different scale levels, yielding macroscopic characteristic parameters that are more consistent with measured values. Using data from a specific oil reservoir, the model developed in this study was analyzed and validated, demonstrating that the variation patterns of transport flow rate and permeability align with the distribution patterns of actual reservoirs.

Full Text

Study on Fractal Transport Characteristics of Reservoir Pores Based on the Principle of Dimensional Unity

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Abstract: Reservoir pore distributions exhibit fractal characteristics. However, current research methods for applying fractal description techniques to fluid transport in reservoir porous media suffer from limitations, primarily the failure to consider dimensional unity principles and scale effects. Based on the principle of dimensional unity, this paper employs fractal description methods to derive permeability and seepage flow in reservoir porous media, obtaining unit scale conversion coefficients for transport characteristic parameters. This enables macroscopic transport processes to be analyzed based on microscopic fractal characteristics. The proposed fractal transport equations facilitate conversion between different scale levels, yielding macroscopic characteristic parameters that more closely match measured values. Using reservoir data, the model was validated, demonstrating that the variation patterns of transport flow and permeability align with actual reservoir distributions.

Keywords: Transport characteristics; Fractal; Dimensional unity; Scale conversion; Reservoir pores

Transport in porous media refers to the transfer of liquids, gases, electricity, heat, sound, waves, etc., through pore structures [?]. Research on fluid transport characteristics in porous media is typically associated with reservoir exploitation, as reservoir medium properties can directly or indirectly reflect subsurface conditions, enabling understanding and prediction of reservoir dynamics to enhance oil and gas recovery [?].

Studies of pore structures date back to the 1920s when Nutting [?] first investigated reservoir pore characteristics using capillary curve theory combined with thin section methods. Since then, researchers have recognized that actual reservoir pore structures are extremely complex and cannot be adequately described using traditional mathematical methods. In the 1980s, B.B. Mandelbrot [?] introduced the concept of fractals after studying irregular patterns in nature, providing a method to handle extremely irregular complex geometries. Subsequent studies demonstrated that reservoir rocks exhibit good fractal characteristics at both macroscopic and microscopic scales. Avnir et al. [?] used molecular adsorption methods to prove the fractal nature of reservoir rock pore structures. Katz and Thompson [?] observed rock fractures using scanning electron microscopy, finding that various sandstones, shales, and carbonate rocks were self-similar across three to four orders of magnitude, with fractal dimensions describing pore structural characteristics ranging from 2.55 to 2.87 within a statistically self-similar scale range of 0.2 to 50 m. They subsequently predicted rock porosity values using statistical fractal methods, demonstrating good agreement with measured values. Hansen [?] derived relationships between sandstone volume fractal dimensions, surface fractal dimensions, and rock permeability, proving that permeability depends on fractal dimension magnitude. Kim Tae H [?] developed fractal discrete fracture network code based on fractal theory to establish FDFN (Fractal Discrete Fracture Network) models for obtaining reservoir porosity values. Xie S Y et al. [?] used electron microscopy to present two-dimensional images and calculated fractal dimensions using box-counting and multifractal parameters to establish relationships with reservoir porosity and permeability for evaluating reserves in different formations.

With scientific and technological advances and deepening oil and gas exploration, domestic scholars have also made significant contributions. Tong Dengke et al. [?] used fractal dimensions and fractal exponents to describe seepage characteristics in fractal reservoirs, analyzing the influence of fractal parameters on reservoir pressure and obtaining predictions close to actual reservoir data. Yu Boming et al. [?] established fractal seepage models for reservoir porous media, deriving fractal calculation formulas for various parameters and obtaining analytical solutions. Liu Junliang et al. [?] derived characteristic parameters such as porosity and permeability for fractal porous media based on fundamental fractal concepts using Sierpinski carpets, discussing relationships between permeability and fractal dimension under different models. Kong Xiangyan et al. [?] derived formulas for seepage velocity, permeability, and porosity in dual-fractal-dimension porous media, plotting type curves for reservoir formation analysis. Wang Ziming et al. [?] combined carbonate reservoir fracture fractal characteristics with permeability data to establish permeability variograms for such reservoirs, proving consistency with actual conditions. Liu Bo et al. [?] obtained fractal dimensions of low-permeability reservoir rocks from mercury injection capillary pressure data, comparing them with other heterogeneity parameters and concluding that higher average fractal dimensions correspond to stronger heterogeneity, lower oilfield productivity, and higher water cut. Yu Benfu et al. [?] established fractal seepage models for reservoir pressure distribution, deriving prediction formulas for dynamic reservoir pressure and demonstrating fractal dimension effects on reservoir pressure.

In summary, researchers studying fluid transport in reservoir porous media using fractal theory have primarily established fractal reservoir seepage models to approximate geological pore structures, achieving certain scientific results. However, actual pore structures exhibit non-uniform variations in internal properties and spatial distribution, leading to structural anisotropy in different directions. For instance, some large pores exist at millimeter or centimeter scales, while micro-pores are only nanometer or micrometer scale, spanning several scale levels from molecular to microscopic or microscopic to macroscopic scales. Although reservoir pores exhibit fractal characteristics across different scale ranges, their impact on transport properties varies significantly. Large scales reflect overall reservoir properties and patterns, while small scales represent local characteristics. This creates fractal scale effect problems when simply combining analyses, as patterns summarized at one scale level may not apply at another, causing substantial deviations from actual values. Therefore, this paper employs fractal description methods based on dimensional unity principles to derive transport process model equations for reservoir porous media and conduct practical analysis.

1. General Fractal Description Methods for Seepage

Transport characteristics in porous media are described by a series of characteristic parameters, with porosity and permeability being the two most important physical parameters. Research on these quantities is therefore a primary focus. Permeability, porosity, and their relationships depend on microscopic pore structures, and studies have shown that naturally formed porous media and some artificially manufactured materials exhibit fractal characteristics in their microstructures. Various theoretical models have been proposed using fractal geometry theory to describe porous media microstructures and establish relationships between transport problems and fractal dimensions.

Permeability exhibits functional relationships with porosity ϕ, fractal dimension Df, and other parameters. Based on fluid flow states, permeability expressions can be derived using capillary bundle models [**?**]:

$$K = \frac{\pi D_f \lambda_{max}^{3+D_T}}{128(3 - D_f + D_T)} \cdot \frac{N}{A} \cdot \frac{\lambda_{max}^{1-D_T} - \lambda_{min}^{1-D_T}}{\lambda_{max}^{3-D_T} - \lambda_{min}^{3-D_T}}$$

where N represents the total number of tortuous capillaries or pores in the unit cell, τ denotes the average tortuosity of the porous medium (the ratio of actual capillary/pore length to macroscopic length), and δ represents the ratio of minimum to maximum pore radius. In this equation, certain coefficients in the calculation of N lack clear physical meaning, and the derivation is imprecise due to ambiguous definitions of average tortuosity τ.

The second model is the porous medium seepage mechanics model, which derives permeability expressions from a fluid mechanics perspective using the logarithmic linear relationship between Fanning friction coefficient f and Reynolds number Re under certain conditions [**?**]:

$$K = \frac{c\phi\lambda_{max}^2}{\tau^2}$$

where c is a constant obtainable from friction coefficient f and Reynolds number Re. However, this formula requires restrictive conditions that limit seepage velocity in porous media, and the proposed correlations have not been validated against experimental results.

Yu Boming et al. [**?**] established a fractal analytical solution model combining the Hagen-Poiseuille equation to derive the total flow relationship in porous media:

$$Q = \frac{\pi \Delta P}{128\mu L_0^{D_T}} \cdot \frac{D_f}{3 + D_T - D_f} \cdot \lambda_{max}^{3+D_T}$$

where ΔP is the pressure difference between inlet and outlet,   is fluid viscosity, L_0 is the apparent length of tortuous capillaries or pores, Df is the porous medium fractal dimension, and DT is the tortuosity fractal dimension.

Combining the flow formula with Darcy’ s law yields the permeability function:

$$K = \frac{\pi}{128\mu} \cdot \frac{D_f}{3 + D_T - D_f} \cdot \frac{\lambda_{max}^{3+D_T}}{AL^{D_T-1}\Delta P}$$

where A is the cross-sectional area of a unit cell. Equation (5) contains no empirical constants, and each parameter has clear physical meaning. However, its derivation fails to address dimensional unity requirements. Without understanding pore variation patterns and scale dependencies, parameters are arbitrarily substituted without theoretical basis, creating dimensional inconsistencies. Since permeability, flow rate, and other characteristic parameters describe macroscopic porous media while fractal dimensions characterize microscopic pore structures, a medium is needed to unify the dimensions between microscopic descriptions and macroscopic calculations when determining properties at one scale from another. This represents the key focus of our research: modifying existing formulas by incorporating dimensional unity to derive unit scale conversion coefficients that reduce errors and approach true values.

2. Fractal Seepage Description Based on Dimensional Unity Principle

The aforementioned description methods have certain defects. First, the single-capillary flow expression involves parameters from two different scale levels: capillary length L (macroscopic scale) and capillary diameter λ (microscopic scale), with macroscopic and microscopic scales exerting substantially different influences on pore characteristics. Second, the derivation process fails to define the dimensional scale for each characteristic parameter, resulting in inconsistent dimensions and units on the right side of the equation when reflecting flow distribution patterns. According to this principle, the fundamental seepage formula is incomplete and imperfect. Therefore, when combining these terms in calculations, dimensional unity must be achieved through a scale conversion coefficient.

Starting from the initial equation containing length L and diameter λ, we unify them to macroscopic scale to obtain the relationship:

$$L = L_0 \left(\frac{\lambda}{\lambda_{max}} \right)^{1-D_T}$$

where k represents the order of magnitude required for consistency between arbitrary different scale levels, determined by the relationship between artificially defined scales and internationally unified dimensions.

Substituting the converted capillary length equation into the flow equation yields the flow rate for the unit cell cross-section:

$$q(\lambda) = \frac{\pi\lambda^4}{128\mu} \cdot \frac{\Delta P}{L_0} \left(\frac{\lambda}{\lambda_{max}}\right)^{D_T-1}$$

The total flow rate becomes:

$$Q = \int_{\lambda_{min}}^{\lambda_{max}} q(\lambda) dN = \frac{\pi \Delta P k^{D_T-1}}{128\mu L_0^{D_T}} \cdot \frac{D_f}{3 + D_T - D_f} \cdot \lambda_{max}^{3+D_T}$$

Using Darcy' s law, the permeability formula is:

$$K = \frac{Q\mu L_0}{A\Delta P} = \frac{\pi k^{D_T-1}}{128} \cdot \frac{D_f}{3 + D_T - D_f} \cdot \frac{\lambda_{max}^{3+D_T}}{AL_0^{D_T-1}}$$

where k^{D_T-1} is called the flow scale conversion coefficient.

From Darcy' s law, we know that related physical quantities in the formula are macroscopic, thus having no scale level issues. Substituting the flow rate Q yields the new permeability formula:

$$K' = \frac{Q'\mu L_0}{A\Delta P} = \frac{\pi k^{D_T+2}}{128} \cdot \frac{D_f}{3 + D_T - D_f} \cdot \frac{\lambda_{max}^{3+D_T}}{AL_0^{D_T-1}}$$

From this, we obtain the conversion coefficient for the fractal permeability formula:

$$K' = k^{D_T+2} K$$

The significance of this modified formula lies in prioritizing calculations of values under certain dimensional scales when describing macroscopic physical characteristics, then combining them with scale conversion coefficients to achieve dimensional unity on both sides of the equation.

3. Verification and Analysis of the Modified Fractal Seepage Formula

Based on the established flow and permeability model equations for reservoir porous media, we numerically calculated the macroscopic transport characteristic parameters (flow rate and permeability) for four core samples to analyze the influence of microscopic pore characteristics on macroscopic transport parameters.

3.1 Experimental Data and Basic Parameters This study employs core data from four different oilfields for numerical calculations, comparing modified equation results with original formulas and measured values. The samples include three sandstones and one carbonate, with original three-dimensional data acquired through micro-CT scanning imaging systems that provide intuitive descriptions of core characteristics. [Figure 1: see original paper] shows the three-dimensional image of sandstone B1 digital core, where transparent sections represent pores and colored sections represent solid skeletons (X, Y, Z indicate different directions). [Figure 2: see original paper] presents a two-dimensional cross-section of B1 in the Z direction, where white indicates pores and black indicates solids.

Basic physical parameters were calculated to enable more precise reservoir description and analysis. Experimental data and fundamental parameters are presented in .

3.2 Fractal Dimensions Using core sample B1 as an example to validate the modified formula, scanning equipment provided the following data: minimum pore diameter $\lambda\{min\} = 345.5$ *m, maximum pore diameter* $\lambda\{\max\} = 608.125$ m, porosity $\phi = 0.196$, sample macroscopic length L = 20 mm, pressure difference ΔP = 1 Pa, dynamic viscosity = 1 Pa·s, and permeability K = 1286 mD. Based on microscopic parameters, the pore distribution fractal dimension D_f = 2.484 and tortuosity fractal dimension D_T = 1.198 were calculated. The remaining three core groups were processed similarly.

3.3 Comparative Calculation Analysis Before and After Modification Based on existing formulas and the modified model, comparative calculations of seepage flow and permeability were performed, with results detailed in and . The numerical calculations yield the following conclusions:

1. Comparing flow and permeability values at micrometer and millimeter scales reveals that parameter selection at different scale levels produces varying degrees of error, demonstrating the scale effect problem. This occurs because reservoir porous media heterogeneity creates pore structures of different scales at different spatial positions.
2. For sample in the tables, millimeter-scale values are closer to measured values than micrometer-scale values, yet both deviate significantly from measurements. Analysis suggests that either the extracted core sample contains predominantly large pores in this region (making it more suitable for larger-scale calculations) or that fractal characteristics are not prominent within this scale range, resulting in substantial errors. This indicates that while microscopic pore structures are being analyzed, smaller scales do not necessarily yield more accurate results, and single-scale analysis cannot reflect overall reservoir characteristics.
3. Comparative analysis of permeability and flow values before and after

modification shows that modified results have smaller errors and are closer to actual values, validating the modified formula. The correction applies the principle of dimensional unity by combining microscopic-scale values with their corresponding dimensional conversion coefficients to obtain final macroscopic results.

3.4 Parameter Variation Characteristic Analysis Sample was selected for parameter variation analysis. By varying sample dimensions, minimum and maximum pore diameters, and porosity during calculations, different permeability and flow values were obtained to analyze how microscopic pore structure parameters affect macroscopic transport characteristics. Results are shown in [Figure 3: see original paper] through [Figure 6: see original paper].

[Figure 3: see original paper] demonstrates that under constant pressure, internal flow and permeability increase with porosity. As core porosity ϕ increases, microscopic pore fractal dimension D_f increases, enhancing pore structure heterogeneity and creating uneven pore distributions that affect seepage performance in different directions. Increased porosity reduces average pore tortuosity and tortuosity fractal dimension D_T, causing pore channels to gradually straighten and facilitating fluid flow, thus increasing permeability and flow rate.

[Figure 4: see original paper] shows that under constant porosity, permeability and flow rate decrease as minimum pore diameter increases. When minimum pore diameter λ_{min} gradually increases, microscopic pore fractal dimension D_f decreases, making pore distributions more uniform across different scales. However, tortuosity fractal dimension D_T increases, indicating greater flow path tortuosity that impedes fluid passage, thereby reducing flow and permeability.

[Figure 5: see original paper] indicates that increasing maximum pore diameter gradually increases core permeability and flow rate. With constant porosity ϕ and unchanged minimum pore diameter λ*{min}, increasing maximum pore diameter* λ{max} enlarges pore fractal dimension D_f, intensifying core heterogeneity and increasing the order-of-magnitude difference between maximum and minimum pores. Micro-pores become negligible and have minimal impact on overall flow and permeability.

[Figure 6: see original paper] reveals that flow and permeability gradually decrease as reservoir core sample size increases. Although increasing core length does not change fractal dimension D_f (as core length does not affect internal structure), D_f cannot be used to evaluate its influence on macroscopic transport parameters. While larger sizes reduce tortuosity fractal dimension D_T, flow and permeability still decrease due to increased distance under constant pressure difference.

4. Conclusions

Porous media structures are extremely complex and difficult to describe precisely. Researchers have introduced fractal theory for description and analysis, but scale differences in pore spatial distribution significantly affect macroscopic transport characteristics and should not be ignored. This paper modifies and improves fractal transport models for porous media considering dimensional unity principles and scale effects, yielding the following conclusions:

1. Addressing the unclear units and scales in existing porous media fractal seepage models, this paper re-derives relevant fractal transport analysis models and validates them in reservoir porous media.
2. Combining microscopic pore scales with macroscopic calculation analysis yields relevant transport characteristic parameters for analyzing scale effects between different levels. Practical analysis demonstrates that smaller scales do not necessarily produce more accurate results for reservoirs.
3. Based on the proposed fractal transport model, oilfield reservoir data were analyzed to examine how microscopic pore characteristic parameters affect macroscopic transport parameters. Results show that flow and permeability increase with porosity and maximum pore diameter, while decreasing with minimum pore diameter and sample size. The simulation results align with actual reservoir transport characteristics.

References

[1] Yu Boming, Xu Peng, Zou Mingqing, et al. Transport Physics of Fractal Porous Media[M]. Beijing: Science Press, 2014: 1-5

[2] Yin Chailing. The Study of Reservoir Porous Media Transport Properties Based on Fractal Theory[D]. Daqing: Northeast Petroleum University, 2015: 3-6

[3] Nutting P G. Some Physical Problems in Oil Recovery[J]. Oil and Gas Journal, 1929, 28(27): 44-45

[4] Mandelbrot B B. How Long Is the Coast of Britain: Statistical Self-Similarity and Fractional Dimension[J]. Science, 1967, 156(3775): 636-638

[5] Adler P M, et al. Real Porous Media: Local Geometry and Macroscopic Properties[R]. Applied Mechanics Reviews, 1998, 51(9): 537-585

[6] Thovert J F, Wary F, et al. Thermal Conductivity of Random Media and Regular Fractals[J]. Journal of Applied Physics, 1990, 68(8): 3872-3883

[7] Pfeifer P, Avnir D. Chemistry in Non-Integral Dimensions Between Two and Three[J]. The Journal of Chemical Physics, 1983, 7(7): 3369-3558

[8] Katz, A.J and Thompson, A.H. Fractal Sandstone Pores: Implications for Conductivity and Pore Formation[J]. Physical Review Letters, 1985, 54: 1325-1328

[9] Hansen, J.P. and Skjeltorp, A.T. Fractal Pore Space and Rock Permeability Implications[J]. Physical Review B, 1988, 38(4): 2635-2638

[10] Kim, Tae H, Schechter, David S. Estimation of Fracture Porosity of Naturally Fractured Reservoirs with No Matrix Porosity Using Fractal Discrete Fracture Networks[J]. Reservoir Evaluation and Engineering, 2009, 12(2): 232-242

[11] XIE Shuyun, CHENG Qiuming, et al. Fractal and multifractal analysis of carbonate pore-scale digital images of petroleum reservoirs[J]. Marine and Petroleum Geology, 2010, 27(2): 476-485

[12] Tong Dengke, Ge Jiali. An Exact Solution For Unsteady Seepage Flow Through Fractal Reservoir[J]. Acta Mechanica Sinica, 1998, 30(5): 621-627

[13] Liu Junliang, Tian Changan, et al. Fractal Dimensionality Dependence of Microstructural Parameters and Permeability in Fractal Porous Media[J]. Advances in Water Science, 2006, 17(6): 812-817

[14] Kong Xiangyan, Li Daolun, et al. Basic Formulas of Fractal Seepage and Type-Curves of Fractal Reservoirs[J]. Journal of Xi' an Petroleum University (Natural Science), 2007, 22(2): 1-5

[15] Wang Ziming, Song Wenjie, et al. Application of Fractal Simulation Technique to Modeling of Fractured Carbonate Reservoir[J]. Natural Gas Industry, 2007, 27(11): 73-75

[16] Liu Bo, Fan Xiaodong, et al. Fractal Characterization of Heterogeneity in Low Permeability Rock and Its Effect on Production Performance[J]. Journal of Liaoning Technical University (Natural Science), 2009, 28(1): 150-152

[17] Yu Benfu, Yan Xiangzhen et al. Dynamic Reservoir Pressure Prediction of Depleted Reservoir Gas Storage Considering the Fractal Feature of Porous Medium[J]. Petroleum Science, 2013, 34(5): 1017-1022

Note: Figure translations are in progress. See original paper for figures.