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Nonlinear Dynamics of Thermoacoustic Instability in Rijke Tubes: Postprint of Bifurcation Analysis

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Abstract

A horizontal Rijke tube thermoacoustic model was established, and the Galerkin method was employed to expand the governing equations, enabling numerical solution. The Galerkin mode convergence order was determined to be 10, and nonlinear dynamics theory was utilized to conduct bifurcation analysis of the system. The bifurcation behavior of the Rijke tube thermoacoustic system belongs to subcritical Hopf bifurcation. The system stability regions are divided into globally stable, globally unstable, and bistable regions. Bifurcation diagrams were obtained for parameters including dimensionless heating power K, heater location xf, damping coefficient c1, and time delay τ, revealing that the bifurcation diagram for heater location xf exhibits two Hopf bifurcation points. Within the linearly unstable region, the oscillation amplitude shows a trend of first increasing and then decreasing with the increase of time delay τ.

Full Text

Preamble

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Study on Nonlinear Dynamics of Thermoacoustic Instability in Rijke Tube–Bifurcation Analysis

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Abstract: A thermoacoustic model of a horizontal Rijke tube is established, and the governing equations are expanded and solved numerically using the Galerkin method. The convergence order of Galerkin modes is determined to be

10, and bifurcation analysis of the system is performed using nonlinear dynamics theory. The bifurcation behavior of the Rijke tube thermoacoustic system is found to be subcritical Hopf bifurcation. The system stability regions are divided into globally stable, globally unstable, and bistable regions. Bifurcation diagrams are obtained for parameters including non-dimensional heater power (K), heater position (xf), damping coefficient (c1), and time delay (τ). The bifurcation diagram for heater position (xf) exhibits two Hopf bifurcation points. Within the linearly unstable region, the oscillation amplitude shows a trend of first increasing and then decreasing with increasing time delay τ.

Keywords: thermoacoustic instability; Rijke tube; Galerkin method; bifurcation analysis; nonlinear dynamics

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Author Biography: FENG Jian-Chang (1991-), male, master' s student, mainly engaged in combustion instability research.

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0 Introduction

Combustion instability refers to large-amplitude self-excited oscillations of pressure and velocity in combustors. These oscillations create cyclic loads that cause excessive structural vibration and heat transfer through combustor walls, leading to catastrophic damage to structural components. For a long time, combustion instability has been a major challenge for many high-performance combustion devices, including rocket engines, gas turbines, aero-engines, and power station boilers. Thermoacoustic instability is the most important form of combustion instability, generally believed to be caused by the coupling between unsteady heat release and acoustic field oscillations, which induces large-amplitude pressure oscillations near the natural acoustic frequencies. The essence of this phenomenon is a positive feedback mechanism between heat source release and the acoustic environment.

The mechanisms underlying combustion instability are extremely complex, involving the coupling of combustion, flow, and acoustic fields. Flame surface variations, equivalence ratio fluctuations, vortex shedding caused by hydrodynamic instabilities, and oscillations in fuel atomization and evaporation processes can all drive combustion instability. Meanwhile, combustor geometry, fuel/air mixing degree, fuel type and composition, and operating conditions (injection air temperature, combustor pressure, equivalence ratio, swirl intensity, lean premixing, etc.) all influence combustion instability.

The nonlinear dynamic characteristics of flow and flame are among the most important phenomena in combustion instability. Flow and flame characteristics in combustors can change dramatically as control parameters vary, especially when parameters cross bifurcation points. The combustion process itself may or may not exhibit bifurcation phenomena, but once bifurcation occurs, nonlinear behavior dominates in many combustion devices. Small perturbations in system parameters may cause bifurcation, transitioning the system from a stable state (small or no oscillations) to an unstable state (large-amplitude limit cycles). Conversely, at the same critical parameter values, the transition from unstable to stable states may not occur due to hysteresis phenomena.

Bifurcation, hysteresis, limit cycles, and triggering phenomena observed in combustion instability can all be explained and described using dynamical systems theory, which represents an emerging research hotspot. Nonlinear dynamics theory enables deep characterization, analysis, and diagnosis of nonlinear system features. Most existing research focuses on thermoacoustic instability problems in gas turbines and rocket engines, with study objects typically being various simplified combustors or actual gas turbines and engines.

The Rijke tube is the most convenient and typical experimental system for studying thermoacoustic phenomena and serves as a fundamental platform for investigating the interaction between combustion and acoustic fields in flame zones. Many studies on Rijke tubes have been conducted to understand instability mechanisms and control fundamentals. Unsteady heat release acts as the driving force for thermoacoustic instability and represents a critical link in its occurrence. Energy input in thermoacoustic systems can take various forms, such as chemical reactions with heat release in combustors, heating wires, or hot gauze. Jahnke and Culick first introduced modern dynamical systems theory to combustion instability research to analyze nonlinear combustion instability. They proposed a continuation algorithm for systematic and efficient calculation of steady-state and limit cycle solutions, enabling stability analysis under steady-state and limit cycle conditions. Bifurcation analysis can determine instability onset points, where acoustic wave properties depend on bifurcation type and limit cycle characteristics at the bifurcation point.

Ananthkrishnan et al. established reduced-order models for unstable motions in combustors and analyzed triggering and limit cycle properties. Their analysis showed that for first-order axial oscillation modes, at least fourth-order Galerkin series expansion is required for instability, while second-order instability requires at least eighth-order expansion to ensure computational accuracy. Balasubramanian investigated the role and influence of non-normality in simple thermoacoustic systems by deriving acoustic field governing equations for horizontal Rijke tubes, focusing on results from non-normality and nonlinear effects. Subramanian analyzed the nature of subcritical bifurcation in thermoacoustic systems and the resulting bistability issues. Waugh et al. studied triggering in thermoacoustic systems and drew analogies to bypass transition to turbulence in fluid mechanics—both mechanisms involve small perturbations causing large-amplitude self-sustained oscillations despite linear stability. Their research demonstrated that certain types of noise are more effective at triggering oscillations.

Existing research has provided some understanding of nonlinear dynamics in Rijke tube thermoacoustic instability, but recognition of system bifurcation characteristics remains unclear. Understanding is lacking regarding how system parameters such as non-dimensional heater power, heater position, and damping coefficient affect bifurcation. This paper establishes a simplified model of Rijke tube thermoacoustic instability, derives the one-dimensional acoustic field governing equations, solves them, and performs bifurcation analysis on Rijke tube thermoacoustic instability to deepen understanding of its generation mechanisms and oscillation phenomena.

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1 Thermoacoustic Instability Model and Solution

1.1 Rijke Tube Model

The horizontal Rijke tube model studied in this paper is shown in [Figure 1: see original paper]. In this model, the tube length L is fixed, while the heater position xf can be varied by changing the heater' s location within the duct. The heat source is an electric heating wire device, whose heating intensity can be adjusted by changing the electric current under experimental conditions.

The heat source model adopts the Heckl empirical formula:

$$\dot{Q}(t) = \frac{\pi \lambda_w L_w (T_w - \bar{T})}{S \rho c_v} \left[1 + \frac{d_w}{2u_0} \frac{d\tilde{u}}{dt}(t-\tau) \right]$$

where L_w and d_w represent the length and diameter of the heating wire, T_w is the wire temperature, $\bar{T}$ is the ambient air mean temperature, S is the duct cross-sectional area, λ is the air thermal conductivity, c_v is the specific heat at constant volume per unit mass of air, $\bar{\rho}$ is the mean gas density, and τ is the time delay due to thermal inertia between heat transfer and flow velocity. Note that this heat source model only applies to "compact" heat sources where the heat source thickness is negligible relative to the wavelength.

1.2 Governing Equations

Ignoring mean flow and temperature gradient effects, the momentum and energy equations controlling the acoustic field in the duct are:

$$\bar{\rho} \frac{\partial \tilde{u}}{\partial t} + \frac{\partial \tilde{p}}{\partial x} = 0 \tag{1}$$

$$\frac{\partial \tilde{p}}{\partial t} + \gamma \bar{p} \frac{\partial \tilde{u}}{\partial x} + \zeta \tilde{p} = (\gamma - 1) \dot{Q}(t) \delta(x - x_f) \tag{2}$$

In equations (1)-(5), x is the axial distance, t is time, $\bar{p}$ is the mean pressure in the duct, $\tilde{p}$ is the acoustic pressure, ρ is the fluid density, γ is the specific heat ratio of the medium, a is the speed of sound, and M is the mean flow Mach number. Additionally, ζ is the damping coefficient, $\dot{Q}$ is the fluctuation in heat release rate per unit area generated by the electric heater, δ is the standard Dirac distribution, and x_f is the heat source location. Symbols '~' and '-' denote dimensional and mean quantities, respectively.

Damping can be expressed as:

$$\zeta = c_1 + c_2 \frac{\omega_j^2 - \omega_1^2}{\omega_j^2}$$

where parameters c_1 and c_2 remain constant, and ω_j is the non-dimensional frequency of the j-th acoustic mode.

Substituting equation (7) into the energy equation (4), the energy equation can be rewritten as:

$$\frac{\partial \tilde{p}}{\partial t} + \gamma \bar{p} \frac{\partial \tilde{u}}{\partial x} + \zeta \tilde{p} = -\frac{\pi \lambda_w L_w (T_w - \bar{T})}{S \rho c_v} \left[1 + \frac{d_w}{2u_0} \frac{d\tilde{u}}{dt}(t - \tau) \right] \delta(x - x_f)$$

For simplification and convenient analysis, equations (1) and (2) are non-dimensionalized to obtain the dimensionless system governing equations:

$$\frac{\partial u'}{\partial t} + \frac{\partial p'}{\partial x} = 0 \tag{3}$$

$$\frac{\partial p'}{\partial t} + \gamma \frac{\partial u'}{\partial x} + \zeta p' = -\frac{K}{\gamma M_a} \left[1 + \frac{d_w}{2u_0} \frac{du'}{dt}(t - \tau) \right] \delta(x - x_f) \tag{4}$$

where the non-dimensional scales are:

$$x = \frac{\tilde{x}}{L}, \quad t = \frac{\tilde{t}a}{L}, \quad u' = \frac{\tilde{u}}{u_0}, \quad p' = \frac{\tilde{p}}{\gamma \bar{p} M_a}, \quad \rho = \frac{\tilde{\rho}}{\bar{\rho}}$$

and the non-dimensional heater power is:

$$K = \frac{4(1-\gamma)}{\gamma \pi} \frac{\lambda_w L_w (T_w - \bar{T})}{\rho_0 c_p u_0 d_w M_a S}$$

1.3 Galerkin Expansion

The conventional method for solving thermoacoustic problems in Rijke tube devices uses Galerkin projection and modal expansion. The principle of the Galerkin method is that any function in a domain can be expressed as a superposition of expansion functions in that domain. The basis functions must be selected to satisfy boundary conditions, and this selection is not unique. In this approach, pressure and velocity signals varying in space and time are expanded in terms of spatial basis functions that satisfy boundary conditions. This paper selects basis functions that are arbitrary, not the system' s eigenfunctions, but rather the eigenfunctions of the self-adjoint part of the linearized system. Therefore, the velocity and pressure fields can be expressed in the form of duct natural modes:

$$u'(x,t) = \sum_{j=1}^{N} \eta_j(t)\cos(j\pi x) \tag{9}$$

$$p'(x,t) = \gamma M_a \sum_{j=1}^{N} \mu_j(t)\sin(j\pi x) \tag{10}$$

where $k_j = j\pi$ is the non-dimensional wavenumber of the j-th acoustic mode in the duct. Under the condition $N \to \infty$, these basis functions form a complete basis set. However, considering computational feasibility, only a finite number of modes can be used. The number of Galerkin modes required to accurately capture the system' s linear and nonlinear behavior is called modal convergence.

Substituting equations (9) and (10) into equations (3) and (8) and projecting onto the basis function set yields:

$$\dot{\eta}_j + j\pi\mu_j = 0 \tag{11}$$

$$\dot{\mu}_j - \frac{j\pi}{\gamma}\eta_j + \zeta_j\mu_j = \frac{K}{\gamma M_a}\sin(j\pi x_f)\left[1 + \frac{d_w}{2u_0}\sum_{i=1}^{N}\dot{\eta}_i(t-\tau)\cos(i\pi x_f)\right]$$

The fully coupled Galerkin model can be simplified through linearized time delay, which is only applicable under the Galerkin modal assumption where $T_j = 2\pi/\omega_j$ is the time period of the j-th Galerkin mode. Under this assumption, the delay term can be written as:

$$\dot{\eta}_i(t-\tau) \approx \dot{\eta}_i(t) - \tau\ddot{\eta}_i(t)$$

Through linearized time delay, the final Galerkin-expanded ordinary differential equation system is obtained:

$$\dot{\eta}_j + j\pi\mu_j = 0 \tag{15}$$

$$\dot{\mu}_j - \frac{j\pi}{\gamma}\eta_j + \zeta_j\mu_j = \frac{K}{\gamma M_a}\sin(j\pi x_f)\left[1 + \frac{d_w}{2u_0}\sum_{i=1}^{N}(\dot{\eta}_i - \tau\ddot{\eta}_i)\cos(i\pi x_f)\right]$$

for $j = 1, 2, ..., N$.

1.4 Galerkin Mode Convergence

Numerical solution of equations (15) and (16) was implemented using Matlab programming, and system evolution under different numbers of Galerkin modes was compared. The comparison of time evolution for the system with different numbers of acoustic modes in the linearly unstable region is shown in [Figure 2: see original paper]. It can be observed that the limit cycle amplitude and phase under first-order Galerkin acoustic mode differ significantly from those under tenth-order Galerkin acoustic mode, but as the number of Galerkin modes increases, these differences gradually diminish. According to [Figure 2: see original paper], the difference between numerical solutions obtained with 9th-order and 10th-order acoustic modes is almost negligible, again demonstrating the convergence of 10th-order acoustic mode Galerkin projection. For 10th-order acoustic modes, increasing one Galerkin mode changes the acoustic velocity limit cycle amplitude by less than 1.4%. Therefore, in all subsequent calculations in this paper, we adopt the 10th-order Galerkin mode model for both linear and nonlinear stability analyses to ensure convergence.

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2 Linear Stability Analysis

To investigate the effect of infinitesimal perturbations on system stability, linear stability analysis is performed first. If the system gradually moves away from the stable state, it is ultimately unstable; if it gradually approaches the stable state, it ultimately reaches stability. This refers to local analysis of stability changes near equilibrium states. Nonlinear stability analysis, on the other hand, studies the effect of finite-amplitude perturbations and characterizes the resulting asymptotic system states. The critical point where system equilibrium loses stability and behavior changes is called a bifurcation point. Once a bifurcation point is identified based on one parameter, the bifurcation point itself continues to vary with another system parameter. Bifurcation point branches provide linear stability boundaries that separate linearly stable and unstable regions in parameter space. This stability boundary is a hypersurface in the space of all varying free parameters, but it is convenient to represent appropriate two-dimensional projection curves.

In the Rijke tube, system parameters include non-dimensional heater power, heater position, damping coefficient, and time delay. However, under actual experimental conditions, only non-dimensional heater power and heater position can be precisely varied, while time delay and damping coefficient change based on other conditions. We analyze the linear stability and instability regions with respect to heater position and time delay variation. A typical linear stability boundary between heater position and time delay is shown in [Figure 3: see original paper], illustrating whether the system is linearly unstable for a chosen range of heater positions (xf) depending on time delay (τ), and vice versa, for fixed system parameter values of damping and heater power. For small or reasonably large τ, such as $\tau < 0.15$, the Rijke tube system is linearly stable for any heater position, meaning the system eventually approaches a stable state for small perturbations. Only in the range $0.15 < \tau < 0.85$ does the stability of the system equilibrium solution depend on heater position.

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3 Bifurcation Analysis

Bifurcation analysis studies the characteristics and generation mechanisms of bifurcation phenomena. Bifurcation refers to sudden changes in global behavior (qualitative properties, topological properties, etc.) of certain fully determined nonlinear systems when a system parameter varies continuously to a critical value c. c is called the bifurcation value or branching value of parameter . Bifurcation where a stable state loses stability and an isolated limit cycle periodic solution appears is called Hopf bifurcation. The stability of the emerging limit cycle branch determines the nature or type of Hopf bifurcation. These two Hopf bifurcation types are illustrated in [Figure 4: see original paper]. If the limit cycle is unstable, as shown in [FIGURE:4(a)], the unstable limit cycle branch forms a bistable region where the steady-state solution is stable to small perturbations but unstable to large perturbations. However, this unstable limit cycle branch may undergo a folding bifurcation and stabilize. For parameter values below the folding point, the stable solution is stable to perturbations of any amplitude, making the system globally stable. Therefore, in subcritical bifurcation cases, linear (local) and nonlinear (global) stability boundaries differ. This bifurcation behavior is called subcritical Hopf bifurcation. If the limit cycle is stable, as shown in [FIGURE:4(b), the system transitions smoothly from a stable steady-state solution to an unstable steady-state solution with increasing limit cycle amplitude. This bifurcation type is called supercritical Hopf bifurcation.

Subcritical and supercritical Hopf bifurcations can be determined through qualitative analysis methods. The nature of nonlinear bifurcation with source terms in the form of (1 + X)^α can be judged by expanding the nonlinear term into a series and discarding higher-order terms. The binomial expansion of this expression yields:

$$(1+X)^{\alpha} \approx 1 \pm \alpha X + \frac{\alpha(\alpha-1)}{2!}X^2 \pm \frac{\alpha(\alpha-1)(\alpha-2)}{3!}X^3 + \cdots$$

In this expression, when $0 < \alpha < 1$, the signs of the first-order term and third-order term are the same, while when $\alpha > 1$, the signs of the first-order and third-order terms differ. The signs of the first-order and third-order terms determine the nature of the bifurcation point. Whenever these two terms have the same sign, the bifurcation is subcritical; when they have different signs, the bifurcation is supercritical. For heat release rate fluctuations in the Rijke tube model, $\alpha = 3/2$, meaning this model will exhibit subcritical Hopf bifurcation.

Variable system parameters in the Rijke tube thermoacoustic system include non-dimensional heater power K, heater position xf, damping coefficient c1, and time delay τ. Bifurcation analysis is performed for variations of these parameters.

3.1 Effect of Heater Power

In the Rijke tube thermoacoustic system, the heater is an electric heating wire. The effect of heater power on the system is investigated by varying the K value. Non-dimensional heater power can be increased by providing more power to the heater, representing increased system driving force that makes the system more unstable. Therefore, for small K values, the system equilibrium is stable and decays asymptotically close to zero under all initial perturbations. Increasing K reduces flow stability margin, and near the bifurcation point, the system evolves to linear instability, leading to oscillatory flow patterns in the duct. [Figure 5: see original paper] shows the calculated effect of non-dimensional heater power (K) variation on system evolution. Hollow circles indicate unstable solutions, while solid circles represent stable solutions. The figure shows that small-amplitude limit cycles near the Hopf bifurcation point are unstable and coexist with stable equilibrium solutions. These unstable limit cycle branches further undergo a folding point bifurcation to gain stability, confirming that the bifurcation belongs to the subcritical type, consistent with previous conclusions.

Under fixed other system parameters, the system' s time evolution behavior depends on K values. When $\mathrm{K} < 0.515$, the system eventually approaches zero for perturbations of any amplitude; this region is globally stable. As K increases continuously, when $0.515 < \mathrm{K} < 0.615$, the system has three possible states: stable state, unstable limit cycle, and stable limit cycle. Depending on initial conditions, the system may eventually enter either a stable state or periodic oscillation; this region is called the bistable region. When $\mathrm{K} > 0.615$, the system enters the globally unstable region, where for any small initial perturbation, the system ultimately reaches a limit cycle state. The bifurcation diagram with respect to non-dimensional heater power also demonstrates that increasing non-dimensional heater power is the main cause of thermoacoustic instability in the system.

3.2 Effect of Damping Coefficient

To study the effect of damping variation on system response, one damping parameter (c1) in the damping model can be varied. Under actual experimental conditions, system damping magnitude is changed by altering the duct end conditions. According to nonlinear theory, increasing damping can enhance system stability. [Figure 6: see original paper] shows the bifurcation diagram of system behavior evolution with damping coefficient (c1) variation. As expected, increasing damping indeed enhances system stability because the equilibrium solution is stable for any time delay τ when the damping coefficient is large. Reducing damping causes the system to lose stability. For the illustrated time delay value, non-dimensional heater power, and heater position parameters, there exists a critical value c1 = 0.19. When below this value, the system eventually develops into a limit cycle with stable amplitude for any initial perturbation amplitude; this critical value is the subcritical Hopf bifurcation point. When c1 $<$ 0.19, the system eventually develops into a limit cycle with stable amplitude for any initial perturbation amplitude; when 0.19 $<$ c1 $<$ 0.25, this region is the bistable region, where the system develops into a stable limit cycle for relatively large initial perturbations but eventually approaches equilibrium for small-amplitude perturbations. As the damping coefficient c1 increases further, when c1 $>$ 0.25, the system eventually approaches a stable state for any initial perturbation amplitude. The point c1 = 0.25 is the folding point.

3.3 Effect of Heater Position

Heater position (xf) also significantly affects system dynamics, changed by placing the heater at different locations along the duct length. System stability changes depend on heater position variation in a particular manner. The bifurcation diagram for heater position (xf) variation is shown in [Figure 7: see original paper]. As the heater position changes from the upstream open end, the system is initially linearly stable. At the critical value xf1 of heater position, the system evolves to linear instability. With further changes in heater position, the system remains linearly unstable until the heater position reaches xf2, where another Hopf bifurcation point forms and the system becomes stable again. The bifurcation diagram for heater position (xf) shows that both Hopf bifurcation points at the duct ends are subcritical Hopf bifurcations. Between the two Hopf bifurcation points, there exists a globally unstable region where the system asymptotically reaches a corresponding stable-amplitude limit cycle for any initial condition. From the bifurcation diagram for heater position variation, we can obtain specific values for the two subcritical Hopf bifurcation points. For our selected other system parameters, when xf $<$ 0.11, the system is stable for any initial perturbation amplitude; when 0.11 $<$ xf $<$ 0.14, the system is in the bistable region, where different evolution behaviors occur depending on initial conditions, so the first subcritical Hopf bifurcation point is at xf1 = 0.14; when 0.14 $<$ xf $<$ 0.365, the system is in the globally unstable region, where any small initial perturbation eventually develops into a limit cycle; when 0.365 $<$

xf < 0.39, the system enters the bistable region again, so the second subcritical Hopf bifurcation point is at xf2 = 0.365; as the heater position changes further, when xf > 0.39, the system re-enters the globally stable region, eventually approaching stability for any initial perturbation amplitude.

3.4 Effect of Time Delay

In the Rijke tube, due to thermal inertia between the heater and flowing medium, there exists a time delay τ between heat transfer and flow rate changes, whose magnitude is related to the heating wire diameter and incoming flow velocity. Under actual experimental conditions, the time delay term (τ) is difficult to change precisely. The bifurcation diagram corresponding to time delay (τ) variation is shown in [Figure 8: see original paper]. The subcritical Hopf bifurcation point corresponds to the time delay value $\tau = 0.155$. When $\tau < 0.13$, the system is in a linearly stable state, eventually approaching stability for any initial perturbation amplitude; when $0.13 < \tau < 0.155$, the system eventually reaches equilibrium for small-amplitude perturbations but develops into a limit cycle for large-amplitude perturbations—the bistable region; when $\tau > 0.155$, the system is in the linearly unstable region, where any small initial perturbation eventually develops into a limit cycle.

Notably, within the linearly unstable region of $\tau > 0.155$, as time delay (τ) gradually increases, the amplitude value first increases and then decreases. This phenomenon can be explained by the Rayleigh criterion. The existence of time delay is due to thermal inertia between heat release from the source and flow. When heater power increases, time delay increases accordingly. The increase in time delay changes the phase difference between acoustic oscillations and heat release pulsations. When this phase difference gradually increases away from 0°, the oscillation amplitude gradually decreases, forming this phenomenon. Accordingly, it can also be found that when $\tau = 0.4$, the phase difference between acoustic oscillations and heat release pulsations is exactly 0°, so the oscillation amplitude is maximum at this point.

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4 Conclusions

This paper models the Rijke tube thermoacoustic system, numerically solves the governing equations using the Galerkin method, and analyzes relevant dynamic characteristics, leading to the following conclusions:

1. When solving the Rijke tube thermoacoustic instability model, using 10th-order Galerkin modes is sufficient to ensure convergence.
2. The bifurcation behavior of the Rijke tube thermoacoustic system belongs to subcritical Hopf bifurcation. The system has one linear stability boundary and one nonlinear stability boundary, with a bistable region between them.

3. Bifurcation diagrams are obtained for system parameters including non-dimensional heater power K, heater position xf, damping coefficient c1, and time delay τ. The bifurcation diagrams for non-dimensional heater power K, damping coefficient c1, and time delay τ each have only one Hopf bifurcation point, while the bifurcation diagram for heater position xf has two Hopf bifurcation points.

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